1 Introduction

We’ve gone half of a semester and made almost no mention of money. Isn’t economics all about money? In this set of notes we define money, introduce it into our neoclassical business cycle model, and talk about the connection between real economic activity, money, prices, and inflation.

2 What is Money?

We typically define things according to intrinsic characteristics of those things. For example, apples are round and red. This is not so with money. Rather, we give money a functional definition. The functions that define money are:

1. It is a medium of exchange. This means that, rather than engaging in barter, one can trade money for goods and services.

2. It serves as a store of value. This means that money holds at least some of its value across time, and is therefore a means by which households can transfer resources across time.

3. It is a unit of account. This simplifies things, as we denominate the value of goods and services in money. This makes it easy to compare value across goods. If we were engaging in barter, we would have to compare relative prices.

In principle, anything can serve these functions, and hence anything can serve as money. In fact, in the past, many different things have served as money. For many years commodities have served as money – thinks like cows, cigarettes, and precious metals (e.g. gold and silver). In more recent years, economies have moved towards fiat money. Fiat money are pieces of paper (or electronic entries) that have no intrinsic value – they just have value because a government declares that they do and market participants accept them in exchange.

You can see why certain kinds of commodities can be problematic as money. First, commodities have value independent of their role as money. Fluctuating commodity values (say there is a drought
that kills off cows, or a new discovery of gold deposits) will generate fluctuating nominal prices of all other goods, which can create confusion. This makes commodities less than desirable as a unit of account. Secondly, commodities may not store well, and hence may not be good stores of value (e.g. crops may not store well in extreme temperatures). Third, commodities are not necessarily easily divisible or transferrable, and hence may be less than ideal as media of exchange (think about trying to divide a cow). Fiat money lacks these problems, but does not come without its own problems. Fiat money only has value because people accept that it has value. If people quit believing that it has value, then it ceases to have value. This makes fiat money quite precarious – if people quit believing that the US government’s currency is valuable, this would wreck the world economy.

The most important function of money is the first listed – that as a medium of exchange. Money is a fairly crummy store of value. Lots of other things can serve as stores of values – houses, stocks, bonds, etc. Because of inflation (general decreases in the value of money), these other things are likely better stores of value. As a unit of account money is convenient, but we could make any one good a unit of account without too much hassle. The first role is the really important one. Without money, we’d have to engage in barter, and this would be costly. For example, I am supplying educational services to you. You (or your parents) are paying me money to do so. Without money, we’d have to trade. Your parents may produce legal services. I (currently) have no need for legal services, and hopefully will not for some time. I do need food and shelter. It would be doable, but very difficult and roundabout, for your parents to pay me in legal services, given that I want food and shelter. The existence of money as a medium of exchange solves the double coincidence of wants problem – once we move beyond barter, it is not necessary that I want to consume what you or your parents produce for us to engage in trade. The existence of money thus allows for more trade, and therefore more specialization. Increased specialization leads to productivity gains that ultimately make us all better off. It is no exaggeration to say that a well-functioning medium of exchange is the most important thing to have developed in economic history, and it is difficult to downplay the importance of money in a modern economy.

We haven’t studied money to this point because, as long as it exists and functions well, it shouldn’t matter too much. Money really only becomes interesting if it does not work well or if there is some other friction with which it interacts. We will introduce money in a fairly simple way in a frictionless world for now, and then we’ll add in some frictions and talk about situations in which it might end up playing a more interesting role later.

3 Adding Money to Our Model

The neoclassical model we’ve written down isn’t that great for studying the role of money. This is because there is only one good – fruit. The lack of multiple goods means that there really isn’t a double coincidence of wants problem here. So we have to take a bit of a short cut. In particular, we are going to assume that people get utility from holding money. This is a reduced form way of modeling the double coincidence problem. You can think about it as follows: the more money
people have, the easier it is to conduct transactions, and therefore the better off they are.

The only interesting difference relative to the baseline neoclassical model is the household side. Though firms will use money to pay workers, the solution to their problem will be identical. Hence, we’ll focus on the household problem and then integrate that into the broader model of the economy.

3.1 Households

Let $M_t$ denote the amount of money that people choose to hold between period $t$ and $t+1$. This has a store of value flavor, as money serves as a means by which the household can transfer resources across time. As stated, this is no different than holding bonds, although, unlike bonds, money does not pay interest. Let $P_t$ be the price of goods in terms of money in period $t$ and $P_{t+1}$ be the price of goods in terms of money in period $t + 1$, which the household takes as given. Equivalently, $\frac{1}{P_t}$ is the relative price of money in terms of goods. Let the units of $M_t$ be dollars, so that $P_t$ has units of “dollars per good.” Let $i_t$ be the nominal interest rate $- 1 + i_t$ is the number of dollars that one gets in period $t + 1$ in return for saving one dollar in period $t$. This is distinguished from the real interest rate, in that the real interest rate is the number of goods that one gets in period $t + 1$ in exchange for saving one good in period $t$. Note that, as a “rate,” an interest rate is itself unit-less, but $i_t$ is the rate paid out on saving money, while $r_t$ is the rate of return on saving goods.

