1 Introduction

In the Keynesian model we thought of monetary policy as exogenous in the sense that the money supply, $M_t$, was set exogenously. This is useful for understanding the model but doesn’t really describe how monetary policy works in practice. In the real world, central banks adjust the money supply (and hence interest rates) endogenously in response to changing conditions. In this set of notes we discuss what the objective of the central bank ought to be and how it can adjust the money supply in such a way as to achieve that objective in response to changes in other exogenous variables. We will then briefly talk about monetary policy in practice, mapping our conclusions into the celebrated Taylor rule, which has been taken to be a good standard by which monetary policy should be conducted. We will then discuss the implications of the zero lower bound on nominal interest rates for the Keynesian model.

2 Optimal Monetary Policy

If you remember back to when we studied the neoclassical model, we wrote down the problem of a fictitious social planner and argued that the outcome of the social planner’s problem was identical to the competitive equilibrium allocation. This meant that there was no role for activist economic policies. This is not the case in the Keynesian model, either the sticky wage or sticky price variants. In that model, because of price and/or wage rigidity, there is no guarantee that the equilibrium of the economy left to its own devices will be efficient.

As we argued in the last set of notes, either version of the Keynesian model is really just a special case of the neoclassical model – the sticky wage version where the nominal wage is fixed and labor is determined off of the labor demand curve, and the sticky price version where some firms are unable to adjust their prices and labor is determined from the labor supply curve. If prices or wages were flexible, the models would be identical to the neoclassical model. Since we know that the equilibrium level of output in the neoclassical model is efficient, this tells us what policy should be targeting in the Keynesian model – since the planner’s problem doesn’t have prices in it,
the planner’s solution (which is optimal) is the same in the Keynesian model as in the neoclasical model. In response to a change in an exogenous variable, the central bank therefore ought to adjust the money supply in such a way that the equilibrium of the sticky wage/price model is the same as the equilibrium of the neoclassical model. This constitutes optimal monetary policy. As such, the objective of monetary policy is the same in either the sticky wage or the sticky price version of the model – it wants to adjust the money supply so as to implement the equilibrium of the neoclassical model. I will denote by $Y^f_t$ the equilibrium level of output in the neoclassical model. The objective of policy in the Keynesian model ought therefore to be to adjust the money supply in such a way that $Y_t = Y^f_t$.

For either version of the model, we will see that optimal monetary policy is *contractionary* conditional on “demand” shocks to the IS curve (in the sense that it will be optimal to move the money supply in the opposite direction of output) and *accomodative* conditional on “supply” shocks (either to productivity or labor supply) in the sense that it will be to optimal to move the money supply in the same direction as output. While the models share this implication, there are different implications for the behavior of prices. In the sticky price model, *price stability* is a good goal – if the central bank adjusts the money supply so that the price level is constant, then it automatically implements the flexible price, neoclassical equilibrium. This is not so in the sticky wage model, because in the neoclassical model the real wage adjusts to supply shocks, and if the price level does not move the real wage cannot move. Hence, price stability is not necessarily a good goal in the sticky wage model. Because the flexible price/wage equilibrium level of output that would emerge in the neoclassical model is not necessarily observable (the central bank can only really observe what output is, not what it would be in the absence of price or wage stickiness), monetary policy is “easier” in the sticky price model – the central bank just needs to focus on stabilizing prices. Monetary policy is more difficult in the sticky wage model.

### 2.1 Sticky Price Model

Let’s first consider the sticky price model. There are essentially three kinds of non-monetary shocks – two supply shocks (productivity and labor supply) and demand shocks (things which would shift the IS curve, so changes in $A_{t+1}$, $q$, $G_t$, or $G_{t+1}$). What I want to do here is the following. Consider a change in one of the exogenous variables. See how this would affect both (i) the equilibrium of the sticky price model and (ii) the hypothetical equilibrium if prices were flexible (i.e. the equilibrium of the neoclassical model, what we will call $Y^f_t$). Then figure out how $M_t$ needs to adjust so that the equilibrium of the sticky price model coincides with the equilibrium of the neoclassical flexible price model.

Let’s first consider a change in an exogenous variable which shifts the IS curve to the right. Denote the original equilibrium before the exogenous change with 0 superscripts. Generically, let’s call this a demand shock. The figure below shows what would happen with the money supply fixed in blue lines. The IS curve shifts to the right, which causes the AD curve to shift right. This results in a higher level of $Y_t$ and a higher $P_t$. The higher $P_t$ causes the LM curve to shift in some (green
line), but not so much that output doesn’t still rise. To get more output the real wage has to rise to induce households to work more. Denote the new equilibrium after the demand shock (but with no endogenous reaction of the money supply) with 1 superscripts.

Now, let’s ask ourselves: what happens to $Y_t^f$ after a positive shock to the IS curve? As we saw previously, nothing. We can think about the AS curve in the flexible price version of the model as being vertical, so that output would not react to a demand shock. This means that the equilibrium
level of output in the sticky price Keynesian model after the positive demand shock is higher than the equilibrium of the neoclassical model. Since the objective of a central bank ought to be to adjust the money supply in such a way as to implement the flexible price equilibrium, evidently the central bank will want to adjust the money supply in such a way as to shift the AD curve back in, in essence to undo to the effects of the outward shift of the AD curve resulting from the IS shift. This requires that the central bank reduce the money supply. This is shown with the red lines in the diagram. The central bank will choose a new money supply, $M_{1t}$, such that the AD curve shifts back all the way in. This implies no change in output and no change in the price level (so that the new equilibrium after the money supply change, denoted by superscript 2, features the same level of output as before the IS shock). The price level is also unchanged. The real interest rate ends up higher. So, in response to a positive demand shock, it is evidently optimal for the central bank to counteract it by lowering the money supply (equivalently raising interest rates). We might call this *contractionary* policy – move monetary policy in the “opposite” direction of output.

