Growth

Prof. Eric Sims

University of Notre Dame

Fall 2012
When economists say “growth,” typically mean average rate of growth in real GDP per capita over long horizons.

Not period-to-period fluctuations in the growth rate.

“Once one begins to think about growth, it is difficult to think about anything else” – Robert Lucas, 1995 Nobel Prize winner.
US Real GDP per capita

Real GDP per Capita - Linear trend

Sims (ND) Growth Fall 2012
Stylized Facts: Time Series

1. Output per worker grows at a roughly constant rate over long periods of time
2. Capital per worker grows at a roughly constant rate over long periods of time
3. The capital-output ratio is roughly constant over long periods of time
4. Rate of return on capital is roughly constant over long periods of time
5. The real wage grows at a roughly constant rate over time. The same rate as output per worker
Stylized Facts: Cross-Section

1. Large differences in GDP per capita across countries
2. Some examples where poor countries catch up (growth miracles)
3. Some examples where they don’t (growth disasters)
Solow Model

- After Robert Solow (1956), 1987 Nobel Prize winner
- Model capable of fitting stylized facts well
- Main implication: sustained growth must come from productivity improvements, not factor accumulation
- Implications for domestic policies as well as developing countries
Model Basics

- Time is discreet. $t$ is “current” period
- Two main actors in model: households and firms
- Assume there are large number of *identical* households and firms
- All the same $\Rightarrow$ can treat as though one household and one firm
- Everything real: no money, no nominal prices
Representative Firm

- Firm produces output using capital, $K$, and labor, $N$
- Labor: supplied by households, denominated in units of time (hours)
- Capital: must be produced, used to produce other stuff, does not depreciate completely. Same units as output
- Think about output as fruit. Plant unsold fruit in ground (investment) $\Rightarrow$ a new tree (capital) tomorrow
Production Function

- Mapping between inputs and output:
  \[ Y_t = AF(K_t, N_t) \]

- A: a measure of productivity. “Static efficiency”

- Properties of \( F(\cdot) \):
  \( F_K(\cdot) > 0 \), \( F_N(\cdot) > 0 \), \( F_{KK}(\cdot) < 0 \), \( F_{NN}(\cdot) < 0 \),
  \( F(\gamma K_t, \gamma N_t) = \gamma F(K_t, N_t) \)

- Example: Cobb-Douglas:
  \[ Y_t = AK_t^\alpha N_t^{1-\alpha}, \ 0 \leq \alpha \leq 1 \]
Factor Prices

- Household supplies labor, owns capital and leases to firm
- $w_t$: real wage
- $R_t$: real rental rate on capital
- Units of both of these real prices are fruit
Firm picks inputs to maximize profit:

$$\max_{K_t, N_t} \Pi_t = A_t F(K_t, N_t) - w_t N_t - R_t K_t$$

FOC:

$$AF_N(K_t, N_t) = w_t$$
$$AF_K(K_t, N_t) = R_t$$
Budget constraint:

\[ C_t + I_t \leq w_t N_t + R_t K_t + \Pi_t \]

\[ \Pi_t: \text{remitted profits (dividends)} \]

Current capital, \( K_t \), predetermined. Remember, has to be produced. Accumulation equation:

\[ K_{t+1} = I_t + (1 - \delta) K_t \]

\[ \delta: \text{depreciation rate, fraction of capital (trees) that become obsolete (die) each period} \]
Solow model does *not* model household optimization problem

- Households consume a constant fraction of income each period, 
  \[(1 - s), \ s \text{ is saving rate}\]
- Inelastically supply labor each period. Normalize to 1
- No need to differentiate between population and labor if inelastic supply
Aggregation

- Plug definition of profit from firm into household budget constraint
- Use consumption rule to get:

\[ I_t = sY_t = sAF(K_t, N_t) \]

- Define \( f(K_t) = F(K_t, 1) \)
Central Equation of Solow Model

- Capital accumulation equation only in terms of capital and parameters:
  \[ K_{t+1} = sAf(K_t) + (1 - \delta)K_t \]
- A “difference” equation: relates future values of \( K \) to past values \( K \)
The Steady State

- $K^*$: point at which $K_t = K_{t+1}$
- Once you get there, you are expected to stay there
- Should converge there from any non-zero starting point
Cobb-Douglas: \( f(K_t) = K_t^\alpha \)

\[
K^* = \left( \frac{sA}{\delta} \right)^{1 \over 1 - \alpha}
\]

\[
Y^* = A \left( \frac{sA}{\delta} \right)^{\alpha \over 1 - \alpha}
\]

\[
C^* = (1 - s) A \left( \frac{sA}{\delta} \right)^{\alpha \over 1 - \alpha}
\]
Permanent Increase in $A$
Dynamic Effects of Increase in A
Permanent Increase in $s$
Dynamic Effects of Increase in $s$
Factor Accumulation and Growth

- Increase in $s$ leads to more capital accumulation
- This fuels faster growth for a while, but we end up in a new steady state with no growth
- Increase in saving rate cannot lead to permanent change in growth
Golden Rule

- Households get utility from consumption, not output
- What is “optimal” saving rate?
- Saving rate which maximizes steady state (long run) consumption: Golden rule
- Intuition and “dynamic inefficiency”
- We wrote down a model to study growth
- But model features no growth: model converges to a steady state
- Two realistic remedies: population and technological growth
Inelastic labor supply ⇒ population and labor input growth the same