Let’s begin by writing out the household’s budget constraint in nominal terms. Let $P_tw_t$ be the nominal wage (real wage times the price level), and $P_t\Pi_t$ be nominal profit (real profit times the price level). Let $P_tT_t$ be nominal tax obligations. With this income the household can either consume ($P_tC_t$ dollars), save in bonds that pay nominal interest $i_t$ ($P_tS_t$ dollars), or carry money over into the future period (effectively save via money):

$$P_tC_t + P_tS_t + M_t = P_tw_tN_t - P_tT_t + P_t\Pi_t$$

In the second period, the household has income from labor, distributed profit, interest from saving/borrowing, and money that it brought over from the first period. It must also pay nominal taxes. With that it can consume:\(^1\)

$$P_{t+1}C_{t+1} = P_{t+1}w_{t+1} - P_{t+1}T_{t+1} + P_{t+1}\Pi_{t+1} + (1 + i_t)P_tS_t + M_t$$

If you divide each of these constraints through by the price level in each period, you get:

$$C_t + S_t + \frac{M_t}{P_t} = w_tN_t - T_t + \Pi_t$$

$$C_{t+1} = w_{t+1}N_{t+1} - T_{t+1} + \Pi_{t+1} + (1 + i_t)\frac{P_t}{P_{t+1}}S_t + \frac{M_t}{P_{t+1}}$$

\(^1\)In writing the second period constraint I am imposing (i) the household will leave no stock of savings over after period $t + 1$, so that $S_{t+1} = 0$, and (ii) that the household would not leave any stock of money over after period $t + 1$, i.e. $M_{t+1} = 0$. 

3
These constraints are now in real terms – all of the units are units of goods. The period $t$ constraint says that the household has real income from labor and dividends, and with that it can consume goods, save in bonds, or save in money, where $\frac{M_t}{P_t}$ is the equivalent number of goods the household gives up by holding money. We often refer to $\frac{M_t}{P_t}$ as real balances – the is the number of goods one could purchase with the money one has available. In the second period the household has real income from labor and dividends again, as well as income from saving and the real purchasing power of money that it wakes up with in period $t + 1$, $\frac{M_t}{P_{t+1}}$.

In the period $t + 1$ constraint the term $(1 + i_t)\frac{P_t}{P_{t+1}}$ has the interpretation as the real interest rate. The Fisher relationship relates the real interest rate to the nominal interest rate as:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$$

In words, the real interest rate says how many goods one gets back in period $t + 1$ for saving one good in period $t$. In terms of nominal prices, saving one good in period $t$ requires putting $P_t$ dollars in the bank ($P_t$ is how much money one has to spend to purchase one good). This $P_t$ dollars grows to $(1 + i_t)P_t$ dollars in period $t + 1$ thanks to interest, and with this money the household can purchase $(1 + i_t)\frac{P_t}{P_{t+1}}$ goods. Defining $1 + \pi^e_{t+1} = \frac{P_{t+1}}{P_t}$ as the expected inflation rate (growth rate of the money price of goods) between period $t$ and period $t + 1$, the Fisher relationship can be written:

$$1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}}$$

Taking logs and using the approximation that $\ln(1 + x) \approx x$ for $x$ small, we get:

$$r_t = i_t - \pi^e_{t+1}$$

Put differently, the real interest rate is approximately the nominal interest rate minus the expected rate of inflation.

Making the substitution to write the second period budget constraint in terms of the real interest rate, we can solve the second period budget constraint in terms of saving as:

$$S_t = \frac{C_{t+1}}{1 + r_t} + \frac{w_{t+1}N_{t+1} - T_{t+1} + \Pi_{t+1}}{1 + r_t} - \frac{M_t}{P_t} \frac{1}{1 + i_t}$$

Plugging this into the first period budget constraint and re-arranging, we get:

$$C_t + \frac{C_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t} = w_tN_t - T_t + \Pi_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + \Pi_{t+1}}{1 + r_t}$$

This is the real, intertemporal budget constraint. Everything here is real – $\frac{M_t}{P_t}$ is real money balances, and because the nominal interest rate is a rate and hence unit-less, $\frac{i_t}{1 + i_t}$ is just a number. Minus the inclusion of the real balances term, this is the same intertemporal budget constraint we saw before.

