Growth

Prof. Eric Sims

University of Notre Dame

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When economists say “growth,” typically mean average rate of growth in real GDP per capita over long horizons

Not period-to-period fluctuations in the growth rate

“Once one begins to think about growth, it is difficult to think about anything else” – Robert Lucas, 1995 Nobel Prize winner
US Real GDP per capita

-4.2
-4.0
-3.8
-3.6
-3.4
-3.2
-3.0
-2.8
-2.6

50 55 60 65 70 75 80 85 90 95 00 05 10

Real GDP per Capita
Linear trend

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Growth
Fall 2013
Stylized Facts: Time Series

1. Output per worker grows at a roughly constant rate over long periods of time
2. Capital per worker grows at a roughly constant rate over long periods of time
3. The capital-output ratio is roughly constant over long periods of time
4. Rate of return on capital is roughly constant over long periods of time
5. The real wage grows at a roughly constant rate over time. The same rate as output per worker
Stylized Facts: Cross-Section

1. Large differences in GDP per capita across countries
2. Some examples where poor countries catch up (growth miracles)
3. Some examples where they don’t (growth disasters)
Solow Model

- After Robert Solow (1956), 1987 Nobel Prize winner
- Model capable of fitting stylized facts well
- Main implication: sustained growth must come from productivity improvements, not factor accumulation
- Implications for domestic policies as well as developing countries
Model Basics

- Time is discreet. $t$ is “current” period
- Two main actors in model: households and firms
- Assume there are large number of identical households and firms
- All the same $\Rightarrow$ can treat as though one household and one firm
- Everything real: no money, no nominal prices
Representative Firm

- Firm produces output using capital, $K$, and labor, $N$
- Labor: supplied by households, denominated in units of time (hours)
- Capital: must be produced, used to produce other stuff, does not depreciate completely. Same units as output
- Think about output as fruit. Plant unsold fruit in ground (investment) $\implies$ a new tree (capital) tomorrow
Production Function

- Mapping between inputs and output:
  \[ Y_t = AF(K_t, N_t) \]

- \( A_t \) a measure of productivity. “Static efficiency”

- Properties of \( F(\cdot) \):
  \( F_K(\cdot) > 0, F_N(\cdot) > 0, F_{KK}(\cdot) < 0, F_{NN}(\cdot) < 0, F(\gamma K_t, \gamma N_t) = \gamma F(K_t, N_t) \)

- Example: Cobb-Douglas:
  \[ Y_t = AK_t^\alpha N_t^{1-\alpha}, \ 0 \leq \alpha \leq 1 \]
Factor Prices

- Household supplies labor, owns capital and leases to firm
- $w_t$: real wage
- $R_t$: real rental rate on capital
- Units of both of these real prices are fruit
Firm picks inputs to maximize profit:

$$\max_{K_t, N_t} \Pi_t = AF(K_t, N_t) - w_t N_t - R_t K_t$$

FOC:

$$AF_N(K_t, N_t) = w_t$$
$$AF_K(K_t, N_t) = R_t$$
Representative Household

- Budget constraint:
  \[ C_t + I_t \leq w_t N_t + R_t K_t + \Pi_t \]

- \( \Pi_t \): remitted profits (dividends)

- Current capital, \( K_t \), predetermined. Remember, has to be produced.
  Accumulation equation:
  \[ K_{t+1} = I_t + (1 - \delta) K_t \]

- \( \delta \): depreciation rate, fraction of capital (trees) that become obsolete (die) each period
Consumption and Labor Supply

- Solow model does *not* model household optimization problem
- Households consume a constant fraction of income each period, 
  \((1 - s)\), \(s\) is saving rate
- Inelastically supply labor each period. Normalize to 1
- No need to differentiate between population and labor if inelastic
  supply
Plug definition of profit from firm into household budget constraint
Use consumption rule to get:

\[ I_t = sY_t = sAF(K_t, N_t) \]