Let us now consider how the central bank ought to react to expansionary supply shocks, picked up by changes in $A_t$ or $H_t$. This will end up being a little more tricky because the flexible price, neoclassical level of output will adjust to these shocks. First, consider an increase in $A_t$. This causes the labor demand curve to shift out and the production function to shift up. $Y^F_t$ increases, which can graphically be found at the level of output consistent with the new production function and with the level of $N_t$ where the new labor demand curve intersects the labor supply curve. This causes the AS curve to shift to the right horizontally by the amount of the increase in $Y^F_t$. We end up with output increasing and the price level decreasing. The lower price level causes the LM curve to shift out (green line), so that the real interest rate falls. Graphically we can see that output in the sticky price model increases by *less* than the increase in $Y^F_t$ (only if the AS curve were vertical, or the AD perfectly horizontal, would the change in equilibrium output equal the change in $Y^F_t$). This means that $N_t$ goes up by less than it would if prices were flexible (where labor demand and supply meet); hours could go up, down, or be unchanged. In the figure I show it where hours would actually decline.
So what should monetary policy do in reaction to this increase in $A_t$? The flexible price level of output has increased (from $Y_{t}^{f}$ to $Y_{t}^{f'}$). The actual level of output has increased but by less than this. Since the central bank wants to implement the flexible price equilibrium, it evidently wants to engage in *expansionary* (or *accommodative*) policy. It wants to adjust the money supply in such a way that the AD curve shifts right so that it intersects the new AS curve at the new flexible price level of output. This requires increasing the money supply. This is shown by the red lines.
in the figure. The money supply increases, which shifts the LM and AD curves out such that the equilibrium of the sticky price model coincides with the neoclassical model. The real interest rate falls. The price level is unchanged. Relative to the sticky price equilibrium with no endogenous reaction of the money supply, employment and the real wage both increase.

Consider next the effects of a positive labor supply shock, captured by an increase in $H_t$. This is shown in the figure below:
The effects of the increase in $H_t$ absent a change in $M_t$ are shown with the blue lines. The labor supply curve shifts right. This increases the level of employment at which the labor demand and supply curves intersect. There is no shift in the production function. This means that $Y_t^f$ increases – the level of output in the neoclassical model increases. This causes the AS curve to shift horizontally to the right by the amount of the increase in the flexible price level of output. In equilibrium output rises and the price level falls. The increase in output is smaller than the increase in $Y_t^f$ (which is equal to the horizontal shift in the AS curve). The lower price level causes the LM curve to shift right (green line), which results in a lower real interest rate. Labor hours increase but by less than they would if prices were flexible. Hence, the real wage decreases (by more than it would if prices were flexible).

How ought monetary policy to react? The objective is for the equilibrium level of output to correspond with the hypothetical flexible price level of output which would emerge in the neoclassical model. Since output increases in the sticky price model but by less than it would if prices were flexible, the central bank ought to engage in accommodative policy by increasing the money supply. It should do so in such a way that the AD curve shifts out so as to intersect the AS curve at the new flexible price level of output. This involves increasing the money supply and cutting interest rates. There is no change in the price level if the central bank adjusts the money supply optimally.

The table below summarizes how the money supply and interest rates (both the real and nominal, since we assume that expected inflation is exogenously fixed) ought to react to increases in exogenous variables which would shift the IS curve right, as well as the two supply shocks, $A_t$ and $H_t$. The optimal response of monetary policy would be the opposite signs of what is presented here for decreases in exogenous variables instead of increases.

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We see that in response to an outward shift of the IS curve (a positive demand shock), the central bank ought to engage in contractionary policy by reducing the money supply and increasing interest rates. This is sometimes called “leaning against the wind.” In response to positive supply shocks (either an increase in $A_t$ or an increase in $H_t$) the central bank ought to engage in accommodative policy – it should increase the money supply and reduce interest rates. So a good rule of thumb to remember: conditional on demand shocks, it is optimal to move the money supply in the opposite direction of output; conditional on supply shocks, it is optimal to move the money supply in the same direction as output.

There is something interesting that is worth pointing out in the analysis above. In all of these cases, the central bank wants to endogenously adjust the money supply in such a way that the price level does not change. Mechanically from the expression for the AS curve we can see why this
is the case. Recall that the mathematical relationship for the AS curve is \( P_t = P^*_t + \gamma(Y_t - Y^f_t) \).
If the price level never changes in response to shocks (and here we implicitly are assuming that
the equilibrium before the shock hits is efficient so that \( P^*_t = P_t \)), then \( Y_t = Y^f_t \) in response to
the shock. This is an important result – it tells us that price stability is a good goal, and forms
the basis of many central banks either implicitly or explicitly adopting strict inflation/price level
targets. This is important because \( Y^f_t \) is not directly observable – it is a hypothetical construct
that would emerge in the absence of price rigidity. Since it is hypothetical the central bank can’t
really observe it. So how do you conduct policy in such a way as to make \( Y_t = Y^f_t \) if you don’t know
what \( Y^f_t \) is? This analysis tells us that we don’t need to know what \( Y^f_t \) is – if the central bank
just commits to stabilizing prices, then it automatically implements the flexible price, neoclassical
equilibrium.

