Equilibrium

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General Equilibrium

Three different levels of analysis in economics:

▸ Decision theory: how do agents make decisions, given prices
▸ Partial equilibrium: how does price clear one market, taking all other prices as given
▸ General equilibrium: how do all prices work to clear all markets simultaneously

▸ In the two period optimizing framework, the only price we’ve encountered is the real interest rate

▸ Now we introduce an equilibrium concept and endogenize the real interest rate
Environment

- Economy populated by many identical agents
- Normalize to one: the representative agent
- Lives for two periods, solves standard consumption-saving problem
- Takes its income as given. Sometimes we call this an *endowment* economy
- “Supply-side” of the economy not very interesting for now
Problem

\[
\max_{C_t, C_{t+1}} \quad U = u(C_t) + \beta u(C_{t+1})
\]

s.t.

\[
C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}
\]

\[\Rightarrow\]

\[
C_t = C(Y_t, Y_{t+1}, r_t)
\]
Definition: *set of prices and allocations such that (i) all agents in economy are behaving optimally, taking prices as given; and (ii) all markets clear.*

“Competitive”: price-taking behavior

“Markets clear”: supply equals demand

“Allocations”: quantities (consumption, etc.)

Basic idea: prices adjust to make quantity demanded equal quantity supplied (e.g. for markets to clear) when the agents behave according to optimal decision rules
The demand side of the economy: total desired expenditure equals total consumption (no other actors in the economy)

\[ Y^d = C(Y_t, Y_{t+1}, r_t) \]

Complicating factor: \( Y \) essentially ends up on both sides

In any equilibrium, \( Y_t = Y^d_t \), however (expenditure = income)
Graphical Derivation of $Y^d$ Curve

- We can deal with this complication via a graph with a 45 degree line
Shifts of $Y^d$ Curve

- $Y^d$ curve shifts if desired expenditure changes for a given interest rate and level of current income
- Increase in $Y_{t+1}$ (anticipated future income) $\Rightarrow$ people want to spend more today $\Rightarrow$ $Y^d$ curve shifts out
Supply Side

- Trivial
- Supply is exogenous here – no production
Graphical Equilibrium

\[ Y_t^d = Y_t \]

\[ Y_t^d = C(Y_t, Y_{t+1}^0, r_t^0) \]
Comparative Statics

- Increase in current $Y_t$: “supply shock” $\Rightarrow r_t$ falls
- Increase in $Y_{t+1}$: “demand shock” $\Rightarrow r_t$ rises
- Intuition: market-clearing requires $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$
- Real interest rate has to “undo” the desired changes in consumption so that $C_t = Y_t$ at all times
- For a given real interest rate, people want to smooth. Real rate has to move to prevent smoothing
- Equilibrium real interest rate reflects the expected plentifulness of the future relative to the present
Suppose we have log utility: \( u(C_t) = \ln C_t \)

Then equilibrium real interest rate is

\[ 1 + r_t = \frac{1}{\beta} \frac{Y_{t+1}}{Y_t} \]

In essence, just take Euler equation and replace \( C_t \) and \( C_{t+1} \) with \( Y_t \) and \( Y_{t+1} \).

That real interest rate a measure of how plentiful future is relative to present is explicit and obvious here.

\textit{NOTE:} if \( Y_{t+1} = Y_t \), then \( \beta(1 + r_t) = 1 \).
Adding a Government

- Suppose government consumes resources, $G_t$ and $G_{t+1}$ (exogenously)
- Pays for resources by issuing taxes, paid for by households, $T_t$ and $T_{t+1}$
- Government chooses spending exogenously, taxes must adjust to make constraint hold
Government Budget Constraints

- Two within-period constraints:
  \[ G_t + S_t^G = T_t \]
  \[ G_{t+1} = T_{t+1} + (1 + r_t) S_t^G \]

- Can be combined just like for household:
  \[ G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} \]
Household Budget Constraints

- Two within-period constraints:
  
  \[ C_t + S_t = Y_t - T_t \]
  
  \[ C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t \]

- Can be combined:
  
  \[ C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t} \]
In equilibrium, both of these budget constraints must hold with equality. And households know this.

Also, government borrowing must equal household saving:

\[ S_t^G = -S_t \Rightarrow \text{in the accounting identity taxes drop out} \]

\[ C_t + G_t = Y_t \]

This characterizes aggregate demand side of the economy.
Combining Intertemporal Constraints

- If you combine government intertemporal constraint with household intertemporal constraint, you get:

\[ C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - G_t + \frac{Y_{t+1} - G_{t+1}}{1 + r_t} \]

- Taxes drop out!
- Households behave as though government balances budget each period, even if they don’t really
Ricardian Equivalence

- Ricardian Equivalence: households do not care about timing of taxes, conditional on timing of government spending
- Intuition: households forward-looking. They know low taxes today mean higher taxes tomorrow, absent a change in spending
- Tax cut (with no accompanying spending change) has *no effect* on economy
- Remember: $S_t^G = -S_t$. Tax cut $\downarrow S_t^G$. Households react by increasing their saving, because they anticipate having to pay higher taxes in the future, so $\uparrow S_t$. These effects perfectly offset: effectively demand for borrowing ($S_t^G$) goes up by same amount as supply of saving ($S_t$), meaning no effect on real interest rate
- “Equivalence”: makes no difference whether current spending financed with taxes or debt
Effects of Changes in Government Spending

- Timing of taxes does not matter, but spending does. Household consumption function is:

\[ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]

- Household behaves as though government balances its budget each period (i.e. \( G_t = T_t \)), whether this is the case or not.

- Aggregate demand relationship:

\[ Y^d = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + G_t \]

- ↑ \( G_t \): \( Y^d \) shifts out, \( r \) rises, \( C \) falls

- ↑ \( G_{t+1} \): \( Y^d \) shifts in, \( r \) falls, \( C \) unchanged

- Intuition: \( r \) measure of plentifulness of future relative to present

- Changes in \( T_t \) or \( T_{t+1} \) not matched by changes in spending have no effect on equilibrium
On the Government Spending Multiplier

- From principles you may recall a government spending multiplier of \( \frac{1}{1 - MPC} \).
- Can get that in this framework, but under restrictions: (1) No Ricardian Equivalence (people don’t react to government spending directly) and (2) real interest rate fixed (supply curve flat).
- In our model, the fixed real interest rate multiplier would be 1, and equilibrium multiplier is 0. If you relaxed Ricardian Equivalence you can get \( \frac{1}{1 - MPC} \) with real interest rate fixed.
- “Real world” multiplier probably somewhere between 0 and \( \frac{1}{1 - MPC} \).