Growth or the Gap? Which Measure of Economic Activity Should be Targeted in Interest Rate Rules?*

Eric Sims
University of Notre Dame, NBER, and ifo

July 15, 2013

Abstract

What measure of economic activity, if any, should be targeted in simple interest rate rules? This paper analyzes the welfare consequences of responding to the growth rate of output and the output gap in policy rules in a conventional New Keynesian model. In spite of the fact that it shows up directly in the approximate welfare criterion of the simplest version of the model, responding to the theoretical output gap, even if perfectly observed, never has large benefits and can significantly reduce welfare if the central bank is faced with a non-trivial gap/inflation tradeoff resulting from time-varying price or wage markups. In contrast, responding to the growth rate of output, which is putatively easier to observe than the gap, is often welfare-improving. These conclusions about the relative merits of responding to growth or the gap also obtain in a medium scale version of the model with both price and wage stickiness, capital accumulation, and different real adjustment frictions.

*I am grateful to Ruediger Bachmann, Olivier Coibion, Yuriy Gorodnichenko, Robert Lester, and seminar participants at Notre Dame, the University of Quebec at Montreal, North Carolina State University, and the Texas Monetary Conference for many helpful comments which have substantially improved the paper.
1 Introduction

Interest rate rules, often called “Taylor Rules” after Taylor (1993), have become a ubiquitous feature of mainstream macroeconomic models. These rules call for central banks to adjust interest rates in response to changes in observable macroeconomic conditions. Though not fully optimal in the Ramsey sense, interest rate rules tend to have good normative properties and yield intuitive and well-understood restrictions to guarantee equilibrium determinacy.

In spite of their widespread application in dynamic stochastic general equilibrium (DSGE) models, there is no set or widely agreed upon specification of Taylor-type rules. What is common to most specifications of policy rules is for a strong reaction of interest rates to deviations of inflation from target. There is considerably less agreement on which measure of economic activity, if any, should enter into the policy rule. Taylor’s original specification featured interest rates rising in response to increases in output above a statistical trend. Many papers instead assume that interest rates are set as a function of the output gap, the deviation between the actual level of output and its “natural rate” – the level of output that would obtain in equilibrium in the absence of nominal rigidities. This specification is usually justified on the grounds that it is the output gap that matters for welfare, not output. Still other specifications of policy rules feature a positive response of interest rates to output growth, either in place of, or addition to, the gap. This specification is often justified on the grounds that output growth is putatively easier to observe in real time than the output gap. Some authors argue against paying attention to any measure of economic activity at all, and instead suggest that policy-makers pursue a policy of strict inflation targeting.

The purpose of this paper is to provide some insight into the simple but apparently unsettled question: what measure of economic activity, if any, should appear in interest rate rules? This question is not merely of academic interest for deciding what kind of Taylor rule specification to include in a DSGE model of the economy. It is also of critical importance for thinking about policy in the current zero lower bound environment. Figure 1 plots the actual behavior of the Federal Funds rate over the period 1985-2012 (black line) along with the implied target level of the funds rate for different policy rule specifications: one in which rates react to the output gap (blue line) and one in which rates instead react to output growth (green line).1 The actual funds rate has been at or near zero since the end of 2008. Under the gap specification, the implied target rate has hovered between -1 and -2 percent (at an annualized rate) since that time. Implicitly or explicitly, many who have called for additional monetary stimulus appeal to a picture like this, arguing that nominal rates ought to be negative in the absence of the lower bound, and therefore support non-standard policies like quantitative easing and “forward-guidance.” If the Fed were following the

1To generate the implied target I assume a rule of the form: $i_t = (1 - \rho_i)\bar{i} + \rho_i i_{t-1} + (1 - \rho_i)\phi_\pi (\pi_t - \pi) + \phi_x (x_t - \bar{x}) + \phi_\Delta_x (\log(y_t) - \log(y_{t-1}))$. $\pi_t$ is quarter-over-quarter inflation as defined by the GDP price deflator; $x_t$ is the CBO measure of the log output gap, and $y_t$ is log real GDP. I use parameter values $\rho_i = 0.8$, and $\phi_\pi = 1.5$. In the gap specification $\phi_x = 0.5$ and $\phi_\Delta_x = 0$. In the growth rate specification $\phi_x = 0$ and $\phi_\Delta_x = 0.5$. Variables with a “bar” denote sample averages over the period 1985-2006. The coefficient values are not from any estimation or optimization, but rather are just common values used in the literature and seem “reasonable” and are simply meant to be instructive. A similar difference between the growth rate and gap specification emerges with different policy rule coefficients within a considerable range. See, for example, Carlstrom and Fuerst (2012).
growth rate rule, in contrast, the implied target interest rate never actually goes negative, and only gets particularly close to zero in the beginning of 2009. Under that rule, the target interest rate has been around 1 percent for the last three years, well above the actual funds rate.

In Section 2, I begin by focusing on the textbook, linearized three equation New Keynesian model with sticky prices. The non-policy block consists of an equation representing the demand side of the model (sometimes called the “IS” curve) and a Phillips Curve describing the relationship between inflation and the output gap. The output gap is defined as the difference between actual equilibrium output and the hypothetical output that would emerge if prices were flexible. The model is closed with a partial adjustment policy rule in which the interest rate is set as a convex combination of the lagged and target interest rates, where the target rate is a function of the deviations from steady state of inflation, the output gap, and the period-over-period growth rate of output. A second order approximation to household welfare gives rise to a loss function in the variances of inflation and the output gap. This loss function expresses the welfare losses arising due to price rigidity, and can therefore be used to evaluate different monetary policy rules. There are three exogenous shocks in the basic model: a preference shock to the IS curve, a productivity shock to the flexible price (or “natural rate”) of output, and a “cost-push” shock to the Phillips Curve. The cost-push shock can be given different interpretations, such as time-variation in the degree of market power in price-setting. Its presence ensures that the central bank faces a non-trivial tradeoff between inflation and gap stabilization. Otherwise the “Divine Coincidence” (Blanchard and Gali, 2007) holds and the central bank can stabilize the gap through a policy of strict inflation targeting.

I begin by showing how the welfare loss from price rigidity varies as the parameters of the policy rule are varied one at a time. For a standard numerical parameterization of the model, welfare is increasing in the size of the response coefficient on inflation and when there is more interest smoothing. Welfare is everywhere decreasing, and in a substantial way, in the coefficient on the output gap. In contrast, there are welfare gains from responding to output growth, at least over a range. I then numerically search for optimized policy rule coefficients so as to minimize the welfare loss from price stickiness. The optimized rule features a moderate amount of inertia, a strong response to inflation, a moderate response to output growth, and no response to the output gap. In spite of its relative simplicity, the optimized rule achieves nearly the same level of welfare as the Ramsey optimal targeting policy.

That it seems to be welfare-improving to respond to output growth and welfare-reducing to react to the gap in the policy rule may seem non-intuitive at first. This is because the variance of the gap shows up directly in the welfare function, whereas output growth does not. Moving rates more aggressively in response to the gap works to reduce gap volatility and therefore seems like it ought to be welfare-enhancing. In contrast, because of the “natural rate” property of the model, output growth tends to be high when the gap is negative. Raising interest rates when output is below potential seems to run counter to conventional stabilization logic.

The non-desirability of responding to the output gap depends on the presence of cost-push shocks in the model. These shocks introduce a tradeoff in gap and inflation stabilization: stabilizing one comes at the expense of more volatility in the other. In the approximation to household
welfare, the weight placed on gap variability is low relative to the weight on inflation for reasonable parameter values. Responding more vigorously to the gap in the policy rule reduces gap volatility at the expense of more inflation variability conditional on these shocks; given the relative weights, this works to reduce welfare on net, and potentially by a substantial amount. Conditional on productivity or preference shocks, in contrast, larger response coefficients on the gap are welfare-enhancing, albeit only mildly so. This is because these shocks tend to not produce very large welfare losses in the first place, and, conditional on these shocks, responding more vigorously to the gap has essentially the same effects as a stronger response to inflation. The desirability of targeting the gap in the policy rule thus depends on how important cost-push shocks are on net. Given a baseline parameterization of the rest of the model, I find that if cost-push shocks account for more than five percent of total output volatility, then positive responses to the gap in the policy rule reduce welfare.

The beneficial welfare effects of responding to output growth, in contrast, do not hinge on the presence of cost-push shocks, though the gains from reacting to growth are stronger when cost-push shocks are present. To understand the intuition for why it may be beneficial to respond to output growth, note that the Phillips Curve is forward-looking, with current inflation depending on both the current gap and expected future inflation. Policies which better “anchor” expected inflation permit the attainment of more preferred “menus” of current inflation and the gap. To fix ideas, suppose the economy is hit by a shock which moves output below potential. By responding to output growth, a central bank can lower rates immediately (when growth declines) with an implicit promise to raise them in the future when growth turns positive as the economy heads back to potential. The implicit promise of a future anti-inflationary stance keeps expected inflation in check and presents the central bank with a better inflation/gap tradeoff in the present. By effectively tying current policy to the past, reacting to output growth in the policy rule works to better anchor expected inflation, which is the source of welfare gain from targeting growth.

Section 3 extends the analysis from the basic New Keynesian model to a more realistic medium scale model, similar to those in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). In addition to endogenous capital accumulation, the model features sticky wages and several real frictions. The basic conclusions about the desirability of targeting the output gap or output growth in the policy rule carry over from the simpler model. In particular, reacting positively to output growth tends to be welfare-enhancing, whereas targeting the output gap can significantly reduce welfare. In a standard parameterization of the model with several different shocks, the welfare-optimizing policy rule features a strong response to inflation, no response to the gap, a moderate to strong response to output growth, and little or no inertia.

As in the model without capital, the non-desirability of reacting to the gap depends on the presence of the cost-push shock to the price Phillips Curve and/or a “wage markup” shock to

---

2 A clean loss function in the variances of only a handful of variables does not exist in the model with capital and other real frictions. To measure welfare in the medium scale model, I take a second order approximation to all equilibrium conditions, including a recursive representation of the household’s value function, as in Schmitt-Grohe and Uribe (2004, 2007).
the wage Phillips Curve. These shocks are sometimes called “inefficient,” because their most straightforward interpretation is as time-variation in price- and/or wage-setting power, neither of which would have any effect on a hypothetical efficient allocation. Conditional on shocks other than to the price and wage Phillips curves, it is welfare-enhancing to respond positively to the output gap and large responses to inflation may be welfare-reducing, the latter being quite different from the simpler model where only prices are sticky. The quantitative importance of these “inefficient” shocks is a matter of some debate. Smets and Wouters (2007), for example, find that shocks to the price and wage Phillips Curves account for between 30-50 percent of output fluctuations at business cycle frequencies. Justiniano, Primiceri, and Tambalotti (2013), in contrast, argue that these “inefficient” shocks to the price and wage Phillips curves are relatively unimportant. Optimal monetary policy in their parameterization would come close to stabilizing the gap.

Given the uncertainty surrounding the importance of “inefficient” shocks, how ought one to think about the output gap and its relation to the design of monetary policy rules? In both the simple model without capital as well as in the medium scale model, the potential gains and losses from targeting the gap are asymmetric. When it is beneficial to respond to the gap (i.e. when “inefficient” shocks are unimportant), then the potential gains from including a response to the gap in the policy rule are small, amounting to between 0.01-0.02 percent of steady state consumption. If “inefficient” shocks are as important as in Smets and Wouters (2007), in contrast, the welfare losses from strong responses to the output gap can amount to several percent of steady state consumption. As in the simpler model without capital, shocks to the price and wage Phillips Curves do not have to be very important quantitatively for strong responses to the gap to be welfare-reducing in the medium scale model. Put colloquially, there is not much to gain from targeting the output gap in interest rate rules, and potentially a lot to lose. Coupled with the fact that real time measurement of the output gap is likely to be difficult (e.g. McCallum, 2001; Orphanides, 2002; and Orphanides and Williams, 2006), there seems to be little justification for including responses to the output gap in simple policy rules, at least in the kinds of DSGE models currently popular among central banks.

Relative to the magnitude of the potential losses from targeting the output gap, the gains from responding to output growth in a policy rule are much more modest. There are a couple of reasons why these modest welfare gains from responding to output growth are likely to represent a lower bound on the benefits of growth targeting. First, as I show in the simple New Keynesian model in Section 2.3 and for the medium scale model in Section 3.1, responding to output growth in the policy rule tends to reduce the incidence of hitting the zero lower bound, which can be quite costly from a welfare perspective (see Coibion, Gorodnichenko, and Wieland, 2012). In contrast, if cost-push and wage markup shocks are present, responding to the output gap can significantly increase the incidence of hitting the zero lower bound. Second, my baseline analysis abstracts from

---

3The non-desirability of inflation targeting, and the welfare benefits of strong responses to the gap, are a well-known feature of models with both price and wage rigidity. In these models the simultaneous stabilization of price inflation, wage inflation, and the output gap is in general impossible, even without cost-push or wage markup shocks. With both wages and price rigid, strict inflation targeting tends to induce a level of wage dispersion that lowers welfare on net. Conditional on productivity shocks, gap stabilization, in contrast, tends to do well from a welfare perspective, as shown by Erceg, Henderson, and Levin (2000).
monetary policy shocks. Monetary policy disturbances induce more inflation and gap variability, and therefore lower welfare. As discussed in Section 2.5, aggressive responses to output growth in a policy rule can serve as a mechanism to limit the effects of policy shocks. Third, more aggressive responses to output growth in the policy rule have been shown to significantly expand the region over which such rules induce equilibrium determinacy (e.g., Coibion and Gorodnichenko, 2011b). Larger responses to the gap, in contrast, can make determinacy less likely when steady state inflation is positive (e.g., Ascari and Ropele, 2009).

This paper is closely related to several different papers in the literature on monetary policy design within the New Keynesian framework. Clarida, Gali, and Gertler (1999) provide a comprehensive survey in the context of the basic three equation linearized model. Woodford (2001) and Svensson (2003) examine how basic Taylor type rules perform from the perspective of Ramsey optimal policies. Woodford (1999), Woodford (2003), Carlstrom and Fuerst (2008), and Giannoni (2012) discuss the advantages of inertia in policy rules. Papers that study the empirical fit of Taylor type interest rate rules include Judd and Rudebusch (1998); Clarida, Gali, and Gertler (2000); Orphanides (2001); Rudebusch (2006); Pappell, Molodtsova, and Nikolsko-Rzhevskyy (2008); and Coibion and Gorodnichenko (2012).

Similarly to the exercises conducted in this paper, Schmitt-Grohe and Uribe (2006, 2007) study the properties of “simple and implementable” rules in which interest rates are constrained to react to only handful of easily observable variables. A central result in their papers is that these rules should not react to the level of output. They do not consider the output gap as a potential target variable in the policy rule, putatively because of the difficulty in observing it. They also do not consider shocks to the price and/or wage Phillips Curves in their model, which are of central importance to the desirability (or non-desirability) of gap targeting. Walsh (2003) studies Ramsey optimal monetary policy under commitment and discretion. He shows that a myopic central bank that acts under discretion will implement the socially optimal policy under commitment if it is presented with a loss function that seeks to minimize variation in output gap changes as opposed to the level. The intuition for how this result arises is similar to that discussed above: by making current policy contingent on the past, focusing on growth rates as opposed to levels better anchors inflation expectations. He does not consider including output growth either in place of or in addition to the output gap in a simple interest rate reaction function, however. Faia and Monacelli (2007) look at optimal policy rule coefficients in a model with credit frictions. Levin, Wieland, and Williams (1999) undertake a similar exercise to Schmitt-Grohe and Uribe (2006, 2007) in several different empirically motivated monetary models. They do not consider policy rules which react to the output growth rate.