As I noted earlier, we assume that the household receives utility from holding money. There is a slight complication here in that, the way we’ve written the problem down, money is held “between
periods” and not really within, so it’s not obvious when the household should receive utility. We assume that utility is received in the first period. Let the lifetime utility function be:

\[ U = u(C_t) + v(1 - N_t) + \phi \left( \frac{M_t}{P_t} \right) + \beta u(C_{t+1}) + \beta v(1 - N_{t+1}) \]

\(\phi(\cdot)\) is a function that is increasing and concave, just like \(u(\cdot)\) and \(v(\cdot)\). The argument of that function is real money balances, \(\frac{M_t}{P_t}\), which effectively says how many goods the money the household chooses to hold could buy. The notion here is that the more money one holds in real terms, the more utility one gets. As I said, this is a reduced-form way of getting at the real benefit of money as a medium of exchange.

We can write the household’s problem as one of choosing current and future consumption, current and future labor, and nominal money holdings subject to the intertemporal budget constraint:

\[
\begin{align*}
\max_{C_t, C_{t+1}, N_t, N_{t+1}, M_t} & \quad U = u(C_t) + v(1 - N_t) + \phi \left( \frac{M_t}{P_t} \right) + \beta u(C_{t+1}) + \beta v(1 - N_{t+1}) \\
\text{s.t.} & \quad C_t + \frac{C_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t} = w_t N_t - T_t + \Pi_t + \frac{w_{t+1} N_{t+1} - T_{t+1} + \Pi_{t+1}}{1 + r_t}
\end{align*}
\]

As before, we can characterize the optimum by solving for \(C_{t+1}\) from the constraint and plugging that back into the objective function, transforming the problem into an unconstrained problem:

\[ C_{t+1} = (1 + r_t)(w_t N_t - T_t + \Pi_t - C_t) + w_{t+1} N_{t+1} - T_{t+1} + \Pi_{t+1} - (1 + r_t) \frac{i_t}{1 + i_t} \frac{M_t}{P_t} \]

\[
\begin{align*}
\max_{C_t, N_t, N_{t+1}, M_t} & \quad U = u(C_t) + v(1 - N_t) + \phi \left( \frac{M_t}{P_t} \right) + \ldots + \beta u \left( (1 + r_t)(w_t N_t - T_t + \Pi_t - C_t) + w_{t+1} N_{t+1} - T_{t+1} + \Pi_{t+1} - (1 + r_t) \frac{i_t}{1 + i_t} \frac{M_t}{P_t} \right) + \beta v(1 - N_{t+1}) \\
\end{align*}
\]

The first order conditions are:

\[ \frac{\partial U}{\partial C_t} = 0 \Leftrightarrow u'(C_t) - (1 + r_t)\beta u'(C_{t+1}) = 0 \]

\[ \frac{\partial U}{\partial N_t} = 0 \Leftrightarrow -v'(1 - N_t) + (1 + r_t)\beta w_t u'(C_{t+1}) = 0 \]

\[ \frac{\partial U}{\partial N_{t+1}} = 0 \Leftrightarrow -\beta v'(1 - N_{t+1}) + \beta w_{t+1} u'(C_{t+1}) = 0 \]

\[ \frac{\partial U}{\partial M_t} = 0 \Leftrightarrow \phi' \left( \frac{M_t}{P_t} \right) \frac{1}{P_t} - \beta \frac{i_t}{1 + i_t} \frac{1}{P_t} (1 + r_t) u'(C_{t+1}) = 0 \]

Combining and simplifying, we get:
\[ u'(C_t) = \beta (1 + r_t) u'(C_{t+1}) \]
\[ v'(1 - N_t) = u'(C_t) w_t \]
\[ v'(1 - N_{t+1}) = u'(C_{t+1}) w_{t+1} \]
\[ \phi' \left( \frac{M_t}{P_t} \right) = \frac{i_t}{1 + i_t} u'(C_t) \]