### 2.2 Sticky Wage Model

We next consider optimal monetary policy in the context of the sticky wage model. The objective
of the central bank is the same: in response to exogenous shocks, it would like to adjust the money
supply in such a way as to implement the flexible wage equilibrium, denoted by \( Y^f_t \). This is the
same as the equilibrium of the neoclassical model.

Consider first the case of a positive IS shock which shifts the AD curve to the right. This could
be caused by an increase in \( A_{t+1} \), an increase in \( G_t \), an increase in \( q \), or a decrease in \( G_{t+1} \). The
AD curve shifting to the right along an upward-sloping AS curve results in an increase in both \( Y_t \)
and \( P_t \) absent the central bank doing anything to the money supply. The higher price level causes
the real wage to fall given a fixed nominal wage, which induces firms to hire more workers. These
effects are shown by the blue lines and the 1 superscripts. Since the AS curve would be vertical
if wages were flexible, nothing happens to \( Y^f_t \). Hence, the central bank will want to adjust the
money supply in such a way as to counteract the outward shift of the AD. Evidently, this requires
reducing the money supply in such a way as to shift the AD curve in. This is shown by the red
lines – the money supply drops by an amount sufficient to shift the AD curve all the way back
in. This results in an increase in the real interest rate. There is no change in output or the price
level after the reduction in the money supply. Hence, there is no change in the real wage (since
the nominal wage is fixed, if the price level doesn’t change the real wage doesn’t change). Overall,
the desired monetary policy response and the effects of that on the endogenous variables are very
similar to what we observe in the sticky price model.

\[ ^1 \text{In the model with which we are working I don’t want to make much of a distinction between inflation or price}
\text{level targeting – we can think of both as trying to stabilize prices, which is evidently optimal in the sticky price}
\text{Keynesian model. In a version of the model with more sophisticated dynamics, the distinction between inflation}
\text{and price level targeting comes down to one of discretion versus commitment. Price level targeting is an implicit}
\text{commitment to “fix” past misses of a price target which has the advantage of betting “anchoring” expected inflation.}
\text{This is something you might explore in a more advanced class in macroeconomics but we will not discuss it further}
\text{here.} \]
Next, consider the effects of an increase in productivity, $A_t$. The effects of this exogenous shock without any change in the money supply are shown in the diagram below in blue. The labor demand curve would shift right and the production function up. For a given price level firms would want to hire more workers; combined with higher productivity this results in the AS curve shifting to the right. This results in an increase in $Y_t$ and a decrease in $P_t$. The decrease in $P_t$ would cause the LM curve to shift out, so that the real interest rate falls. The decrease in $P_t$ also results in
an increase in the real wage (given that the nominal wage is fixed by assumption). Note that it is technically ambiguous how $Y_t$ reacts in relation to the flexible wage level of output. The new flexible wage level of output after an increase in $A_t$ is denoted by $Y_t^{f'}$ and is found as the level of output consistent with the new production function and the labor market being in equilibrium. As discussed previously, in this model we could have output rise by more or less than it would in the flexible wage model, depending on how steep the AD curve is. We assume that output in the sticky wage model rises \textit{by less} than it would in the flexible wage equilibrium – this is how the diagram is drawn. This will be the case in this model as long as the AD curve is not too flat. This means that the central bank would like to increase output. To do this it must increase the money supply, which results in an outward shift of the AD curve. This is shown in the red lines in the diagram.

While the qualitative response by the central bank to an increase in $A_t$ is the same in the sticky wage model as in the sticky price model (it wants to increase $M_t$ so as to shift the AD curve right), there is an important difference. In the sticky wage model, the price level needs to fall for the equilibrium of the model to correspond with the hypothetical flexible wage equilibrium (whereas in the sticky price model the price level didn’t change if the central bank reacted optimally). Intuitively the reason why is simple – to implement the flexible wage equilibrium the real wage has to rise. With a fixed nominal wage the only way for the real wage to rise is for the price level to fall. Hence, the optimal monetary policy response to a productivity shock in the sticky wage model is to allow the price level to fall.
Next, consider the effects of an exogenous increase in $H_t$ in the sticky wage model. As we saw before, this shock has no effect on the equilibrium of the sticky wage model (because it is a shock to the labor supply curve and we are not necessarily on the labor supply curve in the sticky wage model). Hence, there is no shift of either the AS or AD curves. But the flexible wage level of output increases to $Y_t^f$, which can graphically be found as the level of output consistent with the intersection of the the labor demand curve and the new labor supply curve (shown in blue).
Since the flexible wage level of output has increased but the equilibrium level of output has not, the central bank would like to stimulate output. It can do this by increasing the money supply by enough so that the AD curve shifts out to the right so as to intersect the AS curve at the new flexible wage level of output – this effect is shown via the red line. In the new optimal equilibrium, output and the price level are both higher. The higher price level drives the real wage down, which is the mechanism by which labor hours increase.
We again see that while the desired monetary policy response is qualitatively the same in response to an increase in $H_t$ in both the sticky wage and sticky price models (a central bank would want to increase the money supply), there is a difference in terms of how the price level reacts. In particular, in the sticky wage model the price level must increase if monetary policy is conducted optimally so as to induce the decline in the real wage necessary to implement the neoclassical equilibrium. This is different than the sticky price model where the price level did not change in response to an increase in $H_t$ under the optimal monetary policy.