The remainder of the paper is organized as follows. Section 2 presents the basic New Keynesian model, analyzes the welfare consequences of different kinds of policy rules, computes optimized policy rules, and considers a number of extensions. Section 3 describes a medium scale version of the model including capital, wage stickiness, and a number of other real frictions, and repeats many of the same exercises as in Section 2. The final section concludes.
2 Interest Rate Rules and Welfare: Basic New Keynesian Model

This section considers the welfare effects of interest rate rules in the context of the textbook three equation, linearized New Keynesian model. For a full description and derivation of the model, refer to the Appendix section A.9 or the treatments in Woodford (2003), Gali (2008), or Walsh (2010).

The non-monetary side of the economy is characterized by two main equations: an equation characterizing aggregate demand and an aggregate supply relation.\(^4\) These are:

\[
y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \frac{1}{\sigma}(1 - \rho)\nu_t
\]

\[
\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u^p_t
\]

Equation (1), sometimes called the “New Keynesian IS curve,” is derived from log-linearizing the representative household’s consumption Euler equation and imposing the aggregate accounting identity that, in a model with no capital, all output must be consumed. The variable \(y_t\) is the log deviation of output from its non-stochastic steady state and \(i_t\) is the nominal interest rate relative to its steady state. The variable \(\nu_t\) is a preference shock to the utility of consumption and serves as a demand shock. \(\sigma\) is the coefficient of relative risk aversion.

The second equation is the New Keynesian Phillips Curve. \(x_t\) is the output gap – the gap between the actual level of output and the level of output that would obtain if prices were fully flexible, e.g. \(x_t = y_t - y^f_t\). \(\beta\) is the household’s discount factor and \(\kappa\) is a reduced form parameter reflecting the degree of price stickiness. Under Calvo (1983) pricing, it is given by: \(\kappa = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}(\sigma + \eta)\), where \(\theta_p\) is the probability that a firm cannot change its price and \(\eta\) is the inverse Frisch labor supply elasticity.\(^5\) The random variable \(u^p_t\) is a “cost-push” shock. One interpretation of \(u^p_t\), discussed further in Ireland (2004), is that it represents exogenous time series variation in desired markups of price over marginal cost. Alternative interpretations include anything which drives a time-varying wedge between the efficient and flexible price levels of output.\(^6\) The presence of \(u^p_t\) is critical for monetary policy to face a non-trivial inflation-output tradeoff: without such shocks, a central bank can close the output gap with a policy of complete inflation stabilization, the so-called “Divine Coincidence” (Blanchard and Gali, 2007).

In addition to the cost-push and preference shocks, there is also a productivity shock, \(z_t\). Each of these shocks follow log AR(1) processes:

---

\(^4\)As is common in much of this literature, I am abstracting from money altogether, so implicitly referring to a “monetary” side of the economy is a bit of a misnomer.

\(^5\)Implicitly in the text, though explicit in the Appendix, I assume additively separable preferences over consumption and labor, with \(\sigma\) the coefficient of relative risk aversion and \(\eta\) the inverse Frisch elasticity. Though I abstract from trend growth, additively separable preferences without a unitary elasticity of substitution, \(\sigma^{-1}\), are not consistent with balanced growth. Using non-separable preferences consistent with balanced growth in the model without capital has little impact on the analysis. In addition to its being popular, the main reason I assume separability is because it facilitates aggregation in the medium scale model when wages are also sticky.

\(^6\)Gilchrist and Leahy (2002) and Carlstrom, Fuerst, and Paustian (2010) show that net worth shocks are another potential source of the cost-push shock. Adam and Woodford (2013) show that shocks to housing demand and housing productivity could also generate what looks to be a cost-push shock.
\[ z_t = \rho z_{t-1} + s_z e_{z,t} \tag{3} \]
\[ \nu_t = \rho \nu_{t-1} + s_\nu e_{\nu,t} \tag{4} \]
\[ u_{\nu t}^p = \rho_{up} u^p_{\nu t-1} + s_{up} e_{up,t} \tag{5} \]

The shocks, \( e_{j,t}, j = z, \nu, up \), are drawn from standard normal distributions, with \( s_j, j = z, \nu, up \), the standard deviations of the shocks. The autoregressive parameters are all assumed to lie strictly between 0 and 1. In general, the flexible price level of output is a second best construct, differing from the efficient level of output due to the monopoly distortion that gives rise to pricing power. I assume that there exist Pigouvian taxes to offset this wedge, bringing the flexible price and efficient levels of output into alignment, e.g. \( y^f_t = y^c_t \). The efficient/flexible price level of output can be related to the exogenous disturbances as follows:

\[ y^f_t = \left( \frac{1 + \eta}{\sigma + \eta} \right) z_t + \left( \frac{1}{\sigma + \eta} \right) \nu_t \tag{6} \]

The model is closed with a description of monetary policy in the form of an interest rate feedback rule. With some abuse of terminology, I will often refer to interest rate rules as “Taylor rules” after Taylor (1993). Taylor’s original specification, which he took to be both descriptive of and prescriptive for actual policy, called for nominal rates to adjust more than one-for-one to deviations of inflation from target and positively to deviations of output from a statistical trend. Though Taylor’s rule pre-dates their full development, it has become a centerpiece of modern New Keynesian models. What is common to most specifications of interest rate rules is a strong response of nominal interest rates to inflation. There is considerably more variation in other elements of the rule. It is common to see rules featuring inertia, with the current interest rate a function of the lagged rate in addition to macroeconomic conditions. Many specifications of the rule replace detrended output with the theoretical output gap, \( x_t \). This is justified on the grounds that it is fluctuations in the output gap that matter for welfare, not output, as we will see below. Finally, it is also common to see rules in which the central bank reacts to the growth rate of output, either in place of, or addition to, the output gap – see, among others, Ireland (2004), Coibion and Gorodnichenko (2011a), Coibion and Gorodnichenko (2011b), and Fernandez-Villaverde (2010).

Interest rate rules are a kind of instrument rule since they describe how a central bank’s main instrument ought to be set as a function of macroeconomic conditions. There is also a substantial literature that studies optimal monetary policy in the form of targeting rules. Targeting rules are the solution to a Ramsey problem – a central picks a time path of the nominal interest rate to minimize a loss function like that to be described below in (8)-(9). The implementation of optimal targeting rules places potentially large informational burdens on central banks, where it is necessary to know the underlying structure of the economy and the specific shocks hitting it. Relatively simple interest rate rules only require central banks to adjust interest rates in response to a handful of more easily observable endogenous variables. In addition, the conditions under which interest rate rules give rise to a determinate rational expectations equilibrium are well understood.
I consider the following generalized specification of a Taylor type interest rate rule:

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) i^T_t, \quad 0 \leq \rho_i < 1 \]
\[ i^T_t = \phi_\pi \pi_t + \phi_x x_t + \phi_\Delta \Delta y_t \]

In this specification the actual interest rate is set as a convex combination of the previous period’s rate and the target rate, \( i^T_t \). All variables are either deviations or log deviations from trend, and are hence mean zero. The parameter \( \rho_i \) measures the degree of interest smoothing and the target rate, \( i^T_t \), is expressed as a linear function of inflation, the output gap, and output growth.\(^7\) One sometimes also sees policy rules which feature a response to the level of output. To the extent to which there are real shocks, the level of output is a poor proxy for the output gap, which is what matters for welfare. Indeed, it turns out that responding to the level of output is always welfare-reducing for any parameterization of the model that I consider. For this reason, I do not consider the level of output as a potential target variable in the policy rule.\(^8\) Though written in terms of current period values of the target variables, this policy rule can easily be amended to accommodate forward- or backward-looking terms. I require that \( \phi_\pi > 1 \) and that all other parameters be non-negative.\(^9\)

Equations (1)-(7) characterize an equilibrium in the variables \( y_t, \pi_t, i_t, y^f_t, z_t, u_t, \) and \( \nu_t \). The linearized policy functions mapping the states into the forward-looking variables can be found using standard techniques. Welfare can be approximated via a second order approximation to the value function of the representative household. Expressed in terms of deviations from an efficient/flexible price allocation, this yields a quadratic loss function in the output gap and inflation:\(^{10}\)

\[ W = -\Omega E_t \sum_{j=0}^{\infty} \beta^j L_{t+j} \]
\[ L_{t+j} = \pi^2_{t+j} + \lambda x^2_{t+j} \]

This loss function measures the average welfare loss due to price rigidity. Its units are constructed such that it measures the fraction of steady state output that one would need to give up in the flexible price economy to have the same welfare as in the sticky price economy. In terms of the underlying structural parameters of the model, the coefficients of the loss function are given by \( \Omega = \frac{\theta_\pi (\sigma + \eta)}{2 \kappa} \) and \( \lambda = \frac{\kappa}{\epsilon_p} \), where \( \epsilon_p \) is the elasticity of substitution among intermediate goods.

Solving the model and analyzing welfare requires picking values for the parameters. The bench-

\(^7\)I later also consider “difference” and “inertial” rules in which \( \rho_i \geq 1 \).

\(^8\)In an earlier version of the paper I also allowed a response to the growth rate of the output gap. This turns out to have similar effects as responding to output growth and is hence omitted.

\(^9\)The so-called “Taylor principle” calls for the central bank to raise nominal interest rates more than one-for-one with movements in inflation. Though originally articulated informally, something similar to the Taylor principle is required for equilibrium determinacy in modern forward-looking New Keynesian models. \( \phi_\pi > 1 \) is a slightly stronger restriction than is necessary to achieve determinacy, as determinacy also depends on the other response coefficients.

\(^{10}\)It is straightforward to verify that this loss function yields nearly identical welfare losses as taking a second order approximation to the non-linear equilibrium conditions of the model, as described in the Appendix and as done for the medium scale model studied in the next section.
mark parameterization is described in Table 1. The unit of time is taken to be a quarter, so $\beta = 0.99$. I set the coefficient of relative risk aversion, $\sigma$, to 2, and the inverse Frisch labor supply elasticity, $\eta$, to 1. The Calvo parameter, $\theta_p$, is set to 0.75, implying an average duration between price changes of one year. The elasticity of substitution among goods is set at $\epsilon_p = 5$. These parameter values imply that the slope of the Phillips Curve is $\kappa = 0.2575$ and the weight on the output gap in the loss function, (8), is $\lambda = 0.05$. I set the persistence parameters governing the exogenous process, (3)-(5), all equal to 0.95. The shock standard deviations are $s_z = 0.01$, $s_{\nu} = 0.02$, and $s_u = 0.002$. The parameterization of the shock processes is roughly consistent with the (equivalent) values frequently used in the literature. As parameterized, each of the shocks contributes about one-third of the total unconditional variance of output when I assume that $\phi_\pi = 1.5$ and all other parameters of the interest rate rule are set to zero. The standard deviation of HP filtered (smoothing parameter 1600) output with this parameterization of the model is 0.016 and its first order autocorrelation coefficient is 0.75, both roughly consistent with post-war US data.

Figure 2 shows how the welfare loss from price stickiness varies as the parameters of the policy rule are varied one at a time relative to a benchmark rule in which $\phi_\pi = 1.5$ and $\rho_i = \phi_x = \phi_{\Delta y} = 0$. The welfare losses on the vertical axes of the panels are multiplied times 100, and have the interpretation as a percentage of output/consumption. There are large welfare gains to be had from moving from a very small response to inflation ($\phi_\pi = 1.01$) to a larger response, as the upper left panel of the figure shows. Though welfare appears to be everywhere increasing in $\phi_\pi$, the welfare gains from a stronger response coefficient to inflation dissipate fairly quickly. In the upper right panel one observes that welfare is everywhere decreasing in the response coefficient on the output gap. The welfare losses from responding to the output gap can be large. For example, the baseline rule with $\phi_\pi = 1.5$ entails a welfare loss of about 0.2 percent of consumption; adding to this a response coefficient to the gap of $\phi_x = 0.5$ increases the welfare loss to 1.5 percent of consumption. In the lower left panel one sees that welfare is increasing in the response coefficient on output growth up until a value of about 0.85. Relative to the potential losses from responding to the gap, the welfare gains from reacting to growth are rather modest. In the lower right panel we see that welfare is also increasing in the coefficient on the lagged nominal interest rate up to a point. Because of the the partial adjustment nature of the rule, at some point the benefits of interest smoothing are offset by the lower short-run response to inflation.\footnote{As noted in Footnote 7, I later consider difference and super-inertial rules which do not impose the partial adjustment specification. Writing the rule with the response coefficient on inflation of $(1 - \rho_i)\phi_x$ imposes that the long run response of the nominal interest to a permanent change in inflation is invariant to $\rho_i$ and held fixed at $\phi_x$. The greater inertia of course means that the short run response is smaller as $\rho_i$ increases and $\phi_x$ is held fixed. For equilibrium determinacy what matters is the long run response of interest rates to inflation, not the short run response.}

Based on Figure 2, it appears beneficial to respond to output growth in the policy rule, while reacting to the output gap seems to be welfare-reducing. To get a better grasp for the intuition for these conclusions, Figure 3 plots impulse responses of the output gap, inflation, one period ahead expected inflation, and the interest rate to the three shocks under different versions of the policy rule: a baseline rule with $\phi_\pi = 1.5$ and $\rho_i = \phi_x = \phi_{\Delta y} = 0$ (solid line); a rule that responds to...
the output gap with $\phi_x = 1.5$, $\phi_\pi = 0.5$ and $\rho_i = \phi_{\Delta_\pi} = 0$ (dotted line); and a rule which instead responds to output growth, with $\phi_x = 1.5$, $\phi_{\Delta_\pi} = 0.5$, and $\rho_i = \phi_x = 0$ (dashed line).

For the baseline rule which only moves interest rates in response to inflation, both the gap and inflation fall in response to a positive productivity shock and both rise after a positive preference shock. After a positive cost-push shock inflation rises and the gap declines. The interest rate falls after a positive productivity shock but rises after a cost-push or preference shock. Conditional on either a productivity or preference shock, reacting to the output gap results in impulse responses of inflation and the gap that are smaller at all horizons than in the baseline policy rule which only reacts to inflation. Following a cost-push shock, a positive coefficient on the gap leads to a “better” (in the sense of smaller) response of the output gap relative to the baseline rule, but at the expense of a significantly larger movement in inflation. Reacting to the gap also results in a much larger increases in the interest rate after a cost-push shock than under the baseline rule. The dashed lines show impulse responses when the policy rule features a positive reaction to output growth. Responding to growth leads to larger immediate movements in both the gap and inflation relative to the baseline rule after a productivity shock, but smaller subsequent movements in both variables. Conditional on a cost-push shock, a positive coefficient on growth results in a smaller initial movement in the gap and smaller movements in inflation over most horizons than does the rule which only reacts to inflation. In response to a preference shock, reacting to growth causes the output gap to initially fall (instead of rise as in the baseline policy rule), and results in a larger response of inflation over most horizons. Reacting to growth leads to movements in the interest rate which are quite different than for the other two specifications: rates immediately rise after a positive productivity shock, decline after a cost-push shock, and rise substantially more than in either the baseline or gap rule after a preference shock.