The first three of these conditions are the same that we’ve had before, and each has the interpretation as a “marginal benefit = marginal cost” condition. The fourth condition also has a “marginal benefit = marginal cost” interpretation, but it requires a bit of work to see that. The left hand side is the benefit of holding an additional unit of real balances, \( \frac{M_t}{P_t} \). This is the extra utility of of holding more money. The cost of holding more money is an implicit cost – by holding an additional unit of real money balances, you are saving one fewer unit in bonds. Saving one unit in bonds would mean putting \( P_t \) dollars into the bond, which would yield \((1+i_t)P_t\) dollars available for future consumption. By saving \( P_t \) dollars in money instead you have \( P_t \) dollars available for future consumption. The nominal “cost” of saving in money is thus \((1+i_t)P_t - P_t = i_t P_t \) – this cost is the foregone interest that could have been earned by saving in bonds instead of money. This extra income would yield \( i_t \frac{P_t}{P_{t+1}} \) units of consumption in period \( t + 1 \), which would generate utility of \( \beta \frac{P_t}{P_{t+1}} u'(C_{t+1}) \) extra units of utility, where the \( \beta \) is in there to discount it back to the present. Combing the Euler equation with the Fisher relationship, one sees that \[ u'(C_{t+1}) = \frac{1}{\beta} \frac{1}{1 + i_t} \frac{P_t}{P_{t+1}} u'(C_t). \]

Combing these we see that the marginal cost of holding money is \[ \frac{i_t}{1 + i_t} u'(C_t). \] This marginal cost must be equated with the marginal benefit at the optimum. Put differently, the final first order condition for money also has the interpretation as a marginal rate of substitution equals price ratio condition.

The first order conditions imply (i) a consumption function, (ii) a labor supply function, and (iii) a money demand function. The consumption function and labor supply function are the same as we had before, and can be written in purely real terms independent of money altogether:

\[ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]
\[ N_t = N^*(w_t, H_t) \]

The consumption function is written assuming that Ricardian Equivalence holds, so that households behave as though the government balances its budget every period. Consumption is increasing in current and future perceived net income, and is assumed to be decreasing in the real interest rate. The expression for labor supply expresses labor supply as an increasing function of the real wage and a function of the exogenous labor supply shifter, \( H_t \).

The final condition implies a money demand curve of the sort:
\[ M_t = P_t M^d(i_t, Y_t) \]

I’ve written this where money demand depends on the product of the price level with a function of the nominal interest rate and output. Strictly speaking, we should have this in terms of consumption, and not output, since it is the marginal utility of consumption that appears on the right hand side of the first order condition. To the extent to which consumption and income are closely related there is not much lost by doing it this way, and it is more common to see money demand written as a function of income, not consumption.

Holding the nominal interest rate and consumption fixed, an increase in the price level would lead to a proportional increase in the demand for money. I could equivalently write this as a demand for real balances by dividing both sides by \( P_t \). Money demand is decreasing in the nominal interest rate, \( \frac{\partial M^d}{\partial i_t} < 0 \), and increasing in output, \( \frac{\partial M^d}{\partial Y_t} > 0 \). The nominal interest rate represents the opportunity cost of holding money, and so it makes sense that money demand is decreasing in the nominal rate. Intuitively, the reason why money demand is increasing in \( Y_t \) is that, the more transactions you are doing, the more valuable it is to have money, and therefore the more money you’d like to hold. In terms of the first order condition above, if \( Y_t \) is higher, then \( C_t \) is higher, which makes \( u'(C_t) \) smaller. For the first order condition to hold if must be the case that \( \phi' \left( \frac{M_t}{P_t} \right) \) also gets smaller, which requires \( M_t \) to get bigger.

We can relate the nominal rate to the real rate using the approximate Fisher relationship, so \( i_t = r_t + \pi_{t+1}^e \). Unless stated otherwise, we are going to take expected inflation to be an exogenous variable. Measured inflation expectations have been pretty stable in the US over the last thirty years, so this seems like a reasonable assumption. From a practical perspective, this means that we can treat movements in the nominal and real interest rates as being the same, unless you are told that there is an exogenous change in expected inflation. We can therefore write money demand as:

\[ M_t = P_t M^d(r_t + \pi_{t+1}^e, Y_t) \]

For the purposes of expositional ease, I will often write this by omitting the expected inflation term, so I’ll write \( M_t = P_t M^d(r_t, Y_t) \). Even though I write it this way, note that what really matters for money demand is the nominal interest rate, not the real (whereas what matters for consumption and labor supply is the real interest rate). Again, to the extent to which expected inflation is stable, movements in these are the same.

### 3.2 Firms

The firm problem is identical to what we had before. In particular, the objective of the representative firm is to maximize the present discounted value of profit, subject to the constraint of the capital accumulation equation. We can write the problem either in nominal or real terms. We will get the same solution to what we had before in real terms, but for the sake of completeness I will write it out here in nominal terms. Writing it in nominal terms means that all the real
terms get multiplied by the price level in that period, and future nominal profits are discounted by the nominal interest rate as opposed to the real interest rate. In this specification the units of profit (and hence value) are dollars, whereas in our earlier setup the units were goods (fruit). The problem can be written:

\[
\max_{N_t, N_{t+1}, I_t, I_{t+1}} \quad V_t = P_t A_t F(K_t, N_t) - P_t w_t N_t - P_t I_t + \frac{1}{1 + i_t} \left( P_{t+1} A_{t+1} F(K_{t+1}, N_{t+1}) - P_{t+1} w_{t+1} N_{t+1} - P_{t+1} I_{t+1} \right)
\]

s.t.