Define \( f(K_t) = F(K_t, 1) \)
Central Equation of Solow Model

- Capital accumulation equation only in terms of capital and parameters:

$$K_{t+1} = sA f(K_t) + (1 - \delta)K_t$$

- A “difference” equation: relates future values of $K$ to past values $K$
Graphical Representation

\[ K_{t+1} = K_t + s Af(K_t) + (1-\delta)K_t \]

\[ K^* \]

\[ K_t \]
The Steady State

- $K^*$: point at which $K_t = K_{t+1}$
- Once you get there, you are expected to stay there
- Should converge there from any non-zero starting point
Algebraic Example

- Cobb-Douglas: \( f(K_t) = K_t^\alpha \)

\[
\begin{align*}
K^* &= \left( \frac{sA}{\delta} \right)^{\frac{1}{1-\alpha}} \\
Y^* &= A \left( \frac{sA}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \\
C^* &= (1-s) A \left( \frac{sA}{\delta} \right)^{\frac{\alpha}{1-\alpha}}
\end{align*}
\]
Permanent Increase in $A$

\[ K_{t+1} = K_t + sA_0 f(K_t) + (1-\delta)K_t \]

\[ K_{t+1} = K_t + sA_1 f(K_t) + (1-\delta)K_t \]
Dynamic Effects of Increase in $A$

\[ K_t \]
\[ K_0^* \]
\[ K_1^* \]
\[ Y_t \]
\[ Y_0^* \]
\[ Y_1^* \]
\[ C_t \]
\[ C_0^* \]
\[ C_1^* \]
Permanent Increase in $s$

\[
K_{t+1} = K_t + sAf(K_t) + (1-\delta)K_t
\]

\[
s_1Af(K_t) + (1-\delta)K_t < sAf(K_t) + (1-\delta)K_t
\]

\[
s_0Af(K_t) + (1-\delta)K_t
\]

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Dynamic Effects of Increase in $s$
Factor Accumulation and Growth

- Increase in $s$ leads to more capital accumulation
- This fuels faster growth for a while, but we end up in a new steady state with no growth
- Increase in saving rate cannot lead to permanent change in growth
Golden Rule

- Households get utility from consumption, not output
- What is “optimal” saving rate?
- Saving rate which maximizes steady state (long run) consumption: Golden rule
- Intuition and “dynamic inefficiency”
We wrote down a model to study growth
But model features no growth: model converges to a steady state
Two realistic remedies: population and technological growth
Population Growth

- Inelastic labor supply $\Rightarrow$ population and labor input growth the same
- Grows at rate $g_n$:

$$N_t = (1 + g_n) N_{t-1} \Rightarrow N_t = (1 + g_n)^t N_0$$

- Lowercase variables: per-capita/per-worker, e.g. $k_t = \frac{K_t}{N_t}$
- Model otherwise identical
Algebraic manipulation yields:

\[(1 + g_n)k_{t+1} = sAf(k_t) + (1 - \delta)k_t\]

- Can analyze model in per capita variables exactly the same way
- Same conclusions still hold. Converge to a steady state in which per capita variables don’t grow, level variables grow at \(g_n\)
Exogenous Productivity Growth

- $Z$: level of “labor-augmenting technology”
- Efficiency units of labor: $Z_tN_t$

$$Z_t = (1 + g_z)Z_{t-1} \Rightarrow Z_t = (1 + g_z)^t Z_0$$

- Production function: $Y_t = AF(K_t, Z_tN_t)$
- Define lowercase variables with a “hat” as “per efficiency units of labor,” e.g. $\hat{k}_t = \frac{K_t}{Z_tN_t}$
Modified Central Equation

Manipulation yields:

\[(1 + g_n)(1 + g_z)\hat{k}_{t+1} = sAf(\hat{k}_t) + (1 - \delta)\hat{k}_t\]

Do same analysis, same conclusions go through in terms of per-efficiency units variables.
Steady State Growth