Focusing on the impulse responses of expected inflation helps to gain insight into the welfare consequences of different versions of the policy rule. From the Phillips Curve, (2), one can see that a “better” combination of $(x_t, \pi_t)$ can be achieved the less expected inflation moves in response to a shock. One observes that a positive reaction to the output gap results in smaller movements in expected inflation conditional on productivity and preference shocks, but a much larger response of expected inflation after a cost-push shock. This suggests that reacting to the gap is welfare-enhancing conditional on either productivity or preference shocks, but welfare-reducing conditional on cost-push shocks. The magnified response of expected inflation to a cost-push shock is much larger than than dampened responses after productivity or preference shocks, making responding to the gap welfare-reducing on net, at least for this parameterization of the model.

Responding to output growth leads to smaller responses of expected inflation conditional on both productivity and cost-push shocks. Responding to growth ties current policy to the past, which has the effect of better anchoring expected inflation. Interest rate inertia has similar effects. Conditional on a cost-push shock, for example, output growth initially declines, but then turns positive as the level of output heads back to its pre-shock value. A positive response coefficient to growth allows the central bank to cut interest rates in the period of the shock when output growth declines, thereby providing stimulus, but also serves as an implicit promise to raise interest
rates in the future when output growth turns positive. This has the effect of keeping expected inflation in check. After a preference shock to the IS curve, in contrast, reacting to growth leads to a movement of the interest rate, and hence expected inflation, that is much larger than either the inflation-only rule or the rule which reacts to the gap. From these responses, one can gather that reacting to output growth is beneficial conditional on either productivity or cost-push shocks, but welfare-reducing after preference shocks.

2.1 Optimized Rules

In this subsection I choose the parameters of the policy rule with the explicit intention of minimizing the welfare loss due to price stickiness. Specifically, I search numerically for values of \((\rho, \phi_\pi, \phi_x, \phi_\Delta y)\) to minimize the loss function given in (8). I restrict the parameters such that \(\phi_\pi \in (1.01, 2.5)\), \(\rho_i \in (0, 0.99)\), and the coefficients on the output gap and output growth are restricted to the interval \((0, 2.5)\).\(^{12}\)

Row (a) of Table 2 shows the optimized policy rule coefficients, minimized value of the loss function, and standard deviations of inflation and the gap under the optimal policy rule. The optimal policy rule features a large response to inflation \((\phi_\pi = 2.5)\) and a strong response to the output growth rate, \(\phi_\Delta y = 0.45\). There is no response to the output gap, and there is some modest interest smoothing, with \(\rho_i = 0.52\). The value of the objective function is -0.10, or about 0.1 percent of steady state consumption. In spite of the simplicity of the rule, this compares quite favorable with the Ramsey optimal policies under either commitment or discretion. Under discretion, for example, the Ramsey optimal policy results in a welfare loss of -0.15, and under commitment the loss is -0.09.\(^{13}\)

In row (b) I restrict the central bank to not respond to output growth. The optimal rule compensates with a larger coefficient on the lagged rate, and achieves only a slightly worse outcome than in the unrestricted case. Row (c) shows optimized coefficients when I restrict the central bank to no smoothing. Here the rule compensates with a larger response to output growth and achieves virtually the same welfare loss as when the coefficients are unrestricted. The similar welfare effects of responding to growth and interest smoothing will be taken up in further detail in subsection 2.5. Row (d) shows the optimized policy rule when I require a positive response coefficient to the output gap of 0.5, a value frequently used in the literature. This restriction does not have much effect on the optimal values of the other coefficients, but results in a substantial welfare loss of -0.29. Though reacting to the gap lowers gap volatility, this comes at the expense of substantially more inflation volatility. Given the low relative weight on the variance of the gap in the welfare loss function, this works to lower overall welfare.

Rows (e)-(g) of Table 2 show optimized policy rules conditional on particular shocks. For each

\(^{12}\)In general, higher values of \(\phi_\pi\) than 2.5 will improve welfare, albeit only modestly. A coefficient of 2.5 is on the outer range of empirical estimates, and capping this coefficient at a higher value does not substantially affect the other optimized coefficients.

\(^{13}\)Under “discretion,” a central bank picks \(i_t\) to minimize the loss function from (8) each period. Under commitment, the bank picks a time path of \(i_t\) to minimize the present discounted value of losses.
of the three shocks, I set the innovation standard deviations of the other two shocks to zero and compute the optimal policy rule parameters. It is optimal to have a large response to inflation conditional on each of the shocks. Consistent with the intuition from Figures 2 and 3, it is optimal to have a large coefficient on the output gap conditional on either productivity or preference shocks, but no response to the gap conditional on cost-push shocks. Likewise, the optimal coefficient on output growth is large conditional on productivity and cost-push shocks, but is zero conditional on the preference shock. It is worth noting that cost-push shocks are the only significant source of welfare loss – with productivity and preference shocks, the central bank can nearly completely neutralize the effects of price stickiness and achieves close to an efficient outcome.

Rows (h) and (i) show optimized coefficients for a forward- and backward-looking version of the policy rule, respectively. In the forward-looking version of the rule, current values of inflation, the gap, and growth are replaced by their one period ahead expected values; for the backward-looking version, these variables are instead lagged one period. In terms of welfare, the forward-looking version does slightly worse, and the backward-looking rule a little better, than the contemporaneous version of the rule. In both cases it is optimal to have a large response to inflation, no response to the gap, and a positive response to output growth, with the response coefficient on growth larger in the forward-looking version and smaller in the backward-looking rule relative to the baseline rule.

Woodford (2003) and Giannoni (2012) have argued for the benefits of so-called “super-inertial” rules, with $\rho_i \geq 1$. Rows (j) and (k) of Table 2 amend my specification of the policy rule to accommodate this change. To allow the response coefficients on inflation and economic activity to be positive, this requires dropping the $(1 - \rho_i)$ in front of the target rate specification. Row (j) shows results when $\rho_i$ is restricted to be 1, so that the rule is a “difference” rule. This specification features no response to the output gap and a small response to growth. Row (k) shows results for a so-called “super-inertial” rule, in which $\rho_i$ is allowed to exceed unity. The optimal coefficient on $\rho_i$ is nevertheless close to one. The optimized response to the output gap is again zero and, like the difference rule, the coefficient on output growth is small. Both the difference and super-inertial rules achieve a lower welfare loss than the optimized partial adjustment specification, though the differences are small.

Table 3 reports optimized policy rule parameters for different values of the non-policy parameters of the model. Under any of these parameterizations, it is optimal to have the maximum response to inflation. With the exception of the case when the variance of the cost-push shock is set to zero, it is optimal to have no response to the output gap. It appears always optimal to have at least some positive response to output growth. The optimized response to growth is increasing in the amount of price stickiness, $\theta_p$; increasing in the coefficient of relative risk aversion, $\sigma$; and increasing in the Frisch labor supply elasticity (the inverse of $\eta$). The optimized coefficient on growth is larger the more patient households are and is roughly invariant to the elasticity of substitution among goods, $\epsilon_p$. The desired response to output growth is increasing in the impor-

---

14With $\rho_i \geq 1$ all that is required for determinacy is that $\phi_\pi > 0$. The “long run” response of the interest rate to a movement in inflation is much larger in the difference and super-inertial rules than in the baseline specification. As such, I restrict $\phi_\pi \in (0, 0.5)$ to make the results more comparable.
tance of productivity and cost-push shocks and decreasing in the magnitude of preference shocks. In terms of persistence, the optimal response to output growth is decreasing in the persistence of productivity shocks and increasing in the persistence of cost-push and preference shocks.

2.2 Arbitrary Welfare Weights

In the basic model, the relative weight placed on fluctuations in the output gap in the welfare criterion, $\lambda$, is small. For my baseline parameterization $\lambda = 0.05$, so inflation variability is about twenty times more important for welfare than is gap variability. While the welfare criterion, (8), is derived as an approximation to the value function of the representative household, these relative weights seem too low relative to central bank preferences in the real world.

I therefore consider how alternative, arbitrary welfare weights affect the analysis. Table 4 repeats the exercise of choosing the parameters of the policy rule to minimize the welfare loss for different, arbitrary values of $\lambda$. Because the units of this arbitrary loss function are no longer directly interpretable, I do not report the average losses, though I do show summary statistics on inflation and gap volatility. For values of $\lambda$ less than about 0.5, the optimal response coefficient on output growth is actually larger than it is using the welfare criterion derived from the household’s utility function. The optimal response coefficient on inflation is still large, and it is desirable to have greater interest rate smoothing as $\lambda$ increases. The optimal response coefficient on the gap, while not zero, remains small for values of $\lambda$ less than 1. As $\lambda$ increases and the optimal response to the gap rises, average gap volatility declines at the expense of substantially more inflation volatility. Even with $\lambda = 2$, the optimal response coefficient on the gap is relatively modest and it remains desirable to still have a small, positive response coefficient on the output growth rate.

Figure 4 plots the optimized response coefficients on the output growth rate and the gap for different values of $\lambda$. It only becomes optimal to have a non-zero response to the output gap for values of $\lambda > 0.2$, which is roughly 4 times as large as the value derived from the household utility function. The optimized response to output growth at first rises with $\lambda$ and then declines once it becomes optimal to respond some to the output gap (i.e. at values of $\lambda > 0.2$). For values of $\lambda < 0.8$ the desired response coefficient on output growth is larger than the optimized coefficient on the output gap.

2.3 The Zero Lower Bound

The analysis in this paper has ignored the effects of the zero lower bound on interest rates. Though rare empirically as well as in a baseline parameterization of the model, hitting the zero lower bound can be quite costly, as shown in Coibion, Gorodnichenko, and Wieland (2012).

Figure 5 shows how the frequency of hitting the zero lower bound varies with the parameters of the monetary policy rule. I take as a benchmark a policy rule in which $\phi_\pi = 1.5$ and the other parameters are all set to zero. I consider the baseline model amended to account for positive trend inflation, with $\pi^* = 2.00$ at an annualized percentage rate (0.005 at a quarterly rate).\footnote{Though I still solve the model via linearization about the non-stochastic steady state, allowing for positive trend}
the baseline specification the economy hits the zero lower bound about 5.5 percent of the time.\textsuperscript{16} For the optimized value of the policy rule parameters reported in row (a) of Table 2, in contrast, the incidence of hitting the zero lower bound is less than 1 percent. In the left panel one sees that the frequency of hitting the zero lower bound is decreasing in the response coefficient on output growth until $\phi_{\Delta y} \approx 0.4$ (down to a frequency of about 5 percent), after which it increases somewhat. Incidentally, this value of the response to output growth is roughly equal the optimized value not taking into account the zero lower bound. The frequency of hitting the zero lower bound is everywhere increasing in the response coefficient on the output gap, as the middle panel shows.\textsuperscript{17} Large values of this response coefficient can lead to very high probabilities of interest rates hitting zero. Finally, the frequency of reaching the lower bound is everywhere decreasing in $\rho_i$. Though my welfare calculations do not take into account the costs of hitting the zero lower bound, these figures suggest that there are likely additional advantages to having mild responses to output growth and additional costs to responding to the output gap.

### 2.4 Responding to the Output Gap

An important result of the paper is that responding to the output gap reduces welfare, potentially by a substantial amount.\textsuperscript{18} In the basic model, the non-desirability of responding to the output gap depends on the presence of the cost-push shock. Without this shock, the “divine coincidence” (Blanchard and Gali, 2007) holds, and stabilizing the gap is welfare-enhancing. But conditional on these shocks, further stabilizing the gap comes at the cost of more inflation variability. Given the low relative weight on the gap in the loss function, this works to lower welfare.

How important must cost-push shocks be for it to be non-optimal to respond to the output gap? The overall importance of cost-push shocks depends on both the innovation variance, $s_{up}^2$, and the persistence parameter, $\rho_{up}$. Figure 6 plots in the left panel the combinations of $(s_{up}, \rho_{up})$ for which moving from a zero response to the output gap to a small positive response has no effect on welfare.\textsuperscript{19} At combinations above and to the right of the curve responding to the output gap is welfare-reducing. The baseline parameterization with $s_{up} = 0.002$ and $\rho_{up} = 0.95$ is well above the curve. The right panel plots the contribution of cost-push shocks to the forecast error variance of output at the parameter values along the curve in the left panel. If $(s_{up}, \rho_{up})$ are such that cost-push shocks contribute more than about five percent to the forecast error variance of output, it is welfare-reducing to respond to the output gap. In the baseline parameterization the standard deviation of HP detrended output is 1.6 percent. For this level of total volatility, at a five percent

---

\textsuperscript{16}To compute this frequency, I solve the model with positive trend inflation and simulate it for 20,000 periods, and compute the fraction of time that the nominal interest rate is zero or negative.

\textsuperscript{17}This result depends on the presence of cost-push shocks. If there are no cost-push shocks, then responding to the output gap actually reduces the incidence of hitting the zero lower bound.

\textsuperscript{18}Carlstrom, Fuerst, and Paustian (2010) briefly make a similar point in a New Keynesian model with net worth shocks.

\textsuperscript{19}Specifically, for a grid of values of $\rho_{up} \in (0, 1)$, I numerically search for the value of $s_{up}$ for which moving from $\phi_{x} = 0$ to $\phi_{x} = 0.05$ has no effect on welfare using the baseline parameterization of the model and $\phi_{\pi} = 1.5$, with no interest smoothing and no response to output growth.
variance share cost-push shocks would generate essentially no output volatility. In other words, if cost-push shocks have much importance at all, responding to the output gap in the policy rule is welfare-reducing.

If cost-push shocks are absent or very nearly so, then there are welfare gains to be had from responding to the output gap, but these are very small – in those cases, a policy rule which reacts only to inflation achieves close to the efficient allocation anyway, effectively neutralizing the negative welfare effects of price stickiness. Put into everyday parlance, there is not much gain to be had from targeting the output gap in an interest rate rule, but there is potentially a lot to lose.

There are several papers in the literature which also argue against responding to the output gap – see, for example, McCallum (2001), Orphanides (2002), and Orphanides and Williams (2006). These papers base this argument on the grounds that the central bank may have difficulty in observing the theoretical output gap, particularly in real time. To the extent to which the gap is observed with imprecision, these errors manifest themselves as shocks to the policy rule, which are strictly welfare-reducing. In contrast, I have shown that responding to the output gap is welfare reducing (if cost-push shocks are at all important) even if the output gap is observed perfectly. Introducing measurement error would only serve to strengthen this conclusion.

2.5 Interest Smoothing or Output Growth?

Responding to output growth and the lagged interest rate have similarly beneficial welfare effects in Taylor rules. One can see this at work in rows (a)-(c) of Table 2. A policy rule with no smoothing and a strong response to output growth achieves essentially the same welfare outcome as a more inertial rule with a smaller response to output growth.

The reason that responding to output growth and the lagged rate are fairly substitutable from a welfare perspective becomes clearer by looking at the Euler/IS equation (1). Lagging it one period, ignoring the preference shock (e.g. \( \nu_{t-1} = 0 \)), and replacing expectation operators with a rational expectations error term, \( \epsilon_t \), one observes:

\[
y_t - y_{t-1} = \frac{1}{\sigma} h_{t-1} - \frac{1}{\sigma} \pi_t + \epsilon_t
\]

In other words, current output growth is proportional to the lagged nominal interest rate and current inflation. A positive response coefficient on output growth is therefore like having a larger response to the lagged interest rate (though a smaller response to current inflation). One might be tempted to therefore conclude that there are no welfare gains to be had from targeting output growth in a policy rule – the central bank can simply commit to more smoothing and ignore real economic activity altogether and achieve the same welfare (or even slightly better in the case of super-inertial rules).