\[
K_{t+1} = q I_t + (1 - \delta) K_t \\
K_{t+2} = q I_{t+1} + (1 - \delta) K_{t+1}
\]

This is a constrained problem, with the constraints written in real terms. As before, we can take care of the constraints by substituting them into the objective function. In particular, impose the terminal condition discussed above that \( I_{t+1} = -\frac{(1 - \delta) K_{t+1}}{q} \), and eliminate \( I_t = \frac{K_{t+1} - (1 - \delta) K_t}{q} \). This allows us to write the problem as one of choosing \( K_{t+1} \) instead of investment (similarly to how we wrote the household problem as one of choosing future consumption instead of current saving):

\[
\max_{N_t, N_{t+1}, K_{t+1}} \quad V_t = P_t A_t F(K_t, N_t) - P_t w_t N_t - P_t \frac{K_{t+1}}{q} + (1 - \delta) P_t \frac{K_t}{q} + \ldots
\]

\[
\ldots + \frac{1}{1 + i_t} \left( P_{t+1} A_{t+1} F(K_{t+1}, N_{t+1}) - P_{t+1} w_{t+1} N_{t+1} + (1 - \delta) P_{t+1} \frac{K_{t+1}}{q} \right)
\]

The first order conditions of the problem are:

\[
\frac{\partial V_t}{\partial N_t} = 0 \Leftrightarrow A_t F_N(K_t, N_t) = w_t
\]

\[
\frac{\partial V_t}{\partial N_{t+1}} = 0 \Leftrightarrow A_{t+1} F_N(K_{t+1}, N_{t+1}) = w_{t+1}
\]

\[
\frac{\partial V_t}{\partial K_{t+1}} = 0 \Leftrightarrow -\frac{P_t}{q} + \frac{1}{1 + i_t} \left( P_{t+1} A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta) P_{t+1} \right) = 0
\]

The final first order condition can be simplified:

\[
\frac{\partial V_t}{\partial K_{t+1}} = 0 \Leftrightarrow \frac{1}{q} = \frac{P_{t+1}}{1 + i_t} \left( A_{t+1} F_K(K_{t+1}, N_{t+1}) + \frac{(1 - \delta)}{q} \right)
\]

Using the Fisher relationship, \( 1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \), the final first order condition reduces to exactly what we had before:
\[ 1 = \frac{1}{1 + r_t} (A_{t+1} q F_K(K_{t+1}, N_{t+1}) + (1 - \delta)) \]

Just as in our earlier problem, these first order conditions imply (i) a demand curve for labor, and (ii) a demand curve for investment goods:

\[ N^d = N^d(w_t, A_t, K_t) \]
\[ I^d = I^d(r_t, A_{t+1}, q, K_t) \]

Here labor demand is decreasing in the real wage, shifts out whenever \( A_t \) goes up, and shifts in if \( K_t \) declines. Investment demand is decreasing in the real interest rate, shifts out if \( A_{t+1} \) goes up, shifts out if \( q \) goes up, and shifts out if \( K_t \) declines.

### 3.3 Government

There is also a government. As before, the government chooses its (real) spending, \( G_t \) and \( G_{t+1} \), exogenously, and taxes, \( T_t \) and \( T_{t+1} \), must adjust to make its budget constraint (more below) hold.

In addition to its fiscal responsibility, the government in this set up is also responsible for “printing” money. I put “printing” in quotation marks because in reality the money may just be electronic. The government decides how much money to print exogenously. Let this money supply be \( M^g_t = M_t \).

Because printing money is costless, the government printing money is essentially a form of revenue. The technical term for this is seignorage. The money that the government prints can be used to purchase goods and services, and so functions very much like a tax. The government’s first period budget constraint in nominal terms is:

\[ P_t G_t + P_t S^G_t = P_t T_t + M_t \]

The right hand side reflects the fact that printing money, \( M_t \), is just like revenue for the government. Written in real terms by dividing by the price level, this is:

\[ G_t + S^G_t = T_t + \frac{M_t}{P_t} \]

This looks exactly like the first period government budget constraint we had before, plus the addition of real seignorage revenue, which takes the form of \( \frac{M_t}{P_t} \). The second period budget constraint is:

\[ P_{t+1} G_{t+1} = P_{t+1} T_{t+1} + (1 + i_t) P_t S^G_t \]