The table below summarizes the optimal responses of the money supply and interest rates to the three kinds of shocks in the sticky wage Keynesian model. These effects are qualitatively the same as in the sticky price model – the central bank wants to increase the money supply after a positive supply shock (increase in either $A_t$ or $H_t$) and decrease the money supply / increase in interest rates in response to a positive demand shock (something which would shift the IS curve to the right). But an important difference between the two models concerns the desirability of stabilizing the price level. Conditional on IS/demand shocks, the optimal monetary policy stabilizes the price level in both variants of the model. But this is not true conditional on supply shocks – in the sticky price model the optimal monetary policy completely stabilizes the price level, but in the sticky wage model the optimal monetary policy must allow $P_t$ to change conditional on supply shocks (to fall conditional on an increase in $A_t$ and to rise conditional on an increase in $H_t$).

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3 Monetary Policy in Practice

In either the sticky price or sticky wage versions of the Keynesian model it is optimal to adjust the money supply endogenously in response to shocks so that the equilibrium coincides with the hypothetical, flexible price level of output, i.e. adjust $M_t$ so that $Y_t = Y^f_t$. This sounds simple in theory but is difficult in practice. Why is it difficult in practice? $Y^f_t$ is a hypothetical construct and as such is not directly observable. If the central bank imperfectly observes $Y^f_t$ trying to react to it could induce non-optimal volatility into the economy.

In the sticky price model monetary policy is much easier in that the central bank doesn’t actually need to know what $Y^f_t$ is. As we saw, if the price level is stable (and equal to $P^c_t$), then $Y_t = Y^f_t$, so the central bank doesn’t actually need to know what $Y^f_t$ is – it just needs to adjust the money supply in response to shocks such that the price level doesn’t react. This is not so in the sticky wage version of the model, at least conditional on supply shocks (conditional on demand shocks a policy of price stabilization is optimal). Hence, monetary policy is “harder” in the sticky wage model because you technically need to know what $Y^f_t$ is.
Even though price stabilization is not optimal in the sticky wage model that we’ve written down, there are several potential reasons for why price stabilization may still be a worthwhile goal even in the sticky wage model. When firms set wages, they need to do so with an idea of what the price level will be – they will want to target a real wage, and if they don’t know what the price level will be it is difficult to correctly choose the nominal wage. In either the sticky price or sticky wage models, an additional benefit of price stability is the “anchoring” of expected inflation, $\pi_{t+1}$, which we have taken to be exogenous. If $\pi_{t+1}$ moves around, this will cause the LM curve to shift around, which will induce shifts of the AD and non-optimal fluctuations in output. Even though we have modeled it as exogenous, $\pi_{t+1}$ is much more likely to jump around if we’re in a world where the price level changes a lot than if we’re in a world where the price level is stable.

Given this, we can think of monetary policy as facing a “dual mandate” – it would like both price stability and the output “gap” ($Y_t - Y_f^t$) to be stable. In the sticky price model there is no tradeoff between the two objectives – the so-called “Divine Coincidence” holds. This is not so in the sticky wage model, where conditional on supply shocks it is not possible to simultaneously stabilize both prices and output about its flexible wage/price neoclassical level.

Given that central banks may want to stabilize both prices and output about its flexible price level, John Taylor developed his famous “Taylor rule” for monetary policy. The Taylor rule is a rule for how the nominal interest rate should adjust to deviations of inflation from a target and output from its potential (where we take potential output to be the equilibrium level of output in the neoclassical model). We have thought about monetary policy as adjusting the money supply, but money supply adjustments affect the nominal interest rate (and also the real interest rate to the extent to which we think of expected inflation as fixed). The Taylor rule expresses an interest rate target as functions of deviations of inflation from target and output from potential; the money supply then adjusts to make this nominal interest rate consistent with the money market clearing. The basic form of the Taylor rule is:

$$i_t = i^* + \phi_\pi (\pi_t - \pi^*) + \phi_y (Y_t - Y_f^t)$$

We have talked about price stability in terms of the price level but Taylor’s rule expresses price stability in terms of the deviation of an inflation rate, $\pi_t$, from a target level, $\pi^*$. The reason for this is to account for the fact that there is upward drift in prices (i.e. the average level of inflation is positive), for reasons we’ll talk about more below. Nevertheless, in terms of our two period model we can think about stabilizing inflation and the price level as one in the same. The “constant” term $i^*$ is the average interest rate when prices are stable and output equals its flexible price level. The coefficients $\phi_\pi$ and $\phi_y$ are positive numbers.

This rule embodies the dual mandate we talked about and can be used to think about optimal monetary policy in terms of both the sticky wage and sticky price models. If $Y_t > Y_f^t$ (say, because

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of a demand shock in either model), the rule says to raise the interest rate, which involves cutting the money supply. If \( Y_t < Y_t^f \) (say, due to an expansionary supply shock where output doesn’t rise by enough), then the rule says to lower interest rates, which involves increasing the money supply. If there is a positive demand shock, then \( \pi_t > \pi^* \), and the rule says to raise interest rates (and hence cut the money supply). If there is an expansionary supply shock, then prices fall and \( \pi_t < \pi^* \), so the rule says to cut interest rates (i.e. increase the money supply). In other words, the rule features exactly the kinds of endogenous reactions of monetary policy to shocks that we talked about above. In the sticky price model a big coefficient \( \phi_\pi \) will completely stabilize prices, which by default will force \( Y_t = Y_t^f \) regardless of \( \phi_y \). This will not be the case in the sticky wage model.

Expressing the rule in terms of both inflation and the output gap accounts for the realization that complete price stabilization may not be optimal if wages are sticky.