The conclusion that reacting to output growth and the lagged rate are roughly interchangeable breaks down if one amends the policy rule to include monetary policy shocks. In particular, let the actual interest rate evolve according to:
\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) i_T^T + \varepsilon_{i,t} \]  

\( \varepsilon_{i,t} \) is an exogenous policy shock, representing some deviation from the policy rule unrelated to current economic conditions. One interpretation could be that it represents a mis-measurement of current conditions in the target rule specification. In the model as written down, policy shocks are strictly welfare-reducing, leading to more inflation and gap variability.

If one takes policy shocks as given, interest smoothing begins to lose its desirability. The basic New Keynesian model has no natural endogenous state variable, and therefore no built-in propagation mechanism for shocks. If there is no interest smoothing and no response to output growth, then monetary policy shocks have effects which only last one period. Unless the variance of \( \varepsilon_{i,t} \) is very large, then these shocks contribute little to overall inflation and gap variability and therefore have small effects on welfare. Explicit interest-smoothing serves as a propagation mechanism for these shocks, and large values of \( \rho_i \) will allow even small policy shocks to have substantial effects. By the logic above, positive response coefficients to output growth will also serve as a propagation mechanism for policy shocks. Unlike interest smoothing, however, responding to output growth not only propagates policy shocks but also works to dampen/counteract them.

The bottom panel of Table 2 shows optimized coefficients for two different specifications of the policy rule when I include monetary policy shocks in the analysis. Row (l) shows results for the partial adjustment specification of the policy rule, while row (m) considers the super-inertial specification. I set the standard deviation of monetary policy shocks at 0.0033, which is the central point estimate in Ireland (2004). For the partial adjustment specification, the inclusion of policy shocks has little effect on the optimized rule – the optimized response coefficient on output growth is a little higher (0.60 versus 0.45) and it is no longer optimal to have any smoothing. The overall welfare loss is only slightly worse than the optimized rule when there are no policy shocks (-0.101 versus -0.100). Row (m) shows results for the super-inertial rule. This version of the rule performs worse than the partial-adjustment specification (-0.13 vs -0.10) and worse than the super-inertial rule with no policy shocks (-0.13 vs. -0.09). This exercise suggests that responding to output growth is likely to be preferred to explicit interest smoothing in the presence of policy shocks. Responding to output growth yields the welfare benefits of interest smoothing without allowing policy shocks to have large, welfare-reducing effects.

### 3 A Medium Scale DSGE Model

This section extends the analysis of the previous section to a medium scale version of the New Keynesian model. The basic conclusions about the relative merits of policy rules which target output growth or the gap carry over largely intact.

The medium scale version of the model is similar to the models in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Such a model fits the data well and is popular among central banks. Relative to the simpler model from the previous section, the medium scale model
includes capital accumulation, nominal wage rigidity, full or partial price and/or wage indexation, investment adjustment costs, habit formation in consumption, and variable capital utilization. In addition to the three shocks from the simpler model – a productivity shock, a preference shock to the “IS Curve,” and a “cost-push shock” to the Phillips Curve – in the medium scale model I also consider a “wage markup shock” to the wage Phillips Curve. It plays a role analogous to the cost-push shock for price inflation, but for wage inflation. The wage markup shock is shown to be an important driving force behind US business cycles in Smets and Wouters (2007), though the interpretation of this shock and its importance in driving fluctuations is a matter of some debate (see, e.g. Chari, Kehoe, and McGrattan, 2009, or Justiano, Primiceri, and Tambalotti, 2013).20 The inclusion of wage rigidity means that the central bank cannot simultaneously stabilize the output gap, price inflation, and wage inflation, and thus there exists a non-trivial tradeoff between real and nominal stabilization for a central bank even in the absence of cost-push or wage markup shocks. Nevertheless, the presence (or not) of these shocks plays a crucial role in the design of optimal interest rate rules, as will be discussed further in subsection 3.2.

Because it is fairly standard, details and derivations are relegated to the Appendix. The simpler New Keynesian model from Section 2 can be considered a special case of the more complicated model. I choose parameter values for the non-policy block of the model based on the estimations in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). These parameter values are described in the lower panel of Table 1. The time period is again taken to be a quarter. Preference parameters for household utility are the same as in the simpler model, with the exception of a new parameter governing internal habit formation, which is set to 0.7. I set the capital’s share parameter to 0.3. This is higher than the estimate in Smets and Wouters (2007), but more in line with long run averages from NIPA data. The depreciation rate on capital is set to 0.025. The parameter governing the quadratic investment adjustment cost is set to 3. The parameter governing the convexity of the utilization cost function is 0.05. I set the elasticity of substitution between differentiated labor input at 5, the same for prices. The parameters governing price and wage rigidity are both set to 0.65, implying that prices and wages change on average once every three quarters. I assume that wages and prices are both partially indexed to lagged inflation, with \( \zeta_p = \zeta_w = 0.5 \). Indexation is an important feature to produce empirically plausible hump-shaped dynamics of inflation to shocks.

The exogenous variables all follow AR(1) processes in the log with autoregressive coefficients of 0.95. The standard deviations of the technology shock and cost-push shock are based on Smets and Wouters (2007), as is the standard deviation of the wage markup shock.21 The standard deviation of the preference shock is as in the smaller-scale model. Finally, I assume zero steady state inflation.

---

20 One could, of course, consider additional shocks, such as government spending or investment-specific technology shocks. The inclusion of these shocks does not have much effect on the analysis to follow.

21 The shock standard deviations for the cost-push (or “price markup”) and “wage markup” shocks appear different than in Smets and Wouters (2007). This is because they assume that these exogenous variables follow ARMA(1,1) processes, whereas I assume the more conventional AR(1) specification. The shock standard deviations here are chosen to produce volatility of the cost-push and wage markup shocks that match the overall volatility from the Smets and Wouters (2007) ARMA(1,1) specification.
as a benchmark. This is a reasonable baseline for recent US experience, and is mainly employed to avoid issues of determinacy that can arise with positive trend inflation and positive responses to the output gap (see, e.g. Ascari and Ropele, 2009), though I consider robustness along this dimension below. The basic structure of the monetary policy rule is the same as in the simpler model. There are no taxes/subsidies to correct steady state distortions associated with market power in price and wage-setting.\textsuperscript{22} For a specification of the policy rule with $\phi_{\pi} = 1.5$ and other parameters equal to zero, the model produces a standard deviation of HP filtered output of 1.1 percent, which is approximately equal to the volatility of output in the US data post-1984. Correlations and autocorrelations of the model variables are also in line with the data. At these parameter values, productivity shocks account for 39 percent of the unconditional variance of HP filtered output, cost-push shocks 8 percent, preference shocks 15 percent, and wage-markup shocks 38 percent. These numbers are roughly in line with the variance decomposition in Smets and Wouters (2007).

Measuring welfare in the medium scale model is more complicated than in the simple model without capital. In particular, there does not exist a simple quadratic loss function in inflation and the output gap when capital is in the model.\textsuperscript{23} As such, I measure welfare by applying a second order approximation to all equilibrium conditions of the model, including a recursive representation of the household’s value function as one of the equilibrium conditions. This is the approach taken in Schmitt-Grohe and Uribe (2004, 2007). Applying this approach to measuring welfare in the simpler model without capital yields nearly identical results as when using the quadratic approximation to the value function along with the linearized equilibrium conditions.

As described in Appendix Section A.8, my welfare metric is the fraction of period consumption that would make a household indifferent, on average, between the equilibrium of the sticky price and wage economy and a hypothetical allocation where prices and wages are both flexible. The flexible price and efficient allocations will differ due to monopoly distortions in price and wage-setting, but this difference is independent of monetary policy. This approach is slightly different than the one taken by Schmitt-Grohe and Uribe (2007), who measure welfare of “simple and implementable” policy rules relative to the Ramsey optimal monetary policy. The Ramsey optimal policy is a constrained efficient allocation which takes monopoly power and price rigidity as given. The choice of welfare measure to which to compare a simple policy rule is irrelevant for finding optimal policy rule coefficients, as the welfare from both the efficient and Ramsey constrained optimal allocations are independent of the parameters of monetary policy rules – in either case, minimizing a compensating variation is equivalent to maximizing expected welfare. The magnitudes of the welfare metric will be different, however.

Figure 7 shows how welfare varies in the medium scale model as the parameters of the policy

\textsuperscript{22}In the welfare calculations of the model without capital, in contrast, I implicitly assume that there exist Pigou-vian taxes/subsidies which offset the steady state distortion associated with monopoly power, as is standard in the literature. This means that the flexible price and efficient equilibriums are identical.

\textsuperscript{23}See Edge (2003) for a derivation of a quadratic approximation to welfare in a sticky price model with capital, with the loss function depending on substantially more than just inflation and gap volatility. Erceg, Henderson, and Levin (2000) derive a quadratic loss function in the variances of the output gap, price inflation, and wage inflation for a model with sticky prices and sticky wages, but abstracting from endogenous capital accumulation.
rate are varied one a time relative to a benchmark rule in which $\phi_\pi = 1.5$ and $\phi_x = \phi_{\Delta_y} = \rho_i = 0$. This figure is analogous to Figure 2 for the simpler New Keynesian model, with the units on the vertical axis percentages of steady state consumption. As in the simpler model without capital, there are large welfare gains to be had from having a strong response to inflation. It is again everywhere welfare-reducing to respond to the gap, with potentially large welfare losses associated with moderate gap response coefficients.\(^{24}\) It is also welfare-improving to respond to output growth, at least over a range, though the gains are modest. Relative to the model without capital, the gains from interest smoothing are smaller and the range of parameters over which smoothing is welfare-improving is not as large.

I have also considered a specification of the model in which adjustment costs are not to the growth rate of investment, as in Christiano, Eichenbaum, and Evans (2005), but rather to the level of investment, as in Hayashi (1982). The same exercise of varying the policy rule parameters one at a time yields similar conclusions, though the gains from responding to output growth, and the range over which doing so is welfare-increasing, are larger with the Hayashi (1982) specification. In the model without capital, the welfare gains from responding to output growth come through better anchoring expected inflation. Because of the “natural rate” property of the model, output growth declines in the period of adverse shocks, but turns positive immediately afterwards as output heads back to its pre-shock value. Responding to growth allows a central bank to cut interest rates in the immediate aftermath of a negative shock (when growth declines), while implicitly promising to raise rates in the near future when output growth turns positive, which has the effect of keeping expected inflation in check. Put another way, positive responses to output growth are most desirable when a decline (increase) in output growth signals a reversal in the sign of growth in the very near future – i.e. when the impulse responses of output to shocks are not “hump-shaped.” In the medium scale model, the inclusion of investment adjustment costs tends to lead to hump-shaped responses to shocks, which makes output growth positively autocorrelated. Thus a negative shock portends negative growth for several periods, and responding to growth therefore tends to be less effective at anchoring expected inflation through signaling future tight policies when growth turns positive. Level adjustment costs do not typically induce hump-shaped responses, so it is natural that responding to growth in the policy rule appears to be more beneficial under that specification.

### 3.1 Optimized Rules

Table 5 presents optimized policy rule coefficients for the medium scale model. The table is constructed analogously to Table 2 for the simpler New Keynesian model. The optimized values of the loss function are expressed as percentages of steady state consumption. Overall, unless requiring the policy rule to respond to the output gap, the magnitudes of the optimized loss functions

---

\(^{24}\)With capital in the model, there is some arbitrariness in how one defines the flexible price level of output, and hence how to compute the output gap. One could define the flexible price level of output as the amount of output that would obtain with flexible prices and wages given an inherited capital stock from the actual sticky price and wage economy, or as the level of output that would obtain given a hypothetical capital stock that would be inherited had prices and wages always been flexible. I use the latter definition.
amount to around 0.1 percent of steady state consumption, which is comparable to the simpler model without capital. Even though there is no straightforward quadratic loss function because of the presence of capital, I also present standard deviations of the output gap, price inflation, and wage inflation, which are the three things which enter a quadratic loss function for a version of the model with sticky prices and wages, but no capital (see, e.g., Erceg, Henderson, and Levin, 2000).

The optimized rule features a moderate to strong response to inflation, no interest smoothing, and no response to the output gap. Differently from the simpler model without capital and flexible wages, the optimal coefficient on inflation is lower than its upper bound in the optimization routine. The optimized response to output growth is about 0.33, which is similar to, but nevertheless slightly smaller, than in the model without capital. Restricting the response to growth to zero results in little welfare loss, with the rule compensating with a slightly smaller coefficient on inflation. Requiring a positive response to the output gap results in a large welfare loss relative to the unrestricted rule. Interestingly, requiring more interest smoothing results in a larger desired coefficient on output growth, which is different from the model without capital, where the optimal coefficient on output growth tends to move in the opposite direction of $\rho_i$.

Conditioning on particular shocks, it is optimal to have a strong response to output growth conditional on productivity and cost-push shocks, and no response to growth conditional on preference or wage markup shocks. With the exception of the wage markup shock, these are similar to what was found in the simpler model. Interestingly, the desired coefficient on inflation is very low conditional on productivity shocks. As shown by Erceg, Henderson, and Levin (2000), when both prices and wages are sticky, conditional on productivity shocks strict inflation targeting (which would amount to a large coefficient on inflation in the policy rule) does poorly because this leads to too much wage inflation; output gap targeting tends to do better. One can see this in the optimized policy rule coefficients, where the desired response to inflation is low and the coefficient on the gap high conditional on productivity shocks. The reason that this does not carry over to the more general case is that productivity shocks contribute very little to the overall welfare loss relative to the other shocks, for which higher responses to inflation, and low or no response to the gap, are optimal. I will return to this point in some more detail below in subsection 3.2.

Table 6 shows optimized policy rule coefficients for different values of the non-policy parameters of the model. The only case in which it is optimal to have a significant positive response to the output gap is when the variances of both the cost-push and wage markup shocks are both zero. With few exceptions, it is always optimal to have a moderate to strong response to inflation and a moderate positive response to output growth. The desired response to growth is increasing in the amount of price rigidity and decreasing in the magnitude of wage rigidity. If there is full indexation of both prices and wages to lagged inflation, then it is optimal to have a small response to inflation and no response to output growth. If only one of either prices or wages are fully indexed, but not the other, then it is again best to have a large response to inflation and a modest positive response to output growth. With no indexation at all it is optimal to respond positively to output growth and strongly to inflation. Positive steady state inflation does not have large effects on the optimal policy rule parameters.
The optimal coefficient on output growth is increasing in the coefficient of relative risk aversion, \( \sigma \), and increasing in the Frisch labor supply elasticity, \( \eta^{-1} \). The desired coefficient on output growth is decreasing in the amount of habit formation. The relationship between the magnitude of adjustment costs and desired coefficient on output growth is non-linear. With no adjustment costs, the optimal response coefficient to output growth is about 1. The desired response to growth declines quickly as \( \phi_i \) rises, but after a value of \( \phi_i \approx 2 \) the desired response to growth is increasing in the adjustment cost parameter. I also compute policy rule coefficients in which the adjustment cost is modeled in the level of the investment, as in Hayashi (1982). Consonant with the intuition discussed above, the optimal response to output growth is higher under this specification than with the Christiano, Eichenbaum, and Evans (2005) version of adjustment costs. The desired response to output growth is decreasing in \( \Psi_1 \), which governs the convexity of the utilization cost function. The optimal response to growth is decreasing in \( \epsilon_p \) and \( \epsilon_w \); higher values of either parameter imply lower steady state price or wage markups, respectively. The optimized policy rule coefficients are fairly similar to the baseline in a version of the model with Pigouvian subsidies to offset the steady state distortions associated with price- and wage-setting power.