Written in real terms, we have:
\[ G_{t+1} = T_{t+1} + (1 + i_t) \frac{P_t}{P_{t+1}} S_t^G = T_{t+1} + (1 + r_t) S_t^G \]

The last line uses the Fisher relationship to eliminate the nominal interest rate and the price levels. Solving for government saving:

\[ S_t^G = \frac{G_{t+1}}{1 + r_t} - \frac{T_{t+1}}{1 + r_t} \]

Plugging this back into the first period constraint and re-arranging yields the government’s intertemporal budget constraint:

\[ G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} + \frac{M_t}{P_t} \]

This looks the same as what we had before, plus the addition of the real balance term on the right hand side, which again represents seignorage revenue for the government. As before, we assume that households know that the government budget constraint must hold, and hence Ricardian Equivalence will hold. Given a time path for \( G_t, G_{t+1}, \) and \( M_t, \) households know that taxes must adjust to make the government budget constraint hold.

Note that here we are completely abstracting from lots of interesting stuff related to the central bank and its interaction with the fiscal side of the economy. We are just assuming for simplicity that the government/central bank can exogenously specify the quantity of money in circulation, and that it does not do so in any optimizing way. This is a large abstraction from the real world, where there is a complicated money creation process.

### 3.4 Money Supply and the Money Market

We can think about the “money market” in a partial equilibrium setting by graphically depicting money supply and money demand. Money supply, as noted above, is exogenous. Money demand is a function of the price level, the nominal interest rate, and output. Assuming that expected inflation is constant, we can replace the nominal rate with the real interest rate. In thinking about the money market in partial equilibrium, we are going to take the real interest rate and output as given, and will determine the price level.

The “price” of money in terms of goods is \( \frac{1}{P_t} \) – in other words, one unit of money would buy \( \frac{1}{P_t} \) units of goods, so that is how many goods one has to give up in order to get a unit of money. As above, however, we’re going to write the money demand function as \( M_t = P_t M_d(r_t, Y_t) \), so let’s draw a picture with \( P_t \) on the vertical axis, with money demand increasing in \( P_t \) given \( r_t \) and \( Y_t \). This feels a little weird, since the demand curve here will slope up and we are used to demand curves sloping down. Recall, however, that the demand curve here really is sloping down, we’ve just inverted the “price” of money on the vertical axis:
Here I show money demand as upward-sloping, crossing through the origin. It must cross through the origin because if $P_t = 0$, then there would be no need for money, and so the quantity of money demanded would be 0. The money demand curve as drawn would pivot to the right if $r_t$ were to fall or $Y_t$ were to rise – either of these movements would increase the quantity of money demanded at every price.

The money supply curve is just some vertical line at the exogenously specified quantity of money that the government decides to print:

Equilibrium in the money market occurs at a price, $P_t$, where the quantity of money demanded
is equal to the quantity supplied:

\[ P_t = P_t M_d(r_t^0 + \pi_{t+1}^t, Y_t^0) \]

Holding \( r_t \) and \( Y_t \) fixed (so that money demand is fixed), exogenous increases in money supply will cause the price level to increase – a rightward shift of the \( M^s \) curve causes \( P_t \) to increase.

Increases in money demand – due either to decreases in the real interest rate or increases in output – cause the price level to decline, given a money supply curve. The money demand curve would also shift to the right if expected inflation exogenously decreased, because this would lower the nominal interest rate and therefore lower the opportunity cost of holding money.

### 3.5 Equilibrium and the Classical Dichotomy

General equilibrium requires that all markets in the economy clear. We effectively have three markets now: the labor market, the goods market, and the money market. All three must simultaneously clear. We have three prices: \( r_t, w_t, \) and now \( P_t \) (and \( i_t \), though this is pretty uninteresting since it moves one-for-one with \( r_t \) assuming fixed expected inflation). \( r_t \) is the intertemporal price of goods – how much does it cost (either implicitly or explicitly) to borrow from the future? \( w_t \) is the goods price of labor – how many goods must a firm give up to hire an additional unit of labor. \( P_t \) is the “money price” of goods – how many dollars does it take to buy one good (fruit). We could also include the nominal interest rate as a price. Like the real interest rate, this says how much it costs to borrow from the future, but the cost is in terms of money (dollars), not goods as for the real interest rate.