Grows at rate $g_n$:

$$N_t = (1 + n)N_{t-1}$$

$$N_t = (1 + n)^t N_0$$

Lowercase variables: per-capita/per-worker, e.g. $k_t = \frac{K_t}{N_t}$

Model otherwise identical
Algebraic manipulation yields:

$$(1 + g_n)k_{t+1} = sAf(k_t) + (1 - \delta)k_t$$

- Can analyze model in per capita variables exactly the same way
- Same conclusions still hold. Converge to a steady state in which per capita variables don’t grow, level variables grow at $g_n$
Exogenous Productivity Growth

- \( Z \): level of “labor-augmenting technology”
- Efficiency units of labor: \( Z_t N_t \)

\[
Z_t = (1 + g_z)Z_{t-1} \Rightarrow Z_t = (1 + g_z)^t Z_0
\]

- Production function: \( Y_t = AF(K_t, Z_t N_t) \)
- Define lowercase variables with a “hat” as “per efficiency units of labor,” e.g. \( \hat{k}_t = \frac{K_t}{Z_t N_t} \)
Manipulation yields:

\[(1 + g_n)(1 + g_z)\hat{k}_{t+1} = sAf(\hat{k}_t) + (1 - \delta)\hat{k}_t\]

Do same analysis, same conclusions go through in terms of per-efficiency units variables
Steady State Growth

- Per efficiency units variables go to a steady state
- In steady state, per capita variables all grow at rate $g_z$
- In steady state, level variables grow at approximate rate $g_z + g_n$
- Real wage grows at rate $g_z$
- Return on capital is constant
- Consistent with stylized facts
Quantitative Experiment

- Frequency annual
- $\alpha = 0.33$
- $g_n = 0.01, g_z = 0.02$
- $\delta = 0.1$
- $s = 0.15$
- $A = 1$
- Increase $s$ to 0.20 permanently
Per Efficiency Units

Capital per Effective Worker

- with s = 0.2
- with s = 0.15

Output per Effective Worker

Consumption per Effective Worker

Investment per Effective Worker

Sims (ND) Growth Fall 2012 32 / 39
Log Levels

![Graphs showing Capital, Output, Consumption, and Investment over time with different slopes for different values of s.](image)

- **Capital**: With $s = 0.2$ and $s = 0.15$.
- **Output**: Showing growth patterns.
- **Consumption**: Showing consumption levels over time.
- **Investment**: Showing investment levels with different slopes.
Convergence

- If countries are poor only because they don’t have enough capital, Solow model predicts that they should grow faster than normal to reach steady state.
- Countries would all end up looking the same.
- Clearly not true – large, persistent differences in standards of living.
- Some evidence of *conditional* convergence – Japan and Germany post WWII.
<table>
<thead>
<tr>
<th>Country</th>
<th>Relative GDP in 1970</th>
<th>Relative GDP in 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>13.6</td>
<td>15.5</td>
</tr>
<tr>
<td>Barbados</td>
<td>135.7</td>
<td>63.8</td>
</tr>
<tr>
<td>Bolivia</td>
<td>13.3</td>
<td>9.5</td>
</tr>
<tr>
<td>Brazil</td>
<td>18.9</td>
<td>20.9</td>
</tr>
<tr>
<td>Cambodia</td>
<td>4.8</td>
<td>5.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>81.8</td>
<td>83.2</td>
</tr>
<tr>
<td>Ecuador</td>
<td>15.6</td>
<td>15.8</td>
</tr>
<tr>
<td>France</td>
<td>77.5</td>
<td>75.6</td>
</tr>
<tr>
<td>Ghana</td>
<td>9.2</td>
<td>4.9</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>32.2</td>
<td>90.0</td>
</tr>
<tr>
<td>Jamaica</td>
<td>40.7</td>
<td>20.8</td>
</tr>
<tr>
<td>South Korea</td>
<td>13.0</td>
<td>61.8</td>
</tr>
<tr>
<td>Liberia</td>
<td>7.5</td>
<td>1.1</td>
</tr>
<tr>
<td>Portugal</td>
<td>36.6</td>
<td>48.5</td>
</tr>
<tr>
<td>Singapore</td>
<td>31.8</td>
<td>128.0</td>
</tr>
<tr>
<td>Spain</td>
<td>57.1</td>
<td>66.1</td>
</tr>
<tr>
<td>Sudan</td>
<td>5.2</td>
<td>5.5</td>
</tr>
<tr>
<td>Taiwan</td>
<td>18.3</td>
<td>69.4</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>1.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Factor Accumulation?

Could differences in saving rates, which lead to different steady state levels of $K$, drive these differences?

No

Suppose US $s = 0.15$. To explain a country with GDP per capita 20% of US, you’d need saving rate of $s = 0.006$

Not at all plausible
Why are some countries poor?

- The main factor economists have identified is A: “static efficiency.”
- What is this? “Total factor productivity” – output that is unexplained by observable inputs
  - Knowledge
  - Climate
  - Geography
  - Institutions
  - Infrastructure
Policy Implications

- Poor countries are *not* poor because they lack capital \( \Rightarrow \) direct aid
  not likely to have a huge effect
- Have to work on institutions and infrastructure:
  - Democracy
  - Rule of law, property protection
  - Infrastructure – roads, bridges, running water, sewage
Beyond Solow

- Solow model does not explain $A$, $Z$, or $g_z$. Takes them as given
- Reasonable policy prescriptions:
  - Patent protection
  - Subsidize research and development
  - Infrastructure
  - Education
  - Openness
  - Encourage more saving (though won’t permanently affect growth, still probably save too little in US)