In terms of shock processes, the optimal response to growth is increasing in the volatility of productivity shocks, decreasing in the volatility of preference shocks, increasing in the volatility of cost-push shocks, and decreasing in the volatility of wage markup shocks. These are similar patterns to the simpler model with no capital. As noted earlier, the only case in which it is optimal to have a strong positive response to the output gap is when there are no cost-push or wage markup shocks.\(^{25}\) Even without those shocks, it is optimal to respond positively to output growth in the policy rule, indeed more strongly so than in the parameterization where these shocks are important. The optimized coefficient on growth is increasing in the persistence of productivity, cost push shocks, and wage markup shocks, and decreasing in the persistence of preference shocks. The latter is the reverse pattern from what obtains in the model without capital.

Though not shown in a table or figure, the basic conclusions about policy rule coefficients and the incidence of hitting the zero lower bound carry over from the simpler model without capital. The inclusion of capital in the model reduces the probability of hitting the zero lower bound for any specification of monetary policy – whereas the probability of interest rates hitting zero in the basic model without capital was about 5 percent for a rule with just a response to inflation, in the medium scale model this probability is less than 2 percent. As in the simpler model without capital, the probability of hitting the zero lower bound is decreasing in the policy rule coefficient on output growth and increasing in the response to the gap. Moderate responses to the gap can lead to substantial increases in the probability of hitting the zero lower bound if cost-push and wage markup shocks are important.

Finally, I also consider different specifications of the policy rule itself, though these are only discussed qualitatively in the text. First, conclusions about optimal policy rule parameters are not greatly affected by considering forward- or backward-looking versions of the rule. It remains

\(^{25}\)For the case in which there are cost-push shocks, but no wage markup shocks, the optimal response to the gap is positive but close to zero at 0.02.
non-optimal to respond to the output gap and to have positive response coefficients to output growth. A difference rule achieves lower welfare the partial adjustment specification, even without the presence of monetary policy shocks, and still features a strong positive response to output growth, both of which are different from the simpler model without capital.

3.2 Inefficient Shocks and Optimal Rules

In both the simpler model without capital as well as in the medium scale model, the structure of an optimal interest rate rule depends on the presence, or lack thereof, of cost-push and wage markup shocks. These shocks are sometimes referred to as “inefficient shocks” in that they are interpreted as shocks to market power, which affect the actual equilibrium but would not affect a hypothetical efficient equilibrium. With these “inefficient” shocks included in the medium-scale model, it is best to have a strong response to inflation and no response to the output gap. If these shocks are absent, in contrast, it is optimal to have a large response to the output gap. The desirability of responding to output growth is largely independent of the presence of these shocks.

In Table 7, I present average welfare losses, both when all shocks are included in the model as well as conditional on each shock individually, for four different specifications of the interest rate rule. The first, labeled “Optimal Rule,” uses the optimized policy rule coefficients reported in Table 5 and presents the same welfare loss statistics that can be found in that table. The second column, labeled “Optimal w/ $\phi_x = 0$,” presents average welfare losses for optimized policy rules under the restriction that there be no response to the gap.

The third and fourth columns consider “inflation targeting” and “gap targeting.” For inflation targeting, I use an interest rate rule with $\phi_\pi = 100$ and $\rho_i = \phi_x = \phi_{\Delta y} = 0$; for the gap targeting rule I set the policy parameters to $\phi_\pi = 1.01$, $\phi_x = 100$, and $\rho_i = \phi_{\Delta y} = 0$. These specifications result in essentially zero volatility in inflation or the output gap, respectively.

Focusing on the case in which all shocks are included, one can see that pure inflation targeting does poorly, with the welfare loss about twice as big relative to the optimal policy rule for both adjustment cost specifications (e.g. about 0.2 percent of consumption, instead of 0.1). Output gap targeting does significantly worse – gap targeting results in a welfare loss amounting to around 6 percent of steady state consumption. The only shock conditional on which inflation targeting performs well is the wage-markup shock; conditional on all other shocks inflation targeting performs significantly worse than the optimized policy rule. Gap targeting does well conditional on productivity and preference shocks. It also performs well when cost-push and wage markup shocks are absent. Gap targeting leads to very large welfare losses conditional on cost-push and wage markup.

---

26 The second column is redundant for cases in which there is no desired response to the gap in the unrestricted optimization; e.g. when all shocks are present and conditional on cost-push and wage markup shocks. The restricted optimal parameters conditional on just productivity shocks are $\rho_i = 0$, $\phi_x = 2.5$, and $\phi_{\Delta y} = 0$; conditional on preference shocks; and are $\rho_i = 0$, $\phi_x = 2.5$, and $\phi_{\Delta y} = 0.91$ when there are no cost-push or wage markup shocks.

27 It is worth noting that in a couple of cases inflation or gap targeting result in better welfare than the optimal rule. This results because in the optimization I restrict $\phi_x \leq 2.5$ and $\phi_{\Delta y} \leq 2.5$, whereas in inflation or gap targeting these coefficients are set to 100.
shocks, however. The fact that gap targeting does well, and inflation targeting poorly, conditional on productivity and preference shocks is a well-known result dating back to Erceg, Henderson, and Levin (2000), who first demonstrate this result in a model with sticky prices and wages, but without capital or shocks to either the price or wage Phillips Curves.

In the column of Table 7 labeled “Optimal w/ $\phi_x = 0$,” I present average welfare losses for optimized interest rate rules under the restriction that there be no response to the gap. For cases in which there is no desired response to the gap, such as when all shocks are included in the model, this column is obviously redundant. But for cases in which it is optimal to respond to the gap in the policy rule (conditional on productivity or preference shocks, or in the model without cost-push and wage markup shocks altogether), it is instructive to look at the welfare losses that obtain by restricting the rule to not react to the output gap. Productivity and preference shocks typically generate small welfare losses – the optimized rule which reacts to the output gap results in a welfare loss amounting to about 0.01 percent of steady state consumption. Restricting the policy rule to not react to the output gap results in lower welfare by up to a factor of two. A factor of two seems large, but upon closer inspection one sees that the absolute welfare loss from not responding to the gap is actually quite small – conditional on productivity shocks, for example, not reacting to the gap in the policy rule adds less than 0.01 percent to the welfare loss from wage and price rigidity. In other words, if it is beneficial to respond to the gap in the policy rule, then the costs of not doing so are quite small.

The desirability of targeting the output gap in the monetary policy rule thus hinges on how important cost-push and wage markup shocks are. The quantitative significance of these shocks has been the matter of some heated debate. In Smets and Wouters (2007), cost-push and wage markup shocks account for some 30-50 percent of output fluctuations at business cycle frequencies. Based on these numbers, positive response coefficients to the output gap could have very large negative welfare effects, as it does in my baseline parameterization. Chari, Kehoe, and McGrattan (2009), among others, have been vocal critics of this finding, arguing that cost-push and wage markup shocks are dubiously structural. In a recent contribution, Justiniano, Primiceri, and Tambalotti (2013), estimate a DSGE model similar to Smets and Wouters (2007), and find that wage markup shocks are relatively unimportant.\(^{28}\) This has the normative implication that there is not much of a tradeoff between gap and inflation stabilization, and that optimal monetary policy would come close to stabilizing the output gap – i.e. optimal and potential output move closely with one another.\(^{29}\)

Given the uncertainty concerning the importance of price and wage markup shocks, how ought

\[^{28}\] Their finding that wage markup shocks are unimportant obtains for two reasons. First, Justiniano, Primiceri, and Tambalotti (2013) have a different approach to wage measurement, which ends up attributing most of the movements that would be identified as wage markup shocks to measurement error. Second, they place a different prior on the importance of fluctuations in market power relative to a time-varying preference for leisure in their Bayesian estimation.

\[^{29}\] These authors compute optimal monetary policy by finding the time path of interest rates that would maximize welfare – in other words, their optimal policy is a Ramsey optimal policy, not based on a relatively simple interest rate rule. In their estimated model, the gap between optimal and potential output varies very little, though the observed movement in the output gap from their model based on a Taylor rule moves quite a lot, suggesting that historical policy has been far from optimal. What drives this non-optimality in their policy rule is not the policy rule itself, but rather estimated time-variation in the inflation target, which manifests itself as a monetary policy shock.
one to interpret the design of optimal interest rate rules, and in particular the desirability of targeting the output gap? The potential gains from responding to the output gap in the absence of these shocks are quite small, amounting to less than 0.02 percent of steady state consumption when these shocks are altogether absent. In contrast, the potential losses from targeting the gap are quite large if these shocks are important. Rows labeled “‘Small’ wage markup / cost-push shocks” in Table 7 show welfare losses from different policy rules when the cost-push and wage markup shocks are made considerably smaller than in the baseline, with innovation standard deviations of one-fifth their benchmark values. At this magnitude these two shocks together account for about five percent of the unconditional variance of output in the model (as opposed to close to 50 percent in the baseline calibration based on Smets and Wouters, 2007). With these magnitudes for the two shocks, the welfare loss from an optimized policy rule amounts to about 0.02 percent of steady state consumption, there is no desired response to the output gap, and the optimized response to output growth is higher than in the baseline.\footnote{The optimal policy parameters conditional on small wage and price markup shocks are $\rho_i = 0$, $\phi_{\pi} = 2.31$, $\phi_x = 0$, and $\phi_{\Delta y} = 0.83$.} Even with these shocks being relatively unimportant, there are large welfare losses from strong responses to the gap, with pure gap targeting resulting in losses on the order of 0.25 percent of steady state consumption, about ten times worse than the optimized rule. Echoing the findings from the simpler New Keynesian model (see, e.g., Figure 6), even if cost-push and wage markup shocks are rather unimportant (though nevertheless not altogether absent), responding strongly to the output gap tends to reduce welfare.

3.3 Historical Fed Reaction Functions, Welfare, and Determinacy

The analysis in this paper has shown that responding to the gap in a policy rule can significantly reduce welfare, while targeting output growth may be welfare-improving. How do these normative results square with a positive description of actual central bank practice in the US? And how do they relate to the important issue of equilibrium determinacy?

Ever since Taylor (1993), estimation of interest rate rules has been an important area of empirical research in macroeconomics. Much of this work has sought to attribute the decline in volatility since the early 1980s to improved monetary policy. Clarida, Gali, and Gertler (2000), for example, estimate a version of (7) and find an increased response coefficient to inflation, but do not jointly consider responses to both output growth and the gap. Smets and Wouters (2007) estimate the parameters of equation (7), allowing for a reaction to both the gap and growth, in a Bayesian structural estimation of a medium scale DSGE model. Splitting the sample to pre- and post-Volcker, they find that monetary policy post-1984 has been characterized by a stronger response to inflation, a lower response to the output gap, and essentially the same response to output growth. Coibion and Gorodnichenko (2011b) estimate a version of the policy rule also allowing for responses to both growth and the gap. They find that the post-Volcker period has been characterized by a smaller response coefficient to the gap and a significantly larger reaction to output growth. The normative analysis in this paper suggests that this empirically observed movement away from
targeting the gap toward focusing on growth has likely been welfare-improving.

Coibion and Gorodnichenko (2011b) emphasize another channel by which this shift from gap to growth targeting might have been beneficial: by making it more likely that there exists a determinate equilibrium. It is well-known that interest rate rules must react sufficiently to endogenous variables like inflation in order for there to exist a unique rational expectations equilibrium. In a basic New Keynesian model with no trend inflation, determinacy is more likely the larger are the long run response coefficients to both inflation and the output gap. They demonstrate that, once taking into account positive trend inflation, responding to the output gap in the policy rule can be de-stabilizing, while reacting to output growth can help induce determinacy. This result dovetails nicely with the conclusions in the present paper: shifting from a focus on the gap to growth in the interest rate rule is likely to be welfare-improving conditional on being in a determinate equilibrium and is also likely to help ensure that such a determinate equilibrium exists in the first place.

4 Conclusion

Simple interest rate rules have become a ubiquitous feature of models of the business cycle with monetary non-neutralities. Though there is substantial agreement that such rules should react strongly to inflation, there appears to be no practical consensus on what measure of economic activity, if any, ought to appear in these rules.

This paper has sought to provide an answer to this unresolved question. Two principle conclusions emerge. First, if “inefficient” shocks to the price and/or wage Phillips curves are important, then responding to the output gap can lead to substantial welfare losses. Second, moving interest rates in reaction to output growth often has beneficial welfare effects. Several authors have previously argued in favor of targeting growth over the gap on the grounds that the gap is difficult to observe in real time (e.g. McCallum, 2001; Orphanides, 2002; and Orphanides and Williams, 2006). The results in this paper suggest that it is likely better to respond to output growth than the gap, even if the latter is observed perfectly.

The results of this paper are not merely of interest for academic discussions of what kind of rule to write down in a DSGE model. Rather, they are of critical importance for understanding desirable policy responses in the current zero lower bound environment. As shown in Figure 1, while the implied target interest rate under the gap version of the rule is negative, the current desired interest rate under a growth rate policy rule is actually positive. Since the results of this paper suggest that the growth rate rule performs better from a welfare perspective in normal times, it may be that the current Federal Reserve policy of keeping the Federal Funds rate at or near zero may actually be too stimulative.
References


Table 1: Benchmark Parameter Values

<table>
<thead>
<tr>
<th>No capital:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
</tr>
<tr>
<td>$\eta = 1$</td>
</tr>
<tr>
<td>$\theta_p = 0.75$</td>
</tr>
<tr>
<td>$\epsilon_p = 5$</td>
</tr>
<tr>
<td>$\rho_z = 0.95$</td>
</tr>
<tr>
<td>$\rho_{up} = 0.95$</td>
</tr>
<tr>
<td>$s_{up} = 0.002$</td>
</tr>
<tr>
<td>$s_{\nu} = 0.02$</td>
</tr>
<tr>
<td>$\rho_{up} = 0.95$</td>
</tr>
<tr>
<td>$\rho_{\nu} = 0.95$</td>
</tr>
<tr>
<td>$s_{\nu} = 0.002$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Medium scale:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.3$</td>
</tr>
<tr>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>$\phi_i = 3.0$</td>
</tr>
<tr>
<td>$\gamma = 0.7$</td>
</tr>
<tr>
<td>$\theta_p = 0.65$</td>
</tr>
<tr>
<td>$\theta_w = 0.65$</td>
</tr>
<tr>
<td>$\zeta_p = 0.5$</td>
</tr>
<tr>
<td>$\zeta_w = 0.5$</td>
</tr>
<tr>
<td>$\epsilon_w = 5$</td>
</tr>
<tr>
<td>$\Psi_0 = \frac{1}{\beta} - (1 - \delta)$</td>
</tr>
<tr>
<td>$\Psi_1 = 0.05$</td>
</tr>
<tr>
<td>$\rho_{uw} = 0.95$</td>
</tr>
<tr>
<td>$s_{uw} = 0.00075$</td>
</tr>
<tr>
<td>$s_{ez} = 0.006$</td>
</tr>
<tr>
<td>$s_{up} = 0.00075$</td>
</tr>
<tr>
<td>$\pi^* = 0.00$</td>
</tr>
</tbody>
</table>

Notes: This table shows the benchmark values of the parameters in the basic model without capital in the upper panel. In the lower panel are the parameter values used in the medium scale model with capital. Unless otherwise indicated, parameter values not listed in the lower panel are equal to the values listed in the upper panel.