The labor market clearing requires that the quantity of labor demanded equal the quantity supplied: \( N^d_t = N^s_t \). Mathematically this requires:

\[ N^d(w_t, A_t, K_t) = N^s(w_t, H_t) \]
The goods market clearing requires that desired expenditure equal total income: \( Y^d_t = Y_t \) when both households and firms are behaving optimally. Total desired expenditure is once again \( Y^d_t = C_t + I_t + G_t \). This can be derived by imposing that \( S_t = -S^G_t \) (household saving equals government borrowing) and combining the definition of profit/dividend with the household budget constraint. Mathematically, the goods market clearing requires:

\[
Y^d_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I(r_t, A_{t+1}, q, K_t) + G_t
\]

Total income is equal to total production, which is given by the production function:

\[
Y_t = A_t F(K_t, N_t)
\]

The money market-clearing requires that the quantity of money demanded equals the quantity supplied. Mathematically:

\[
M_t = P_t M^d(r_t + \pi^e_{t+1}, Y_t)
\]

The exogenous variables of the model are: \( A_t, A_{t+1}, K_t, G_t, G_{t+1}, q \), and now \( \pi^e_t, M_t \) (the money supply set by the central bank). These are the same exogenous variables in the neoclassical model, appended to include the two nominal exogenous variables of expected inflation and the money supply. The endogenous variables are \( Y_t, C_t, I_t, N_t, r_t, w_t, i_t, \) and \( P_t \). These are again the same endogenous variables that we had in the neoclassical model without money, now appended to include two endogenous nominal variables – the nominal interest rate, \( i_t \), and the price level, \( P_t \).

We know have a total of eight conditions that all must simultaneously hold. These are:

\[
N_t = N^d(w_t, A_t, K_t)
\]
\[
N_t = N^s(w_t, H_t)
\]
\[
C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)
\]
\[
I_t = I(r_t, A_{t+1}, q, K_t)
\]
\[
Y_t = C_t + I_t + G_t
\]
\[
Y_t = A_t F(K_t, N_t)
\]
\[
M_t = P_t M^d(r_t + \pi^e_{t+1}, Y_t)
\]
\[
i_t = r_t + \pi^e_{t+1}
\]

This is now eight equations in eight endogenous variables. The endogenous variables are \( Y_t, C_t, N_t, r_t, w_t, I_t, P_t, \) and \( i_t \). The first six of these equations are exactly the same as we had
earlier when there was no money in the model. There are six equations (labor demand, labor supply, consumption demand, investment demand, total goods demand, and total production) and six endogenous variables (consumption, investment, output, labor, the real interest rate, and the real wage). This is a critical point: mathematically, we can solve for the equilibrium values of all of these “real variables” uniquely without any mention of the nominal variables. In other words, we can use the same graphical apparatus we used before to solve for the “real” side of the equilibrium without knowing anything about nominal variables. Once we know the values of the real variables in the economy, in conjunction with an exogenous money supply we can figure out the values of nominal endogenous variables.

The way this works is as follows: the real side of the economy determines $r_t$ and $Y_t$. The money supply and expected inflation are exogenously given. Knowing $r_t$ and being given $\pi_{t+1}^e$, we can determine a value for $i_t$. Knowing $i_t$ and $Y_t$ we can determine a position of the money demand curve. Knowing a position of the money demand curve in conjunction with the exogenously given position of the money supply curve determines the price level, $P_t$.

The above paragraphs lay out the gist of an important concept known as the classical dichotomy. The classical dichotomy says that real variables (output, the real interest rate, the real wage, and the components of output) are determined independently of nominal variables (money, prices, nominal interest rates). This means that we can solve for the equilibrium of the real side of the economy exactly as we did before without reference to any nominal variables. Then, knowing the equilibrium of the real side of the economy, we can solve for the equilibrium values of the nominal variables. The classical dichotomy also means that “money is neutral” in the sense that changes in the money supply have no real effects. This is an issue to which we will return below.

We can work through the effects of changes in the exogenous variables on all the endogenous variables, including the nominal ones. We can determine the real variables exactly as we did before. Consider first an exogenous increase in $A_t$. We know from earlier that this causes $r_t$ to fall and $Y_t$ to rise (in addition, $C_t$, $I_t$, $w_t$, and $N_t$ rise). Once we know what happens to $r_t$ and $Y_t$, we can determine what happens to the endogenous nominal variables, $P_t$ and $i_t$. Given that we assume expected inflation is exogenous, a falling real interest rate means that the nominal interest rate must also fall. $r_t$ falling and $Y_t$ rising both work to shift money demand to the right. Along a stable money supply curve, equilibrium in the money market requires the price level to fall. This is shown in the graph below showing equilibrium in the money market. In terms of what happens to $i_t$ and $P_t$, the effects of an exogenous labor supply shock, manifested as an increase in $H_t$, will be identical – the nominal interest rate and price level will both fall.
Next, consider a “demand shock” driven by a change in an exogenous variable that shifts the $Y^d$ curve. This could be due to an increase in $A_{t+1}$, an increase in $q$, an increase in $G_t$, a decrease in $G_{t+1}$, or a decrease in $K_t$. We know from earlier that these changes have no effect on $Y_t$ in equilibrium and cause the real interest rate to rise. Since we assume that expected inflation is constant, a rising real interest rate means that the nominal interest rate also rises. A rising real interest rate causes the money demand curve to shift in. For the money market to be in equilibrium with a fixed supply of money, this requires the price level to rise. We can see this in the graph below:
Now let’s work through a change in the money supply. This does not show up in the equations determining the equilibrium of the real side of the economy. This means that changes in $M_t$ have no effect on any of the real variables of the economy. The only effect of an increase in $M_t$ in the economy is to shift the money supply curve. Given that there is no shift in the money demand curve, this just has the effect of increasing the price level, $P_t$. Since we assume that expected inflation between $t$ and $t + 1$ is exogenously given, with no change in the real interest rate there is no change in the nominal interest rate either.