Taylor argued for coefficients of about \( \phi_\pi = 1.5 \) and \( \phi_y = 0.5 \). We can think of justification for the coefficient on inflation being bigger than the coefficient on the output gap as being due to (i) a belief that price stickiness is more important than wage stickiness, which is driven by the empirical finding that real wages are procyclical, (ii) a desire to stabilize prices for the reasons that are outside of our current model listed above, and (iii) the recognition that observing \( Y_t^f \) is difficult and that reacting to an incorrect measure of \( Y_t^f \) could induce undesirable outcomes. An additional reason for a bigger response to inflation relates to equilibrium determinacy to which we will return below.

The Taylor rule is normative in the sense that it roughly describes how monetary policy ought to react to changing economic conditions. It also happens to provide a remarkably accurate positive description of reality in the sense of describing how central banks actually do conduct policy. The figure below shows the actual behavior of the Federal Funds Rate since 1985 (solid line) and the implied value of the interest rate using the Taylor rule (dashed line). The two series track each other remarkably well, implying that the Taylor rule is a pretty good guide for how actual policy is conducted. There is some obvious divergence after 2009, when the Fed Funds Rate got close to zero but the implied rate from the Taylor rule went negative. We will return to the issue of the zero lower bound in more depth below.

\[ \text{For an example of this, see Athanasios Orphanides (2003): “The Quest for Prosperity without Inflation,” Journal of Monetary Economics 50(3), 633-663. Orphanides argues that the high inflation of the 1970s was due to the central bank misperceiving } Y_t^f. \text{ In particular, they observed output falling and attributed this to demand. But in fact we had suffered adverse supply shocks (high oil prices, which could manifest themselves like reductions in } A_t. \text{ The optimal policy response would have been to move the money supply in the same direction as output (and hence interest rates in the opposite direction), but because they didn’t recognize the fall in } Y_t^f \text{ they cut interest rates instead (increased the money supply), which caused high inflation.} \]

\[ \text{There is one minor modification of the Taylor rule I use to produce this graph in that I assume some interest rate smoothing, wherein the interest rate also depends on its lagged value. This reflects a real-world desire for gradualism in the conduct of monetary policy. In generating this figure I measure inflation via the GDP deflator and the output gap using the CBO measure of potential output.} \]
4 The Zero Lower Bound

We have studied optimal monetary policy in the Keynesian model. Optimal monetary policy consists of adjusting the money supply (and hence interest rates) in such a way as to implement the neoclassical equilibrium level of output. A Taylor rule is a particularly simple description that captures many features of optimal monetary policy, and seems to fit the data well.

A potential problem with this discussion of monetary policy is that nominal interest rates cannot go below zero – we call this the zero lower bound (ZLB). Note that real interest rates can go negative, but nominal rates cannot. Why is this? The nominal interest rate is the return on saving in money. Since money is storable across time (by definition, storability is one of the functions which defines money), no one would ever accept a negative nominal interest rate. A nominal interest rate of -5 percent would mean, for example, that in exchange for saving one dollar today you’d get back 95 cents tomorrow. No one would ever take that – you could just hold on to the dollar and still have one dollar tomorrow. Because of non-storability of real goods (e.g. fruit), you might accept negative real rates of return – e.g. you might give up one fruit today in exchange for .95 of a fruit tomorrow, which you might take because you don’t have the option of simply storing the fruit over time. The zero lower bound has important implications for the conduct of monetary policy, and is particularly relevant now since nominal interest rates are at or near zero and have been since the end of 2008. The zero lower bound is sometimes referred to as a “liquidity trap.”

What is economically relevant about the ZLB is not that nominal interest rates are zero per se but rather that nominal interest rates become fixed. This changes the IS-LM-AD-AS model of the economy in important ways. Hence, for understanding the ZLB we need to first understand what the model looks like if the central bank pursues a policy of fixing the nominal interest rate
at some exogenous value, call this \( i \) (without a time subscript). A situation like this is sometimes called an “interest rate peg.” Since we have assumed that expected one period ahead inflation is exogenous, the real interest rate becomes fixed in a situation like this, since by the Fisher relationship \( r_t = i - \pi_{t+1}^e \). This means that we can functionally think of the LM curve becoming perfectly horizontal under an interest rate peg. In this situation, there is no effect of the current price level on the position of the LM curve, which makes the AD curve perfectly vertical. In essence, if the real interest rate is fixed, the equilibrium level of output can be read directly off of the IS curve and is therefore independent of the price level. We can see IS-LM-AD graphs under an interest rate peg below:

We can think about how “demand” and “supply” shocks impact the economy when monetary policy is characterized by an interest rate peg. For understanding the basic effects we do not need to worry about whether prices or wages are sticky (or both), we just need the AS curve to be upward-sloping. Suppose first that there is a positive supply shock (say, due to an increase in \( A_t \)) which shifts the AS curve to the right. If the AD curve is vertical, there is no change in output. The price level just falls and there is no change in interest rates nor any change in any components
of spending.\footnote{Though I don’t show the labor market here, if $A_t$ increased with no change in $Y_t$, then $N_t$ would have to fall.}

Next, consider a positive shock to demand coming from some exogenous variable which shifts the IS curve to the right. This results in an outward shift of the AD curve and increases in output and the price level. There is no change in the real interest rate. This can be seen in the figure below.
We can see here something rather neat. An interest rate peg *exacerbates* the effects of nominal rigidity (either in the form of price or wage stickiness) on the relative effects of demand/supply shocks. In the standard Keynesian model we argued that supply shocks have smaller effects on output and demand shocks bigger effects on output relative to the neoclassical model. Here these effects are exacerbated – under an interest rate peg, supply shocks have no effect on output and demand shocks have even bigger effects on output. The intuition for why is the behavior of the real interest rate. For supply shocks to impact output the real interest rate must fall. Positive shocks to the IS curve result in higher real interest under normal policy, which partially undoes some of the expansionary effects of the IS shock on output. An interest rate peg exacerbates both of these effects. Since the real interest rate is fixed, it cannot fall after an expansionary supply shock, and therefore output cannot rise. Conditional on a positive demand shock, a fixed real interest rate means that there is no “undoing” effect of the real interest rate rising, and so output expands by even more than it would under normal circumstances. If the objective of a central bank is to implement the neoclassical equilibrium, we can conclude that an interest rate peg (or an exogenous interest rate target, if you prefer) is a very bad policy choice – the gap between equilibrium output
and the hypothetical flexible price/wage level of output after either demand or supply shocks is even bigger than in a situation where the central bank sets the money supply exogenously.