Table 2: Optimal Policy Rule Parameters

<table>
<thead>
<tr>
<th>Specification</th>
<th>$L$</th>
<th>std($x$)</th>
<th>std($\pi$)</th>
<th>$\rho_i$</th>
<th>$\phi_\pi$</th>
<th>$\phi_x$</th>
<th>$\phi_{\Delta y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Unrestricted</td>
<td>-0.100</td>
<td>2.40</td>
<td>0.22</td>
<td>0.52</td>
<td>2.50</td>
<td>0.00</td>
<td>0.45</td>
</tr>
<tr>
<td>(b) $\phi_{\Delta y} = 0$</td>
<td>-0.101</td>
<td>2.41</td>
<td>0.22</td>
<td>0.78</td>
<td>2.50</td>
<td>0.00</td>
<td>n/a</td>
</tr>
<tr>
<td>(c) $\rho_i = 0$</td>
<td>-0.101</td>
<td>2.41</td>
<td>0.21</td>
<td>n/a</td>
<td>2.50</td>
<td>0.00</td>
<td>0.61</td>
</tr>
<tr>
<td>(d) $\phi_x = 0.5$</td>
<td>-0.287</td>
<td>2.16</td>
<td>0.86</td>
<td>0.79</td>
<td>2.50</td>
<td>0.00</td>
<td>0.61</td>
</tr>
<tr>
<td>(e) Only prod. shocks</td>
<td>-0.002</td>
<td>0.13</td>
<td>0.07</td>
<td>0.00</td>
<td>2.50</td>
<td>2.50</td>
<td>1.07</td>
</tr>
<tr>
<td>(f) Only cost-push shocks</td>
<td>-0.091</td>
<td>2.35</td>
<td>0.16</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>1.18</td>
</tr>
<tr>
<td>(g) Only pref. shocks</td>
<td>-0.005</td>
<td>0.04</td>
<td>0.04</td>
<td>0.87</td>
<td>2.50</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>(h) Forward-looking rule</td>
<td>-0.101</td>
<td>2.43</td>
<td>0.20</td>
<td>0.44</td>
<td>2.50</td>
<td>0.00</td>
<td>0.87</td>
</tr>
<tr>
<td>(i) Backward-looking rule</td>
<td>-0.099</td>
<td>2.40</td>
<td>0.21</td>
<td>0.00</td>
<td>2.00</td>
<td>0.00</td>
<td>0.24</td>
</tr>
<tr>
<td>(j) Difference rule, $\rho_i = 1$</td>
<td>-0.093</td>
<td>2.41</td>
<td>0.10</td>
<td>n/a</td>
<td>0.50</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>(k) Super-inertial rule, $\rho_i &gt; 1$</td>
<td>-0.090</td>
<td>2.43</td>
<td>0.09</td>
<td>1.08</td>
<td>0.50</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>(l) Partial-adjustment w/ policy shocks</td>
<td>-0.101</td>
<td>2.41</td>
<td>0.21</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.60</td>
</tr>
<tr>
<td>(m) Super-inertial w/ policy shocks</td>
<td>-0.126</td>
<td>2.44</td>
<td>0.35</td>
<td>1.00</td>
<td>0.50</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note: This table shows optimized values for the policy rule coefficients. Also included are the minimized value of the objective function and the standard deviations of inflation and the output gap (all multiplied by 100). Row (a) shows optimized parameter values for an unrestricted version of the Taylor rule. Rows (b)-(d) show optimized parameter values for different restricted versions of the rule. Rows (e)-(g) show optimized policy rule conditioning on one shock at a time (e.g. the other shock standard deviations are set to zero). Rows (h)-(k) show optimized parameter values for different versions of the rule. Rows (l)-(m) show optimized policy rule coefficients for the partial adjustment and super-inertial specifications when there are monetary policy shocks with standard deviation of 0.0033. With the exception of conditioning on particular shocks in rows (e)-(g), parameters of the model are set at their benchmark values given in Table 1.
Table 3: Optimal Policy Rule Parameter Values: Robustness

<table>
<thead>
<tr>
<th>Parameter Difference</th>
<th>$\rho_{i}$</th>
<th>$\phi_{\pi}$</th>
<th>$\phi_{x}$</th>
<th>$\phi_{\Delta y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{p} = 0.9$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$1.69$</td>
</tr>
<tr>
<td>$\theta_{p} = 0.6$</td>
<td>$0.39$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.05$</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>$0.73$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.27$</td>
</tr>
<tr>
<td>$\sigma = 5$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$1.52$</td>
</tr>
<tr>
<td>$\eta = 0$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.83$</td>
</tr>
<tr>
<td>$\eta = 5$</td>
<td>$0.31$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.15$</td>
</tr>
<tr>
<td>$\epsilon_{p} = 3$</td>
<td>$0.72$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.46$</td>
</tr>
<tr>
<td>$\epsilon_{p} = 15$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.48$</td>
</tr>
<tr>
<td>$\beta = 0.999$</td>
<td>$0.44$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.51$</td>
</tr>
<tr>
<td>$\beta = 0.96$</td>
<td>$0.59$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.33$</td>
</tr>
<tr>
<td>$s_{z} = 0.02$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.54$</td>
</tr>
<tr>
<td>$s_{z} = 0.00$</td>
<td>$0.79$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.42$</td>
</tr>
<tr>
<td>$s_{up} = 0.004$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.94$</td>
</tr>
<tr>
<td>$s_{up} = 0.000$</td>
<td>$0.87$</td>
<td>$2.50$</td>
<td>$2.50$</td>
<td>$0.03$</td>
</tr>
<tr>
<td>$s_{up} = 0.001$</td>
<td>$0.60$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.16$</td>
</tr>
<tr>
<td>$s_{\nu} = 0.04$</td>
<td>$0.73$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.19$</td>
</tr>
<tr>
<td>$s_{\nu} = 0.01$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.80$</td>
</tr>
<tr>
<td>$\rho_{z} = 0.99$</td>
<td>$0.80$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.22$</td>
</tr>
<tr>
<td>$\rho_{z} = 0.90$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.68$</td>
</tr>
<tr>
<td>$\rho_{u} = 0.99$</td>
<td>$0.41$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.48$</td>
</tr>
<tr>
<td>$\rho_{u} = 0.90$</td>
<td>$0.66$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.29$</td>
</tr>
<tr>
<td>$\rho_{\nu} = 0.99$</td>
<td>$0.00$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.66$</td>
</tr>
<tr>
<td>$\rho_{\nu} = 0.90$</td>
<td>$0.64$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.37$</td>
</tr>
</tbody>
</table>

Notes: This table shows optimized values for policy rule coefficients over different values of the non-policy parameters of the model. Unless otherwise stated, parameters are set at their values given in Table 1.

Table 4: Optimal Policy Rule Parameter Values: Arbitrary Welfare Weights

<table>
<thead>
<tr>
<th>Weight on Output Gap</th>
<th>std($x$)</th>
<th>std($\pi$)</th>
<th>$\rho_{i}$</th>
<th>$\phi_{\pi}$</th>
<th>$\phi_{x}$</th>
<th>$\phi_{\Delta y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.10$</td>
<td>$2.37$</td>
<td>$0.24$</td>
<td>$0.75$</td>
<td>$2.50$</td>
<td>$0.00$</td>
<td>$0.49$</td>
</tr>
<tr>
<td>$\lambda = 0.25$</td>
<td>$2.30$</td>
<td>$0.35$</td>
<td>$0.90$</td>
<td>$2.50$</td>
<td>$0.04$</td>
<td>$0.70$</td>
</tr>
<tr>
<td>$\lambda = 0.50$</td>
<td>$2.16$</td>
<td>$0.58$</td>
<td>$0.95$</td>
<td>$2.50$</td>
<td>$0.16$</td>
<td>$0.55$</td>
</tr>
<tr>
<td>$\lambda = 1.00$</td>
<td>$2.03$</td>
<td>$0.85$</td>
<td>$0.97$</td>
<td>$2.50$</td>
<td>$0.34$</td>
<td>$0.17$</td>
</tr>
<tr>
<td>$\lambda = 2.00$</td>
<td>$1.93$</td>
<td>$1.15$</td>
<td>$0.97$</td>
<td>$2.50$</td>
<td>$0.59$</td>
<td>$0.02$</td>
</tr>
</tbody>
</table>

Notes: This table shows optimized parameters of the baseline policy rule for different arbitrary weights on the variance of the output gap in the loss function.
Table 5: Optimal Policy Rule Parameters: Medium Scale Model

<table>
<thead>
<tr>
<th>Specification</th>
<th>$L$</th>
<th>std($x$)</th>
<th>std($\pi$)</th>
<th>std($\pi^w$)</th>
<th>$\rho_i$</th>
<th>$\phi_\pi$</th>
<th>$\phi_x$</th>
<th>$\phi_{\Delta u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Unrestricted</td>
<td>-0.105</td>
<td>3.02</td>
<td>0.35</td>
<td>0.24</td>
<td>0.00</td>
<td>2.45</td>
<td>0.00</td>
<td>0.32</td>
</tr>
<tr>
<td>(ii) $\phi_{\Delta u} = 0$</td>
<td>-0.106</td>
<td>3.05</td>
<td>0.34</td>
<td>0.24</td>
<td>0.00</td>
<td>2.38</td>
<td>0.00</td>
<td>n/a</td>
</tr>
<tr>
<td>(iii) $\phi_x = 0.5$</td>
<td>-0.345</td>
<td>2.59</td>
<td>0.95</td>
<td>0.90</td>
<td>0.84</td>
<td>2.50</td>
<td>0.00</td>
<td>0.52</td>
</tr>
<tr>
<td>(iv) $\rho_i = 0.75$</td>
<td>-0.112</td>
<td>3.02</td>
<td>0.31</td>
<td>0.24</td>
<td>n/a</td>
<td>2.50</td>
<td>0.00</td>
<td>0.54</td>
</tr>
<tr>
<td>(v) Only prod. shocks</td>
<td>-0.009</td>
<td>0.16</td>
<td>0.40</td>
<td>0.17</td>
<td>0.20</td>
<td>1.01</td>
<td>2.50</td>
<td>1.29</td>
</tr>
<tr>
<td>(vi) Only pref. shocks</td>
<td>-3.7e-4</td>
<td>0.02</td>
<td>0.06</td>
<td>0.04</td>
<td>0.93</td>
<td>2.50</td>
<td>2.26</td>
<td>0.00</td>
</tr>
<tr>
<td>(vii) Only cost-push shocks</td>
<td>-0.007</td>
<td>0.83</td>
<td>0.24</td>
<td>0.12</td>
<td>0.00</td>
<td>1.66</td>
<td>0.00</td>
<td>2.50</td>
</tr>
<tr>
<td>(viii) Only wage markup shocks</td>
<td>-0.071</td>
<td>2.88</td>
<td>0.07</td>
<td>0.14</td>
<td>0.74</td>
<td>2.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table shows optimized values for the policy rule coefficients in the medium scale model. Also included are the minimized value of the objective function and the standard deviations of price inflation, wage inflation, and the output gap (all multiplied by 100). Parameters of the model are set at their benchmark values given in Table 1.

Table 6: Optimal Policy Rule Parameters: Medium Scale Model, Robustness

<table>
<thead>
<tr>
<th>Specification</th>
<th>$\rho_i$</th>
<th>$\phi_\pi$</th>
<th>$\phi_x$</th>
<th>$\phi_{\Delta u}$</th>
<th>$\rho_i$</th>
<th>$\phi_\pi$</th>
<th>$\phi_x$</th>
<th>$\phi_{\Delta u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_p = 0.50$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.20</td>
<td>$\epsilon_p = 3$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_w = 0.50$</td>
<td>0.00</td>
<td>2.27</td>
<td>0.00</td>
<td>0.60</td>
<td>$\epsilon_w = 3$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_p = \theta_w = 0.50$</td>
<td>0.00</td>
<td>2.13</td>
<td>0.00</td>
<td>0.49</td>
<td>$\epsilon_p = \epsilon_w = 3$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_p = 0.75$</td>
<td>0.00</td>
<td>2.19</td>
<td>0.00</td>
<td>0.69</td>
<td>$\epsilon_w = 15$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_w = 0.75$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.02</td>
<td>$\epsilon_p = \epsilon_w = 3$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_p = \theta_w = 0.75$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.22</td>
<td>$\epsilon_p = \epsilon_w = 15$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\zeta_p = \zeta_w = 1$</td>
<td>0.00</td>
<td>1.14</td>
<td>0.00</td>
<td>0.00</td>
<td>$s_z = 0.012$</td>
<td>0.00</td>
<td>2.32</td>
<td>0.00</td>
</tr>
<tr>
<td>$\zeta_p = 0$, $\zeta_w = 1$</td>
<td>0.34</td>
<td>2.50</td>
<td>0.00</td>
<td>0.23</td>
<td>$s_z = 0.00$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\zeta_p = 1$, $\zeta_w = 0$</td>
<td>0.32</td>
<td>2.50</td>
<td>0.00</td>
<td>0.30</td>
<td>$s_v = 0.04$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma = 0.005$</td>
<td>0.00</td>
<td>2.36</td>
<td>0.00</td>
<td>0.20</td>
<td>$s_{uw} = 0.0015$</td>
<td>0.00</td>
<td>2.08</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.00</td>
<td>2.33</td>
<td>0.00</td>
<td>0.17</td>
<td>$s_{up} = 0.0000$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\eta = 0$</td>
<td>0.98</td>
<td>2.50</td>
<td>0.00</td>
<td>0.45</td>
<td>$s_{uw} = 0.0000$</td>
<td>0.00</td>
<td>2.27</td>
<td>0.02</td>
</tr>
<tr>
<td>$\eta = 5$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.45</td>
<td>$s_{uw} = 0.0000$</td>
<td>0.45</td>
<td>2.50</td>
<td>2.50</td>
</tr>
<tr>
<td>$\gamma = 0.9$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.06</td>
<td>$\rho_z = 0.99$</td>
<td>0.00</td>
<td>2.21</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma = 0.0$</td>
<td>0.00</td>
<td>2.06</td>
<td>0.00</td>
<td>0.58</td>
<td>$\rho_z = 0.90$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi_k = 0$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.96</td>
<td>$\rho_v = 0.99$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi_k = 5$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.37</td>
<td>$\rho_v = 0.90$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Cap. Adjust Cost, $\phi_k = 3$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>1.09</td>
<td>$\rho_{up} = 0.99$</td>
<td>0.00</td>
<td>2.43</td>
<td>0.00</td>
</tr>
<tr>
<td>$\Psi_1 = 100$</td>
<td>0.00</td>
<td>2.46</td>
<td>0.00</td>
<td>0.03</td>
<td>$\rho_{up} = 0.90$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>$\Psi_1 = 0.01$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
<td>0.36</td>
<td>$\rho_{uw} = 0.99$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
<tr>
<td>w/ Pigouvian taxes</td>
<td>0.00</td>
<td>2.08</td>
<td>0.00</td>
<td>0.14</td>
<td>$\rho_{uw} = 0.90$</td>
<td>0.00</td>
<td>2.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table shows optimized values for policy rule coefficients over different values of the non-policy parameters of the medium scale model. Unless otherwise stated, parameters are set at their values given in Table 1.
Table 7: Welfare Losses Under Different Policies, Medium Scale Model

<table>
<thead>
<tr>
<th></th>
<th>Optimal</th>
<th>Optimal w/ $\phi_x = 0$</th>
<th>Inf. Target</th>
<th>Gap Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) All shocks</td>
<td>-0.106</td>
<td>-0.106</td>
<td>-0.226</td>
<td>-6.073</td>
</tr>
<tr>
<td>(ii) Only prod. shocks</td>
<td>-0.008</td>
<td>-0.014</td>
<td>-0.111</td>
<td>-0.008</td>
</tr>
<tr>
<td>(iii) Only pref. shocks</td>
<td>-3.8e-04</td>
<td>-3.8e-04</td>
<td>-9.8e-04</td>
<td>-1.4e-04</td>
</tr>
<tr>
<td>(iv) Only cost-push shocks</td>
<td>-0.007</td>
<td>-0.007</td>
<td>-0.048</td>
<td>-0.428</td>
</tr>
<tr>
<td>(v) Only wage markup shocks</td>
<td>-0.071</td>
<td>-0.071</td>
<td>-0.065</td>
<td>-5.581</td>
</tr>
<tr>
<td>(vi) No wage markup / cost-push shocks</td>
<td>-0.013</td>
<td>-0.016</td>
<td>-0.113</td>
<td>-0.008</td>
</tr>
<tr>
<td>(vii) “Small” wage markup / cost-push shocks</td>
<td>-0.020</td>
<td>-0.020</td>
<td>-0.117</td>
<td>-0.238</td>
</tr>
</tbody>
</table>

Notes: This table shows optimized values for the welfare loss under different policies. I show results when all shocks are set at their benchmark values from Table 1, as well as conditional on one shock at a time. The loss for the column titled “Optimal Rule” is the same as that shown in Table 5. For the column titled “Optimal w/ $\phi_x = 0$,” I find optimal parameter values for $\phi_i$, $\phi_{\pi}$, and $\phi_{\Delta y}$ conditional on the restriction that $\phi_x = 0$. For the column titled “Inf. Target” I calculate the loss based on a policy rule with $\phi_{\pi} = 100$ and $\rho_i = \phi_x = \phi_{\Delta y} = 0$. For the column titled “Gap Target” I calculate the loss based on a policy rule with $\phi_{\pi} = 1.01$, $\phi_x = 100$, and $\rho_i = \phi_{\Delta y} = 0$. For the rows titled “‘small’ wage markup / cost-push shocks,” I set the standard deviations of both the cost-push and wage markup shocks to 0.00015.