- Per efficiency units variables go to a steady state
- In steady state, per capita variables all grow at rate $g_z$
- In steady state, level variables grow at approximate rate $g_z + g_n$
- Real wage grows at rate $g_z$
- Return on capital is constant
- Consistent with stylized facts
Quantitative Experiment

- Frequency annual
- $\alpha = 0.33$
- $g_n = 0.01, \ g_z = 0.02$
- $\delta = 0.1$
- $s = 0.15$
- $A = 1$
- Increase $s$ to 0.20 permanently
Per Efficiency Units

Capital per Effective Worker

- with $s = 0.2$
- with $s = 0.15$

Output per Effective Worker

Consumption per Effective Worker

Investment per Effective Worker

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Log Levels

- Capital
  - with $s = 0.2$
  - with $s = 0.15$

- Output

- Consumption

- Investment

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Sims (ND)
If countries are poor only because they don’t have enough capital, Solow model predicts that they should grow faster than normal to reach steady state.

Countries would all end up looking the same.

Clearly not true – large, persistent differences in standards of living.

Some evidence of conditional convergence – Japan and Germany post WWII.
### Per Capita GDP Relative to US

<table>
<thead>
<tr>
<th>Country</th>
<th>Relative GDP in 1970</th>
<th>Relative GDP in 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>13.6</td>
<td>15.5</td>
</tr>
<tr>
<td>Barbados</td>
<td>135.7</td>
<td>63.8</td>
</tr>
<tr>
<td>Bolivia</td>
<td>13.3</td>
<td>9.5</td>
</tr>
<tr>
<td>Brazil</td>
<td>18.9</td>
<td>20.9</td>
</tr>
<tr>
<td>Cambodia</td>
<td>4.8</td>
<td>5.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>81.8</td>
<td>83.2</td>
</tr>
<tr>
<td>Ecuador</td>
<td>15.6</td>
<td>15.8</td>
</tr>
<tr>
<td>France</td>
<td>77.5</td>
<td>75.6</td>
</tr>
<tr>
<td>Ghana</td>
<td>9.2</td>
<td>4.9</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>32.2</td>
<td>90.0</td>
</tr>
<tr>
<td>Jamaica</td>
<td>40.7</td>
<td>20.8</td>
</tr>
<tr>
<td>South Korea</td>
<td>13.0</td>
<td>61.8</td>
</tr>
<tr>
<td>Liberia</td>
<td>7.5</td>
<td>1.1</td>
</tr>
<tr>
<td>Portugal</td>
<td>36.6</td>
<td>48.5</td>
</tr>
<tr>
<td>Singapore</td>
<td>31.8</td>
<td>128.0</td>
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<tr>
<td>Spain</td>
<td>57.1</td>
<td>66.1</td>
</tr>
<tr>
<td>Sudan</td>
<td>5.2</td>
<td>5.5</td>
</tr>
<tr>
<td>Taiwan</td>
<td>18.3</td>
<td>69.4</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>1.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Could differences in saving rates, which lead to different steady state levels of $K$, drive these differences?

No

Suppose US $s = 0.15$. To explain a country with GDP per capita 20% of US, you’d need saving rate of $s = 0.006$

Not at all plausible
Why are some countries poor?

- The main factor economists have identified is $A$: “static efficiency.”
- What is this? “Total factor productivity” – output that is unexplained by observable inputs
  - Knowledge
  - Climate
  - Geography
  - Institutions
  - Infrastructure
Policy Implications

- Poor countries are *not* poor because they lack capital \(\Rightarrow\) direct aid not likely to have a huge effect
- Have to work on institutions and infrastructure:
  - Democracy
  - Rule of law, property protection
  - Infrastructure – roads, bridges, running water, sewage
Beyond Solow

- Solow model does not explain $A$, $Z$, or $g_z$. Takes them as given
- Reasonable policy prescriptions:
  - Patent protection
  - Subsidize research and development
  - Infrastructure
  - Education
  - Openness
  - Encourage more saving (though won’t permanently affect growth, still probably save too little in US)