The undesirability of an interest rate peg forms the basis of why the zero lower bound is problematic for policy. In a more general setting, we can think about the zero lower bound as introducing a kink into an otherwise standard LM curve. When the nominal interest rate hits zero, the real interest rate gets fixed at \(-\pi_{t+1}^e\) (which we take to be exogenous). This means that the LM curve is perfectly flat in this region. Once the real interest rate gets big enough so that \(i_t > 0\), the LM curve has its usual upward-slope. So we can think about two regions for monetary policy: the “normal” region where the LM curve is upward-sloping and the ZLB region where we can think about monetary policy as effectively following an interest rate peg (so that the LM curve is flat). The picture below shows a diagram with the IS-LM-AD-AS curves. I assume that the initial equilibrium is such that the zero lower bound does not “bind” – we are in the region where the LM curve is upward-sloping, so the flat region where the nominal interest rate is zero (and therefore it’s like the interest rate is pegged) is not relevant.

Now let’s suppose that there is a shock which causes the zero lower bound to bind. Assume that
we are initially in an equilibrium in the “normal” part of the LM curve and that output equals its flexible price/wage level, $Y_t^f$. Let’s conceptualize the shock as a change in an exogenous variable which causes the IS curve to shift inward (shown by the blue line below, and labeled IS’), and hence does not affect $Y_t^f$. I assume that this shift is sufficiently big such that the new IS curve intersects the LM curve in the horizontal region. This results in an inward shift of the AD curve and the AD curve becomes vertical at AD’. Output and the price level fall.

A binding ZLB occurs when the nominal interest rate gets to zero, so we think of this is a situation where interest rates are low. Paradoxically, it is actually a situation in which the real interest rate is too high – if the LM curve were upward-sloping instead of horizontal, the real interest rate would be lower than it is as shown above. A situation like this is problematic for two reasons. First, normal optimal monetary policy is not available. Under normal circumstances, an optimizing central bank would counteract the contractionary IS shock with expansionary monetary policy – raising the money supply and therefore cutting interest rates. But since the nominal interest rate cannot fall, the central bank cannot cut interest rates, even if it were to increase the money supply.\footnote{Sometimes the ZLB is referred to as a “liquidity trap” which refers to a situation in which an increase in the}
Hence, when the ZLB binds, the central bank cannot optimally react to a shock like this. But a binding ZLB is also problematic for a second reason – it can get worse. Why is this? We discussed previously that a situation in which $Y_t < Y_t^f$ will naturally be followed by an outward shift of the AS curve as we transition from short run to medium run. This is shown below with the purple line representing an outward shift of the AS curve. With a downward-sloping AD this outward shift of the AS curve would cause output to rise. But with the AD vertical because of the binding ZLB the outward shift of the AS won’t do anything to output, it will just result in lower prices.

This can become especially problematic to the extent to which agents in the economy anticipate this future decline in prices. So as to simplify matters we have thought of expected inflation, $\pi_{t+1}$, money supply does not affect interest rates.

\[ LM' \quad LM \quad IS \quad LM' \]

(1) Negative IS shock which makes ZLB bind (blue)

(2) $Y_t^1 < Y_t^f$: AS curve shifts right due to normal dynamics $\rightarrow$ further price decline, no change in output (purple)

(3) People begin to expect deflation. $\downarrow \pi_{t+1}^e$ to $\pi_{t+1}^e'$. Leads to increase in $r_t$ and inward shift of AD, causing further reduction in $Y_t$ (orange)
as exogenous and fixed. But what if agents in the economy are forward-looking and anticipate that the price level will fall in the future? This would cause expected inflation to go down to something like $\pi_{t+1}^e$. A decrease in expected inflation would shift the LM curve in/up. We can see this with the orange lines in the figure above – both the flat part of the LM curve and the “normal” part shift up. But along a fixed IS curve, an upward shift of the LM curve due to a decrease in expected inflation causes the real interest rate to rise – for example, if the nominal interest rate is zero, and expected inflation goes from 2 percent to 0 percent, then the real interest rate goes from -2 percent to 0. This reduces spending along the IS curve and shifts the AD curve in (orange line), which causes output to fall further (and the price level to decline). So, over time the ZLB can actually get worse – output is low, people start to expect lower inflation going forward, which causes the real interest rate to rise, which leads to further output declines. A situation like this is sometimes called a deflationary spiral in the sense that prices and output can keep falling. This is the second reason why the ZLB is undesirable – not only is normal monetary policy not available when the nominal interest rate gets constrained from below, the situation will tend to naturally get worse on its own.