Figure 1: Actual and Target FFR Under Different Hypothetical Rules

Notes: The black line plots the actual effective Federal Funds rate, average to a quarterly frequency and expressed at an annualized rate. The blue line plots the implied target funds rate under the rule $i_t = (1 - \rho_i)\bar{i} + \rho_i i_{t-1} + (1 - \rho_i)(\phi_{\pi}(\pi_t - \bar{\pi}) + \phi_x(x_t - \bar{x})$. $\pi_t$ is quarter-over-quarter inflation as defined by the GDP price deflator and $x_t$ is the CBO measure of the output gap. I use parameter values $\rho_i = 0.8$, $\phi_{\pi} = 1.5$, and $\phi_x = 0.5$. Variables with a “bar” denote sample averages over the period 1985-2006. The green line plots the implied target funds rate using a growth rule, where the $\phi_x(x_t - \bar{x})$ term is replaced by $\phi_{\Delta y} (\Delta y_t - \Delta \bar{y}_t)$, and I use a value of $\phi_{\Delta y} = 0.5$.
Figure 2: Welfare and Parameters

Note: This figure shows how welfare varies as the parameters of the policy rule are changed one at a time in the basic New Keynesian model, relative to a benchmark rule in which $\phi_{\pi} = 1.5$ and other parameters are set to zero.

Figure 3: Impulse Responses

Note: This figure plots impulse responses of the output gap (left column) and inflation (right column) to each of the three shocks in the basic New Keynesian model for three different policy rule specifications. The solid black line shows responses for a benchmark rule in which $\phi_{\pi} = 1.5$ and other parameters are set to zero. The dotted line shows responses when $\phi_x = 0.5$ ($\phi_{\pi} = 1.5$ and $\phi_{\Delta y} = 0$), while the dashed line shows responses with $\phi_{\Delta y} = 0.5$ ($\phi_{\pi} = 1.5$ and $\phi_x = 0$).
Figure 4: Optimal Parameter Configurations with Arbitrary Welfare Weights

Note: This figure plots the optimized policy rule coefficients for different, arbitrary values of \( \lambda \), the weight on the variance of the output gap in the loss function.

Figure 5: Frequency of Hitting ZLB

Note: This figure plots the frequency of the simulated nominal interest rate going below 0 in the baseline New Keynesian model with steady state inflation of 2 percent at an annual rate over different parameter configurations of the policy rule. The baseline policy rule is taken to have \( \phi_{\pi} = 1.5 \) and all other parameters at zero. The plots show the frequency of hitting the ZLB as the coefficients on output growth, the gap, and the lagged rate are varied one at a time relative to the baseline.
Figure 6: Cost-Push Shocks and Desirability of Responding to Output Gap

Notes: The left panel plots the combinations of $\rho_u$ and $s_u$ (the persistence and innovation standard deviation of the cost-push process, $u_t$) for which welfare is unaffected by a movement from $\phi_x = 0$ to $\phi_x = 0.05$, relative to a baseline Taylor rule in which $\phi_{\pi} = 1.5$ and all other parameters are zero. The remainder of the parameters in the model are set at their benchmark values from Table 1. The plot can be thought of analogously to an indifference map – for $(\rho_u, s_u)$ pairs below the curve, welfare is increasing in $\phi_x$; above the curve, welfare is decreasing in the response coefficient on the output gap. The right panel plots the overall percentage contribution of cost-push shocks to the unconditional variance of output for the $(\rho_u, s_u)$ pairs on in the indifference map in the left panel.

Figure 7: Welfare and Parameters, Medium Scale Model

Note: This figure shows how welfare varies as the parameters of the policy rule are changed one at a time in the medium scale New Keynesian model, relative to a benchmark rule in which $\phi_{\pi} = 1.5$ and other parameters are set to zero.
The Appendix lays out the decision problems of a medium scale DSGE model with both price and wage rigidity and several real frictions. The model is similar to those in Smets and Wouters (2007) or Christiano, Eichenbaum, and Evans (2005). Simpler versions of the model (e.g. the textbook New Keynesian model without capital) can be considered special cases of the medium scale model under parameter restrictions. The model abstracts from trend growth.

A The Model

The model consists of several actors: a union, who combines differentiated labor input from households and sells it to firms; households, who consume, supply differentiated labor, save through bonds, and accumulate capital; a representative final good firm, who aggregates intermediate outputs to produce a final good available for consumption or investment; intermediate good firms, who produce output using capital and labor and set prices; and a government, which sets interest rates.

A.1 Union

There is a competitive union that combines differentiated labor input from a continuum of households indexed by \( h \in (0, 1) \). Labor input available to lease to firms, \( N_{d,t} \), is:

\[
N_{d,t} = \left( \int_0^1 N_t(h)^{\frac{1}{\epsilon_w} - 1} \, dh \right)^{\frac{1}{\epsilon_w - 1}}
\]

The parameter \( \epsilon_w > 1 \) is the elasticity of substitution between different types of labor. The union takes both the real wage on labor it sells to firms, \( W_t \), and the real wage charged by households, \( W_t(h) \), as given. Profit maximization gives rise to a downward-sloping demand curve for each type of labor and an aggregate real wage index:

\[
N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\epsilon_w} N_{d,t}
\]

\[
W_t = \left( \int_0^1 W_t(h)^{1-\epsilon_w} \, dh \right)^{\frac{1}{1-\epsilon_w}}
\]

A.2 Households

There are a continuum of households, indexed by \( h \in (0, 1) \). They get utility from consumption and disutility from working. They supply differentiated labor to unions as described above. As long as \( \epsilon_w < \infty \), households have some market-power in wage setting. As in Erceg, Henderson, and Levin (2000), households are not freely able to adjust their wages every period, with \( 1 - \theta_w \) being the probability in each period that a household can adjust its wage. Non-updating households
can partially index nominal wages to inflation at $\zeta_w \in (0, 1)$. Households can save through non-contingent one-period bonds or capital. Households can also make a capital utilization decision, and rent capital services (the product of physical capital and utilization) to intermediate goods producers. Households have access to a full set of state contingent securities which insure them against idiosyncratic income risk. I abstract from money. The household problem is:

$$\max_{C_t(h), N_t(h), u_t(h), I_t(h)} E_0 \sum_{t=0}^{\infty} \beta^t \left( \nu_t \frac{(C_t(h) - \gamma C_{t-1}(h))^{1-\sigma}}{1-\sigma} - \Psi N_t(h)^{1+\eta} \right)$$

s.t.

$$C_t(h) + I_t(h) + \frac{B_{t+1}(h)}{P_t} + \int \Gamma_t(h) b_{t+1}(h) d\theta_{t+1} \leq W_t(h) N_t(h) + R_t u_t(h) K_t(h) - \left( \Psi_0(u_t(h) - 1) + \frac{\Psi_1}{2}(u_t(h) - 1)^2 \right) K_t(h) + (1+i_{t-1}) \frac{B_t(h)}{P_t} + b_t(h) + \Pi_t(h) + T_t(h)$$  \hspace{1cm} (A.3)

$$K_{t+1}(h) = \left( 1 - \frac{\phi_i}{2} \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t(h) + (1-\delta)K_t(h)$$  \hspace{1cm} (A.4)

$\beta$ is a discount factor, $\sigma$ and $\eta$ are positive constants, and $\gamma \in (0, 1)$ is a measure of internal habit formation. $\nu_t$ is a (common) preference shock to utility from consumption. In the resource constraint, (A.3), $C_t(h)$ is consumption, $I_t(h)$ is investment in new physical capital, and $P_t$ is the nominal price of goods. $W_t(h)$ is the real wage that household $h$ charges for its labor, $N_t(h)$. Households lease capital services, $K_{s,t}(h) = u_t(h) K_t(h)$, to firms at competitive rental rate $R_t$. There is a convex resource cost of utilization governed by the parameters $\Psi_0$ and $\Psi_1$. Upper-case $B_t(h)$ are holdings of one-period nominal bonds. Lower-case $b_t(h)$ denote holdings of state-contingent claims priced at $\Gamma$; events are indexed by $\vartheta$. It is assumed that there exist a full set of state contingent claims that can completely insure households against idiosyncratic risk arising from earning different wages. $i_t$ is the nominal interest rate on aggregate one-period bonds, and $\Pi_t$ is real profit distributed back to households. $T_t(h)$ denotes lump sum transfers from the government back to the household. In (A.4), I follow Christiano, Eichenbaum, and Evans (2005) in having a convex adjustment cost to the growth rate of investment, governed by the parameter $\phi_i$.

Due to the presence of a complete set of state-contingent securities, households will only differ in their choices of labor and wages; along all other dimensions they will be identical. Optimality conditions for non-labor choices are (dropping $h$ subscripts):

$$\mu_t = \nu_t(C_t - \gamma C_{t-1})^{-\sigma} - \beta \gamma E_t \nu_{t+1}(C_{t+1} - \gamma C_t)^{-\sigma}$$  \hspace{1cm} (A.5)

$$\mu_t = \beta E_t \mu_{t+1}(1+i_t)(1+\pi_{t+1})^{-1}$$  \hspace{1cm} (A.6)

$$R_t = \Psi_0 + \Psi_1(u_t - 1)$$  \hspace{1cm} (A.7)

$$q_t = \beta E_t \frac{\mu_{t+1}}{\mu_t} \left( R_{t+1} u_{t+1} - \Psi_0(u_{t+1} - 1) - \frac{\Psi_1}{2}(u_{t+1} - 1)^2 + (1-\delta)q_{t+1} \right)$$  \hspace{1cm} (A.8)

$$1 = q_t \left( 1 - \frac{\phi_i}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_i \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_t}{I_t} \right) \right) + \beta E_t \frac{\mu_{t+1}}{\mu_t} \phi_i \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 q_{t+1}$$  \hspace{1cm} (A.9)
The first expression defines the marginal utility of consumption as $\mu_t$. The second is the standard Euler equation for bonds, and third is the first order condition for capital utilization. $q_t$ is the ratio of the multiplier on the accumulation equation to the multiplier on the resource constraint, and has the interpretation as the marginal value (measured in units of consumption) of an additional unit of installed capital.

The labor choice can be considered separately. Households are not freely able to adjust nominal wages each period; it is assumed that households have probability $1 - \theta_w$ of being allowed to update their nominal wage each period. Non-updating households can partially index their nominal wage to lagged aggregate inflation at $\zeta_w \in (0, 1)$. The problem of a household who gets to update its wage in period $t$ as:

$$\max_{W_t(h)} E_t \sum_{m=0}^{\infty} (\theta_w \beta)^m \left(-\psi N_t(h) \frac{1 + \eta}{1 + \eta} + \mu_{t+m} \prod_{n=1}^{m} \frac{(1 + \pi_{t+n-1})\zeta_w}{(1 + \pi_{t+n})} W_t(h) N_{t+m}(h)\right)$$

s.t.

$$N_{t+m}(h) = \left(\prod_{n=1}^{m} \frac{(1 + \pi_{t+n-1})\zeta_w}{(1 + \pi_{t+n})} W_t(h) W_{t+m}\right)^{-\epsilon_w} N_{d,t+m}$$

Here $\mu_{t+m}$ is the marginal utility of consumption. The first order condition is:

$$W_t^{\#_t+\epsilon_w \eta} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{L_t}{J_t} \exp(u_w^{\#_t}) \frac{\theta_w^{1+\epsilon_w \eta}}{1-\theta_w}$$ (A.10)

$$L_t = \psi W_t^{\epsilon_w (1+\eta)} N_{d,t}^{1+\eta} + \theta_w \beta (1 + \pi_t)^{-\zeta_w \epsilon_w (1+\eta)} E_t (1 + \pi_{t+1})^{-\epsilon_w (1+\eta)} L_{t+1}$$ (A.11)

$$J_t = \mu_t W_t^{\epsilon_w} N_{d,t} + \theta_w \beta (1 + \pi_t)^{\zeta_w (1-\epsilon_w)} E_t (1 + \pi_{t+1})^{-\epsilon_w - 1} J_{t+1}$$ (A.12)

The variable $u_w^{\#_t}$ is a “wage-markup” shock. Raising to the power $\frac{\theta_w (1+\epsilon_w \eta)}{1-\theta_w}$ is a normalization. Under zero steady state inflation (or with non-zero inflation and perfect wage indexation), these expressions can be linearized to yield a wage Phillips Curve:

$$\pi_t^{w} = \frac{(1-\theta_w)(1-\theta_w \beta)}{\theta_w (1+\epsilon_w \eta)} (mrs_t - w_t) + \beta E_t \pi_{t+1}^{w} + u_t^{w}$$ (A.13)

Here $mrs_t$ is the log deviation of the marginal rate of substitution from the deterministic steady state, e.g. $mrs_t = \phi \tilde{N}_t - \tilde{\mu}_t$, and $w_t$ is the log-deviation of the aggregate real wage from steady state.

