1. Write down the definition of a competitive equilibrium.

2. Discuss two different empirical studies that have looked at empirical predictions of the life cycle / permanent income hypothesis. Discuss, using both words and indifference curve-budget line graphs, how borrowing constraints can potentially help reconcile theory with evidence.

3. Suppose that a household expects to live for $T = 50$ periods, and to work for the first $R = 40$ of these periods. Assume that there is a fixed, constant real interest rate across time, $r$.
   
   (a) Write down the Euler equations that must hold between any two adjacent periods of time.
   (b) If $\beta (1 + r) > 1$, what will be true about the time path of consumption?
   (c) If $\beta (1 + r) < 1$, what will be true about the time path of consumption?
   (d) If $\beta (1 + r) = 1$, what will be true about the time path of consumption?
   (e) Regardless of what $\beta (1 + r)$ is, how much savings will the household end life with? In what period will the stock of savings reach its peak?
   (f) Suppose that $r = 0$ and $\beta = 1$. Suppose that income is 1 in each of the first $R = 40$ periods of life, and 0 during each period of retirement. Provide solutions for (i) the value of consumption in each period of life, (ii) the value of saving in each period of life, and (iii) the stock of savings at the date of retirement.
   (g) Suppose that the household (at the beginning of time) becomes aware that it will win the lottery in period 20, with income in that period equal to 10 (and income in every other non-retirement period still equal to 1). How will this affect your answers for the time paths of consumption and saving?

4. Write down the definition of Ricardian equivalence and explain its implications.

5. Suppose that we have an endowment economy with a government. Optimal behavior by the household implies that:

   $$ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) $$

   The household takes income, government spending, and the real interest rate as given. Total expenditure demand in the economy is given by:

   $$ Y_t^d = C_t + G_t $$
(a) Write down the definition of the $Y^d$ curve.
(b) Graphically derive the $Y^d$ curve as we did in class and in the notes.
(c) Since this is an endowment economy, the $Y^s$ curve is vertical. Graphically depict the equilibrium of the economy.
(d) Suppose that there is an increase in the current endowment, $Y_t$. Show how this will affect the equilibrium graphically. Provide some intuition for the effect on the equilibrium real interest rate.
(e) Suppose that there is an increase in the future endowment, $Y_{t+1}$. Show how this will affect the equilibrium graphically. Provide some intuition for the effect on the equilibrium real interest rate.
(f) Suppose that there is an increase in current government spending, $G_t$. Show how this will affect the equilibrium graphically. Provide some intuition for the effect on the equilibrium real interest rate.
(g) Suppose that there is an anticipated increase in future government spending, $G_{t+1}$. Show how this will affect the equilibrium graphically. Provide some intuition for the effect on the equilibrium real interest rate.

Suppose that $u(C_t) = \ln C_t$, so that the consumption function is:

$$C_t = \frac{Y_t - G_t}{1 + \beta} + \frac{Y_{t+1} - G_{t+1}}{(1 + \beta)(1 + r_t)}$$

(h) Use this specification of the consumption function to algebraically derive an expression for the $Y^d$ curve.
(i) Use this algebraic expression to verify your qualitative answers to parts (d)-(g).

(j) Use your algebraic expression to verify that the horizontal shift in the $Y^d$ curve for a one unit change in $G_t$ is 1.

6. Suppose you have two types of agents, $i = 1$ or $i = 2$, each with identical preferences, with $u(C_{i,t}) = \ln C_{i,t}$ for $i = 1$ or $i = 2$ (the two types of agents), and they have the same discount factor, $\beta$. They are each exogenously endowed with a known stream of income, $(Y_{i,t}, Y_{i,t+1})$. Suppose there are a large number of agents, and that the number of type 1 agents is the same as the number of type 2 agents (call the number of each type of agent $N$). They live for two periods and solve a standard consumption-saving problem, facing a common interest rate $r_t$.

(a) Solve for the consumption function as a function of a generic endowment stream, $(Y_{i,t}, Y_{i,t+1})$ for agent of type $i$.
(b) Write down the aggregate market-clearing condition. What must be true about saving for each type of agent?
(c) Suppose that $\beta = 0.9$. Suppose that $(Y_{1,t}, Y_{1,t+1}) = (1, 1)$ and $(Y_{2,t}, Y_{2,t+1}) = (1, 1)$. Solve for the equilibrium real interest rate and the equilibrium consumption allocations for both types of agents. Also compute the lifetime utility of both types of agents.
(d) Instead suppose that type 2 has endowment stream $(Y_{2,t}, Y_{2,t+1}) = (3, 1)$. Solve for the equilibrium real interest rate and the consumption allocations for both types of agents, as well as the lifetime utility of both types of agents.
(e) What happens to the lifetime utility of type 1 agents when I make type 2 agents have a bigger period $t$ endowment? What is the intuition for this?

7. Describe in some detail the “stylized business cycle facts” as presented in class.

8. Write down the 6 equations characterizing the competitive equilibrium of the neoclassical business cycle model. List the exogenous variables and the endogenous variables. Describe in words where each equation comes from.

9. Graphically depict the equilibrium of the neoclassical model, using the five part graph. Write down, in words, the definition of the $Y^d$ and $Y^s$ curves. Explain why the $Y^d$ curve slopes down and why the $Y^s$ curve is vertical. Identify which exogenous variables will affect which curve and in which direction.

10. Separately consider exogenous changes in each of the exogenous variables in the neoclassical model. Graphically show how they affect the equilibrium values of the endogenous variables.

11. In our baseline neoclassical model, write down the maximization problem of a hypothetical “social planner” who desires to maximize the utility of the representative household subject to the scarcity that the economy as a whole faces. Discuss the main way in which this problem differs from the competitive equilibrium setup. Find the first order conditions characterizing the solution to the planner’s problem, and compare them to the first order conditions of the competitive equilibrium solution.

12. In the context of the neoclassical business cycle model, what is the only exogenous variable that can be the main source of business cycle fluctuations? Explain why. Are declines in that exogenous variable welfare improving or welfare reducing? Should policy react to changes in this exogenous variable? Why or why not?