For these reasons, the ZLB is considered highly undesirable. So what can policy do to escape? Normal monetary policy is not available – there is no room to cut interest rates via an increase in the money supply and outward shift of the LM curve. There are in essence two options for policy. One involves non-standard monetary policy and the other involves fiscal policy. Let’s focus on non-standard monetary policy first. The chief problem with the deflationary spiral situation highlighted above is that agents can start to expect future deflation. Non-standard policy involves trying to convince people on the contrary to expect higher future inflation. Not only would doing so mitigate any deflationary spiral problem, it might also eliminate the ZLB altogether. In essence, a central bank could commit to future expansionary policy in such a way as to convince people to expect higher future inflation in such a way that $\pi_{t+1}^e$ increases. An increase in $\pi_{t+1}^e$ would work like the opposite of a decrease – it would shift the LM curve down/out (both the flat and upward-sloping regions of the curve). In essence, an increase in expected inflation for a fixed nominal interest rate lowers the real interest rate which stimulates spending. If the increase in expected inflation is sufficiently big the ZLB could cease to bind altogether, resulting in a new equilibrium with a lower real interest rate but one where we are in the “normal” upward-sloping region of the LM curve. This is shown below with the blue curves.
Another option for escaping the ZLB is to use policy so as to shift the IS curve to the right. This would involve fiscal policy increasing government spending (or reducing current taxes, to the extent to which Ricardian Equivalence does not hold). As shown above, demand shocks such as this have larger effects on output if the interest rate is fixed than they would under normal monetary policy – this is because if there is no increase in interest rates, there is no “crowding out” due to higher interest rates. Even if the fiscal expansion is sufficiently large so that the ZLB ceases to bind (which is what I’ve shown below) and the real interest rate rises, the increase in the real interest rate will be smaller than it would if we were starting from the “normal” part of the LM curve, so output will expand by more than it would if we were in a normal monetary policy position. For this reason, it is often thought that fiscal policy is particularly effective at the ZLB. A potential added benefit of fiscal expansion is that it will raise prices, which might fend off a deflationary
Given that the zero lower bound on nominal interest rates is costly, it makes sense to try to conduct policy in normal times so as to try and avoid it altogether. How can a central bank do this? Recall that the Fisher relationship says that \( r_t = i_t - \pi_t^{e} \). Over the long run, the value of \( r_t \) is determined by how impatient households are (i.e. the magnitude of \( \beta \)) and how fast the inflation rate goes up.

Another potential advantage of fiscal expansion at the ZLB is that borrowing costs are low because nominal interest rates are low, which means that debt-financed government spending (or tax cuts) have smaller effects on outstanding debt than they would in a more normal interest rate environment. In our baseline world with Ricardian Equivalence this effect wouldn’t be operative, since the mix between debt and taxes is irrelevant for understanding the effects of changes in government expenditure. But in a world where Ricardian Equivalence does not hold (say due to distortionary tax rates or different generations) this effect might matter and it therefore might make relative sense for fiscal expansion in a low interest rate environment.

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Footnote 8: Another potential advantage of fiscal expansion at the ZLB is that borrowing costs are low because nominal interest rates are low, which means that debt-financed government spending (or tax cuts) have smaller effects on outstanding debt than they would in a more normal interest rate environment. In our baseline world with Ricardian Equivalence this effect wouldn’t be operative, since the mix between debt and taxes is irrelevant for understanding the effects of changes in government expenditure. But in a world where Ricardian Equivalence does not hold (say due to distortionary tax rates or different generations) this effect might matter and it therefore might make relative sense for fiscal expansion in a low interest rate environment.
economy grows – its value is independent of monetary policy. Also, over the long run, we would expect \( \pi_{t+1}^e \) to equal the average inflation rate. This means that the long run average value of the nominal interest rate ought to equal the real interest rate plus the average level of inflation. We saw in the notes on money that this relationship holds quite well – over long periods, the nominal interest rate and inflation are positively correlated. This suggests that having higher average rates of inflation might reduce the incidence of zero lower bound episodes. Why is this? If \( \pi^* \) (the long run average rate of inflation) goes from, say, 2 percent to 4 percent, we would expect the average value of the nominal interest rate to increase by 2 percent. The higher is the average value of the nominal interest rate, the less likely we ought to be for the nominal interest rate to hit zero, and therefore the less likely we ought to be run into the problem of the zero lower bound.

This reasoning suggests that a potential policy proposal to reduce the problems of the zero lower bound going forward would be to raise the average level of inflation (i.e. to raise the central bank’s inflation target). There have been many proposals to do just that – an additional benefit of announcing a higher long run inflation target today is that it would raise expected inflation, which might help us escape the ZLB via the logic above. Other economists have countered that this would be too costly outside of the zero lower bound – for a variety of reasons, high inflation ends up being costly. Coibion, Gorodnichenko, and Wieland (2012) find that the optimal rate of inflation in a slightly more sophisticated model than the one with which we are working is about 2 percent per year, which is roughly what the average inflation rate has been in the US over the last three decades.

5 Replacing the LM Curve with the MP Curve (Optional)

Given that the Taylor rule has become so ubiquitous and that it provides such a good description of policy, many have proposed replacing the LM curve (which shows the set of \( (r_t, Y_t) \) pairs where the money market clears for given values of \( M_t \) and \( P_t \)) with an “MP Curve,” where the MP stands for “Monetary Policy.” The MP curve describes the endogenous formulation of policy via the Taylor rule. I will briefly sketch this formulation and use it to think about the desirability of responding strongly to inflation.

Formally, to make this operational we need to move away from the assumption that expected inflation is exogenously given at \( \pi_{t+1}^e \). Rather, let’s assume that agents in the economy have adaptive expectations in the sense that they expect future inflation to equal current inflation, \( \pi_{t+1}^e = \pi_t \).