### A.3 Final Good Firm

There is a competitive, representative firm that produces a final good available for consumption, $Y_t$. It is a composite of a continuum of intermediate goods, $Y_t(j)$, $j \in (0, 1)$:

$$Y_t = \left(\int_0^1 Y_t(j) \frac{e_p - 1}{e_p} dj\right)^{e_p} \frac{e_p}{e_p - 1}$$
The parameter $\epsilon_p > 1$ is the elasticity of substitution. The final good firm maximizes profits. The first order conditions are a demand curve for each intermediate good and an aggregate price index:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t$$

$$P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$$

(A.14)

(A.15)

A.4 Intermediate Goods Firms

There are a continuum of intermediate goods firms index by $j \in (0, 1)$. They produce output according to:

$$Y_t(j) = Z_t K_{s,t}(j)^\alpha N_{d,t}(j)^{1-\alpha}$$

$K_{s,t}(j)$ is the quantity of capital services (the product of physical capital and utilization) which firms rent from households. $N_{d,t}(j)$ is the quantity of labor demanded by firm $j$ at time $t$. $Z_t$ is a productivity disturbance common across firms. Firms behave atomistically and take real factor prices, $R_t$ and $W_t$, as well as aggregates, $P_t$ and $Y_t$, as given. Conditional on any price of its product, an intermediate producer will seek to minimize cost subject to producing as much as is demanded at its price:

$$\min_{K_{s,t}(j),N_{d,t}} W_t N_{d,t}(j) + R_t K_{s,t}(j)$$

s.t.

$$Z_t K_{s,t}(j)^\alpha N_{d,t}(j)^{1-\alpha} \geq \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t$$

Cost-minimization implies that all firms hire capital services and labor in the same ratio and all have the same real marginal cost, $mc_t$ (the multiplier on the constraint):

$$\frac{K_{s,t}}{N_{d,t}} = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t}$$

(A.16)

$$mc_t = \frac{W_t^{1-\alpha} R_t^\alpha}{Z_t} \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)$$

(A.17)

Given cost-minimization, an intermediate goods firm wants to pick its price, $P_t(j)$, to maximize the present discounted value of flow profits, $\Pi_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - mc_t Y_t(j)$. Firms are not freely able to adjust their price every period. Rather, following Calvo (1983), each period they face a fixed probability, $1 - \theta_p$, of being able to change their price. Non-updating firms can partially index prices to lagged aggregate inflation, $\pi_t = \frac{P_t}{P_{t-1}} - 1$, at $\zeta_p \in (0, 1)$. Updating firms discount future
profit flows by the probability of being stuck with a price as well as the stochastic discount factor of the household. The problem of an updating firm is:

$$\max_{P_t(j)} E_t \sum_{m=0}^{\infty} (\theta_p \beta)^m \mu_t \left( \prod_{n=1}^{M} (1 + \pi_{t+n-1})^{\epsilon_p} \frac{P_t(j)}{P_{t+m}} - m c_{t+m} \right) \left( \prod_{n=1}^{M} (1 + \pi_{t+n-1})^{\epsilon_p} \frac{P_t(j)}{P_{t+m}} \right) \epsilon_p Y_{t+m}$$

The first order condition can be simplified to an optimal reset price, $P_t^\#$. Because of the assumption of price-taking in factor markets, this reset price will be the same across all updating firms. Expressed in terms of reset price inflation, $1 + \pi_t^\# = \frac{P_t^\#}{P_{t-1}}$, we have:

$$1 + \pi_t^\# = \frac{\epsilon_p}{\epsilon_p - 1} (1 + \pi_t) \frac{X_t}{H_t} \exp \left( u_t^p \right)^{\frac{\theta_p}{1 - \theta_p}}$$  \hspace{1cm} (A.18)

$$X_t = \mu_t Y_t m c_t + \theta_p \beta (1 + \pi_t)^{-\epsilon_p} E_t (1 + \pi_{t+1})^{\epsilon_p} X_{t+1}$$  \hspace{1cm} (A.19)

$$H_t = \mu_t Y_t + \theta_p \beta (1 + \pi_t)^{\epsilon_p} E_t (1 + \pi_{t+1})^{\epsilon_p - 1} H_{t+1}$$  \hspace{1cm} (A.20)

The random variable $u_t^p$ in the expression does not directly show up in the firm pricing problem. Rather, it is considered a reduced-form “cost-push” shock. Raising to the power $\frac{\theta_p}{1 - \theta_p}$ is simply a normalization. When linearizing about a zero inflation steady state with no indexation, one derives the standard Phillips Curve expression between inflation, real marginal cost, expected future inflation, and the cost-push shock:

$$\pi_t = \frac{(1 - \theta_p)(1 - \theta_p \beta)}{\theta_p} m c_t + \beta E_t \pi_{t+1} + u_t^p$$  \hspace{1cm} (A.21)

A.5 Government

The government sets monetary policy according to a generalized Taylor rule and collects lump sum taxes. The policy rule is:

$$i_t = (1 - \rho_i) \left( \frac{1}{\beta} (1 + \pi^*) \right) + \rho_i i_{t-1} + (1 - \rho_i) \left( \phi_\pi (\pi_t - \pi^*) + \phi_x (x_t - x^*) + \phi_\Delta_y (y_t - y_{t-1}) \right)$$

Because there is no government spending in the model, the sum of lump sum transfers over households must be zero:

$$\int_0^1 T_t(h) dh = 0$$  \hspace{1cm} (A.22)

A.6 Exogenous Processes

The exogenous processes for productivity and preferences are assumed to follow mean zero stationary AR(1) processes in the log:
\ln Z_t = \rho_z \ln Z_{t-1} + s_z e_{z,t} \tag{A.23}

\ln \nu_t = \rho_\nu \ln \nu_{t-1} + s_\nu e_{\nu,t} \tag{A.24}

The processes for the cost-push and wage markup shocks follow mean zero stationary AR(1) processes:

\begin{align*}
u^p_t &= \rho_{up} \nu^p_{t-1} + s_{up} e_{up,t} \tag{A.25} \\
u^w_t &= \rho_{uw} \nu^w_{t-1} + s_{uw} e_{uw,t} \tag{A.26}
\end{align*}

The innovations are all drawn from \( N(0, 1) \) distributions and the \( s_i \) are the innovation standard deviations.

### A.7 Aggregation

The definition of equilibrium is standard – a set of prices and allocations such that all optimality conditions hold and all markets-simultaneously clear.

By the properties of Calvo (1983) pricing, we can write the equations describing the evolution of the aggregate nominal price in terms of inflation as:

\[ 1 + \pi_t = \left( (1 - \theta_p)(1 + \pi^#_t)^{1-\epsilon_p} + \theta_p(1 + \pi_{t-1})^{\epsilon_p(1-\epsilon_p)} \right)^{1/1-\epsilon_p} \tag{A.27} \]

Similarly, the aggregate real wage evolves according to:

\[ W_t = \left( (1 - \theta_w)u^i_t(1-\epsilon_w) + \theta_w W_{t-1}^{1-\epsilon_w}(1 + \pi_t)^{\epsilon_w-1}(1 + \pi_{t-1})^{\epsilon_w(1-\epsilon_w)} \right)^{1/1-\epsilon_w} \tag{A.28} \]

Market-clearing in factor markets requires:

\[ u_t K_t = \int_0^1 K_{s,t}(j) dj \]

\[ N_{d,t} = \int_0^1 N_{d,t}(j) dj \]

Since all firms hire capital and labor in the same ratio, the aggregate production function is:

\[ Y_t = \frac{Z_t(u_t K_t)^{\alpha} N_{d,t}^{1-\alpha}}{v^p_t} \tag{A.29} \]

\( v^p_t \) is a term related to inefficient price dispersion, and can be written recursively as:

\[ v^p_t = (1 + \pi_t)^{\epsilon_p} \left( (1 - \theta_p)(1 + \pi^#_t)^{-\epsilon_p} + \theta_p(1 + \pi_{t-1})^{-\epsilon_p(1-\epsilon_p)} v^p_{t-1} \right) \tag{A.30} \]
Integrating over household budget constraints and using the definition of intermediate firm profit, the aggregate resource constraint is:

\[ Y_t = C_t + I_t + \left( \Psi_0(u_t - 1) + \frac{\Psi_1}{2}(u_t - 1)^2 \right) K_t \]  

(A.31)

The aggregate capital accumulation equation is:

\[ K_{t+1} = \left( 1 - \frac{\phi_i}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t + (1 - \delta)K_t \]  

(A.32)

A.8 Evaluating Welfare

Suppressing explicit dependence on the state, the value function of household \( h \) at time \( t \) can be written:

\[ V_t(h) = E_t \sum_{j=0}^{\infty} \beta^j \left( v_t \frac{C_{t+j}(h) - \gamma C_{t+j-1}(h)}{1 - \sigma} - \psi N_{t+j}(h)^{1+\eta} \right) \]

Broke down into separate components from consumption and labor and written recursively, the value function is:

\[ V_t(h) = V_t^c(h) + V_t^n(h) \]  

(A.33)

\[ V_t^c(h) = v_t \frac{(C_t(h) - \gamma C_{t-1}(h))^{1-\sigma}}{1 - \sigma} + \beta E_t V_{t+1}^c(h) \]  

(A.34)

\[ V_t^n(h) = -\psi N_t(h)^{1+\eta} + \beta E_t V_{t+1}^n(h) \]  

(A.35)

In evaluating aggregate welfare a complication arises due to staggered wage-setting: though all households will have the same consumption, they will have different labor. I assume that the aggregate welfare function places equal weight on all households. That is:

\[ V_t = \int_0^1 V_t(h)dh = \int_0^1 V_t^c(h)dh + \int_0^1 V_t^n(h)dh \]

Since households will be the same with respect to their choice of consumption, aggregate welfare can be written:

\[ V_t = V_t^c + \int_0^1 V_t^n(h)dh \]

Where the value function over consumption is the same as above, but using aggregate consumption and dropping \( h \) subscripts:

\[ V_t^c = v_t \frac{(C_t - \gamma C_{t-1})^{1-\sigma}}{1 - \sigma} + \beta E_t V_{t+1}^c \]  

(A.36)
The sum of the value functions over labor input can be written:

\[ V^n_t = \int_0^1 V^n_t dh = \int_0^1 \frac{W_t(h)}{W_t} \left( N^{1+\eta}_{d,t} \right] dh + \beta E_t \int_0^1 V^n_{t+1}(h) dh \]

Define \[ v^w_t = \int_0^1 \left( \frac{W_t(h)}{W_t} \right)^{-\epsilon_w(1+\eta)} dh. \] This can be written recursively as:

\[ v^w_t = (1 - \theta_w) \left( \frac{W^w_t}{W_t} \right)^{-\epsilon_w(1+\eta)} + \theta_w \left( \frac{W_{t-1}}{W_t} \right)^{-\epsilon_w} \left( \frac{1 + \pi_{t-1} \zeta_{t}}{1 + \pi_{t}} \right)^{-\epsilon_w(1+\eta)} v^w_{t-1} \] (A.37)

Using the recursive representation of wage dispersion given above, the value function over employment can be written without reference to \( h \) subscripts:

\[ V^n_t = -\psi_1 + \beta E_t V^n_{t+1} \]

To examine the welfare consequences of different policy rules, I first compute the hypothetical value functions that would emerge in a flexible price and wage allocation. With flexible prices and wages, there is no wage dispersion, so there is no household heterogeneity. Flexible price welfare is:

\[ V^f_t = V^c_t + V^n_t \]

\[ V^f_t = \nu_t \frac{(C_{f,t} - \gamma C_{f,t-1})^{1-\sigma}}{1 - \sigma} + \beta E_t V^c_{t+1} \] (A.38)

\[ V^n_t = -\psi \frac{N^{1+\eta}_{d,t}}{1 + \eta} + \beta E_t V^n_{t+1} \] (A.39)

Let \( \omega_c \) denote the fraction of consumption that a household would need to have the same welfare in the distorted, sticky price and wage equilibrium relative to the flexible price and wage equilibrium. Formally, this satisfies:

\[ \tilde{V}^c_t = \nu_t \frac{(1 + \omega_c) (1 - \gamma C_{t-1})^{1-\sigma}}{1 - \sigma} + \beta E_t \tilde{V}^c_{t+1} = \omega_c^{1-\sigma} V^c_t \]

One could compute either conditional or unconditional values of \( \omega_c \). In the conditional exercise one would condition on a particular realization of the state; in the unconditional one simply takes unconditional expectations. I focus on the unconditional metric. Taking unconditional expectations and solving for \( \omega_c \):

\[ \omega_c = \left( \frac{E(V_f) - E(V^n)}{E(V^c)} \right)^{1-\sigma} - 1 \] (A.41)
A.9 The Simple New Keynesian Model

The simplest New Keynesian model abstracts from capital, wage rigidity, habit formation, indexation, and steady state inflation. Relative to the medium scale model described above, this amounts to restrictions that $\alpha = 0$, $\gamma = 0$, $\theta_w = 0$, $\zeta_p = 0$, $\pi^* = 0$, and that the economy begins with zero physical capital, $K_0 = 0$. It also requires that households have no power in wage-setting, so $\epsilon_w \to \infty$. In addition, it is assumed that there is a wage tax/subsidy which corrects the steady state distortion associated with market-power, $\tau$. With these assumptions, the equilibrium conditions can be reduced to:

$$\nu_t Y_t^{-\sigma} = \beta E_t \nu_{t+1} Y_{t+1}^{-\sigma} (1 + i_t)(1 + \pi_{t+1})^{-1}$$  \hspace{1cm} (A.42)

$$\psi N_t^\theta = \nu_t Y_t^{-\sigma} W_t$$  \hspace{1cm} (A.43)

$$mc_t = \frac{(1 + \tau) W_t}{Z_t}$$  \hspace{1cm} (A.44)

$$Y_t = \frac{Z_t N_t}{u_t^p}$$  \hspace{1cm} (A.45)

$$u_t^p = (1 + \pi_t)^{\epsilon_p} \left( (1 - \theta_p)(1 + \pi_t^{\#})^{-\epsilon_p} + \theta_p u_{t-1}^p \right)$$  \hspace{1cm} (A.46)

$$1 + \pi_t^{\#} = \frac{\epsilon_p}{\epsilon_p - 1} (1 + \pi_t) \frac{X_t}{H_t} \exp(u_t^p) \frac{\theta_p}{1 - \theta_p}$$  \hspace{1cm} (A.47)

$$X_t = \mu_t Y_t mc_t + \theta_p \beta E_t (1 + \pi_{t+1})^{\epsilon_p} X_{t+1}$$  \hspace{1cm} (A.48)

$$H_t = \mu_t Y_t + \theta_p \beta E_t (1 + \pi_{t+1})^{\epsilon_p - 1} H_{t+1}$$  \hspace{1cm} (A.49)

$$1 + \pi_t = \left( (1 - \theta_p)(1 + \pi_t^{\#})^{1-\epsilon_p} + \theta_p \right)^{\frac{1}{1-\epsilon_p}}$$  \hspace{1cm} (A.50)

The labor tax to offset the steady state price-setting distortion is $1 + \tau = \frac{\epsilon_p - 1}{\epsilon_p}$, which means that labor is subsidized with $\tau < 0$. The processes for the exogenous variables are the same as described above.

Log-linearizing these expressions about a zero inflation steady state yields the equations in the text:

$$y_t = E_t y_{t+1} - \frac{1}{1-\rho}(i_t - E_t \pi_{t+1}) + \frac{1}{1-\rho}(1-\rho)\nu_t$$  \hspace{1cm} (A.51)

$$\pi_t = \frac{(1 - \theta_p)(1 - \theta_p)}{\theta_p} \left( (\sigma + \eta)x_t + \beta E_t \pi_{t+1} + u_t^p \right)$$  \hspace{1cm} (A.52)

Here $x_t = y_t - y_t^f$, where $y_t^f$ is the output which would obtain in equilibrium with flexible prices. Because of the labor subsidy, this coincides with the efficient allocation.