Introduction and Math

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Logistics

- Syllabus and related issues
About Me

- Associate Professor, Dept. of Economics
  - B.A. Trinity University, 2003
    - “Miracle in Mississippi” October 27, 2007
  - Ph.D. University of Michigan, 2009
    - Don’t get the wrong picture
    - Wife proud ND graduate – Lewis chicken
    - Signed Charlie Weis picture in office (oops?)
    - *Michigan Sucks*
Out of Style
In Style I
In Style III
My Effect on Notre Dame Football

![Graph showing Notre Dame Winning % by Decade](attachment:ND_WP.png)

- Sims arrival

Notre Dame Winning % By Decade

ND WP

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Introduction

- Macroeconomics:
  - Better called *aggregate economics*
  - Focus on *dynamic* and *intertemporal* nature of economic decision-making
  - Economics is “micro”: “macro” just studies issues at aggregated (country) level

- Key questions:
  - Why does the economy grow over time?
  - Why are some countries rich and others poor?
  - Why do economies experience recessions?
  - What is the role of government?
  - More recently:
    - What the heck happened in 2007-2009?
    - Why is the recovery weak?
Basic Approach of this Course

- Economists do it with models
  - But the models are used to help understand the real world
- Micro-founded macroeconomics. Agents Optimize
- Equilibrium approach – market-clearing as the relevant benchmark
- Fundamentally dynamic
“It is true that modern macroeconomics uses mathematics and statistics to understand behavior in situations where there is uncertainty about how the future will unfold from the past. But a rule of thumb is that the more dynamic, uncertain and ambiguous is the economic environment that you seek to model, the more you are going to have to roll up your sleeves, and learn and use some math. That’s life.” – Thomas Sargent, 2011 Nobel Prize Winner
Math Topics

- Dynamic notation
- Exponents and logs
- Growth rates
- Calculus
- Optimization
Notation

▶ Variable types: exogenous (determined outside model) and endogenous (determined in the model)
  ▶ Upper case letters
▶ Parameters: fixed values governing mathematical relationships
  ▶ Either low case letters or Greek letters

| \( \alpha \) | “alpha” |
| \( \beta \) | “beta” |
| \( \delta \) | “delta” |
| \( \gamma \) | “gamma” |
| \( \lambda \) | “lambda” |
| \( \theta \) | “theta” |
| \( \sigma \) | “sigma” |
| \( \omega \) | “omega” |


Time

- Time is discreet, with $t = 0, 1, 2, \ldots$
  - $x_t$ is $x$ observed at time $t$. $x_{t+j}$ is $x$ observed $j$ periods away from $t$
- Summation notation:
  \[
  S = x_t + x_{t+1} + x_{t+2} + \ldots x_{t+T}
  \]
  \[
  S = \sum_{j=0}^{T} x_{t+j}
  \]
- Alternatively:
  \[
  S = x_0 + x_1 + x_2 + \ldots x_T
  \]
  \[
  S = \sum_{t=0}^{T} x_t
  \]
- Can go either forward or backward in time
Some basic rules

Growth rate: \( g_t^x = \frac{x_t - x_{t-1}}{x_{t-1}} \)

Fun facts: \( \ln(1 + \alpha) \approx \alpha \) and \( \exp(\alpha) \approx 1 + \alpha \), for \( \alpha \) small

\( g_t^x \approx \ln x_t - \ln x_{t-1} \)

Growth rate of a product \( \approx \) sum of the growth rates
Calculus

- Function $y = f(x)$
- Derivative: how $y$ changes as $x$ changes. Derivative itself a function
- Notation: $\frac{dy}{dx} = f'(x)$
- Second derivative: derivative of a derivative: $\frac{d^2y}{d^2x} = f''(x)$
- Distinction between the derivative, $f'(x)$, which is a function, and the derivative evaluated at a point, $f'(x_0)$, which is a number
Derivative Rules

- **Powers, logs, exponents**
  - \( y = x^a, \frac{dy}{dx} = ax^{a-1} \)
  - \( y = \ln x, \frac{dy}{dx} = \frac{1}{x} \)
  - \( y = \exp(x), \frac{dy}{dx} = \exp(x) \)

- **Derivative of a sum is sum of derivatives**
  - \( y = f(x) + g(x), \frac{dy}{dx} = f'(x) + g'(x) \)

- **Product rule, quotient rule**
  - \( y = f(x)g(x), \frac{dy}{dx} = f(x)g'(x) + f'(x)g(x) \)
  - \( y = \frac{f(x)}{g(x)}, \frac{dy}{dx} = \frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2} \)

- **Chain rule:**
  - Composite function: \( y = f(g(x)) \)
  - Derivative is “derivative of outside times derivative of inside”:
    \( \frac{dy}{dx} = f'(g(x))g'(x) \)
  - \( y = \ln(x^a), \frac{dy}{dx} = \frac{1}{x^a} ax^{a-1} = \frac{a}{x} \)
Multivariate Derivatives

- Function of two variables: \( y = f(x, z) \)
- Partial derivative is change in \( y \) for a change in \( x \) (or \( z \)), holding \( z \) (or \( x \)) fixed
- Notation: \( \frac{\partial y}{\partial x} = f_x(x, z) \)
- Example:

\[
y = x^a z^b
\]

\[
\frac{\partial y}{\partial x} = ax^{a-1} z^b, \quad \frac{\partial y}{\partial z} = bx^a z^{b-1}
\]

- Total derivative: total change in \( y \) is approximately sum of partials evaluated at a point times changes in each variable about that point: \( dy \approx f_x(x_0, z_0) \, dx + f_z(x_0, z_0) \, dz \), where \( dy = y - y_0 \), \( dx = x - x_0 \) and \( dz = z - z_0 \)
- Application: growth rate of a sum
Optimization

- Pick $x$ to either maximize or minimize $f(x)$
- First order condition: $x^*$ needs to satisfy $f'(x^*) = 0$
- Second order condition: sign of $f''(x^*)$ tells you whether you have a maximum or a minimum
- Example: $y = \ln x - 2x$
- Multivariate optimization works the same way: first order conditions set partial derivatives with respect to each choice variable equal to zero
Constrained Optimization

- Want to optimize $f(x, z)$, but there is some constraint that $x$ and $z$ must satisfy, e.g. $x + z \leq 1$
- Our approach: assume constraint holds with equality, eliminate one of the choice variables ($x$ or $z$), and then do an unconstrained optimization problem
- Application: simple consumer problem