Use this assumption and subtract \( \pi_t \) from both sides of the Taylor rule expression above:

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9See, for example, Laurence Ball, “The Case for 4% Inflation,” [http://www.voxeu.org/article/case-4-inflation](http://www.voxeu.org/article/case-4-inflation).

10This is so for a number of reasons. First, inflation acts as a tax on the holders of money, and to the extent to which money is socially useful because it facilitates exchange, taxing it is undesirable. Second, there are things like “shoeleather costs” which involve agents taking actions to avoid this inflation tax which can be costly. Third, higher rates of average inflation can introduce non-optimal distortions to the relative prices of different kinds of goods to the extent to which some firms are unable to adjust their prices each period.

\[ i_t - \pi_t = i^* - \pi^* + (\phi_\pi - 1)(\pi_t - \pi^*) + \phi_y (Y_t - Y^f_t) \]

If \( \pi_{t+1}^e = \pi_t \) at all times, then we can re-interpret this specification as a real interest rate rule, since \( r_t = i_t - \pi_{t+1}^e = i_t - \pi_t \) under adaptive expectations. Hence:

\[ r_t = r^* + (\phi_\pi - 1)(\pi_t - \pi^*) + \phi_y (Y_t - Y^f_t) \]

We can plot this in a graph with \( r_t \) on the vertical axis and \( Y_t \) on the horizontal axis. The plot holds \( Y^f_t \) and \( \pi_t \) fixed – if either changes, then the curve shifts. Since \( \phi_y > 0 \), this is upward-sloping:

The IS curve is the same as before – it is the set of \((r_t, Y_t)\) pairs where desired expenditure equals income and is downward-sloping. We can combine the IS and MP curves to derive a new AD relationship. This time the AD curve will be defined as the set of \((\pi_t, Y_t)\) pairs where IS=MP instead of the set of \((P_t, Y_t)\) pairs where IS=LM, but is otherwise conceptually similar. We can derive the AD curve as follows. Start with a value of \( \pi_t \), call it \( \pi_t^0 \). This will determine the position of the MP curve. The point where the MP and IS curves cross, call it \( Y_t^0 \), is the point where IS=MP for that level of inflation. Then try a different level of inflation, call it \( \pi_t^1 \). This will shift the MP curve. Determine the new value of \( Y_t \) where the IS and new MP curves cross, call it \( Y_t^1 \). Then connect the dots.

The slope of the AD curve will depend on the magnitude of the coefficient \( \phi_\pi \). Suppose for a moment that \( \phi_\pi > 1 \). The picture below derives an AD curve for this specification. I start with a value \( \pi_t^0 \) and determine the \( Y_t^0 \) where the IS and MP curves cross. Then I consider a higher value of inflation, \( \pi_t^1 \). Since \( \phi_\pi > 1 \), the coefficient on \( \pi_t \) in the Taylor rule is positive – an increase in inflation raises the real interest rate for any level of \( Y_t \). This means that the MP curve shifts up,
resulting in a lower value of output, \( Y_t^1 \). I can do the same for a lower value of inflation. If we connect the dots we get a downward-sloping curve that qualitatively looks just like the AD curve we had before, it’s just now in terms of inflation and the LM curve has been replaced by the MP curve.

Now let’s instead consider the case where \( \phi_\pi < 1 \). If we repeat the exercise from above, we see that an increase in inflation raises the level of output at which the MP and IS curves cross. This is because, if \( \phi_\pi < 1 \), an increase in inflation results in a lower real interest rate, which from the IS curve stimulates spending. The reverse happens for a decrease in inflation. If you connect the dots, you see that the AD curve becomes upward-sloping in this setup if \( \phi_\pi < 1 \). This is shown in the diagram below.
We can think about the AS relationship as looking qualitatively the same as before (either driven by sticky prices or sticky wages) in a graph with $\pi_t$ on the vertical axis instead of $P_t$. The AD curve being upward-sloping is problematic. First, if the AD curve is upward-sloping, then there may exist no equilibrium at which $AD=AS$. Second, if the AD curve is upward-sloping, there may be multiple equilibria. This isn’t obvious the way I’ve drawn the AD and AS curves as straight lines. Two straight lines can cross at most once in a plane. But if there is curvature in one or both curves, they could cross multiple times if both the AD and AS curves are upward-sloping (if the AD curve is downward-sloping it doesn’t matter if these relationships are lines or curves; if one is always upward-sloping and the other always downward-sloping, they can cross at most once). I show an example below in which the AS curve is a line but the AD curve is upward-sloping, being relatively flat at low levels of $Y_t$ and steep at higher levels of $Y_t$. There are two equilibria in which we are on both the AD and AS curves. I label one as “bad” in the sense of having a low level of output and the other “good” in the sense of having a high level of output.
When there are multiple equilibria the economy can get stuck in a bad equilibrium like this. In addition, the economy may react very non-optimally to non-monetary demand or supply shocks if the AD curve becomes upward-sloping (in particular, you can think about an upward-sloping AD curve as kind of an extreme case of an interest rate peg discussed above). For these reasons, economists deem it desirable that a monetary policy rule like the Taylor rule satisfy what has come to be known as the “Taylor principle”: the coefficient on inflation should be greater than 1. This coefficient being greater than 1 means that real interest rates increase whenever inflation increases, which results in a downward-sloping AD curve and therefore rules out the existence of multiple equilibria.

You can go through and do the normal exercises where we consider changes in exogenous variables to see how the equilibrium of the economy is affected using this setup. Aside from the fact that \( \pi_t \) shows up on the vertical axis in the AD-AS graph, things are qualitatively similar (though you need to be careful in thinking about how the MP curve will shift in response to inflation and changes in \( Y_t^f \)).

References