In this example, we say that “money is neutral.” Money is neutral in the sense that changes in $M_t$ have no effect on real variables. Many casual observers of the real world don’t buy this – they think that changes in money have real effects. That may be the case, but it requires some kind of friction (some of which we’ll examine soon). I think that monetary neutrality is the right benchmark from which to start. Changes in the quantity of money are essentially changes in the way we measure prices, but there is no reason why such changes should affect the amount of stuff that we produce. For example, a football field is 100 yards (endzone to endzone). One yard is 3 feet. Changing the definition of a foot such that one yard is 6 feet (without changing the “real” length of a yard) would have no effect on the dimension of the football field, and therefore should have no effect on how the game is played. It just changes the units of measurement. Changes in money in this model are similar – such changes don’t affect real quantities, they just change the price level (the units of measurement).

The table below summarizes the effects of changes in exogenous variables on the two endogenous nominal variables in the model:
A useful rule of thumb to remember is this: positive supply shocks (things which shift the $Y^s$ curve to the right) cause the price level to fall, while positive demand shocks (things which shift the $Y^d$ curve to the right) cause the price level to increase.

4 Money, Inflation, and Interest Rates: the Long Run

What determines the rate of inflation in the long run? What about the level of nominal interest rates? Let’s take a step back and combine some elements of our model with money with some facts about the long run behavior of the economy to answer these important questions.

To think about this clearly let’s move beyond the two period framework. Time exists for many periods, but the basic money demand-supply relationship must hold every period. That money market equilibrium is characterized by the following relationship:

$$M_t = P_t M^d (i_t, Y_t)$$

To make things more concrete, let’s use a specific functional form assumption, so that we get:

$$M_t = P_t \frac{1 + i_t}{i_t} Y_t$$

Take logs of both sides:

$$\ln M_t = \ln P_t + \ln (1 + i_t) - \ln i_t + \ln Y_t$$

Now first difference this expression, subtracting off the period $t - 1$ expression:

$$\ln M_t - \ln M_{t-1} = \ln P_t - \ln P_{t-1} + i_t - i_{t-1} + (\ln i_t - \ln i_{t-1}) + \ln Y_t - \ln Y_{t-1}$$

Above I have used the approximation that $\ln (1 + i_t) \approx i_t$. Now, we are going to make use of a couple of facts we’ve already seen from our discussion of long run economic growth. First, over long time horizons output grows at a roughly constant rate. Call this $g^y$; in the data this is about 0.8 percent per quarter, or 3.2 percent per year. Second, real interest rates are roughly constant over time; if expected inflation is roughly constant over time, then $i_t \approx i_{t-1}$ over long periods of time. Using these facts, we can re-write the above expression:

$$\ln M_t - \ln M_{t-1} = \ln P_t - \ln P_{t-1} + g^y$$

Now, the first difference of the price level is just inflation; $\pi_t = \ln P_t - \ln P_{t-1}$. The first difference
of the money supply is just the growth rate of the money supply; call this \( g_t^M \). Since we are talking about these variables over long time horizons, drop the time subscripts. Making these substitutions and re-arranging:

\[
\pi = g^M - g^y
\]

In words, this expression says that inflation ought to be equal to growth in the money supply minus growth in output. We know from the Solow model that the growth rate of output is determined by technology and population growth and ought to be independent of changes in the money supply. As I mentioned, at a quarterly frequency average output growth in the data is about 0.8 percent. If I go to the data and look at the average growth rate of the money supply, where I define the money supply to be M2, I see that on average growth rate of the money supply is 1.6 percent per quarter (or about 6.4 percent per year).

The difference between the growth rate of the money supply and the growth rate of output, on average, is therefore about 0.8 percent per quarter. This turns out to be exactly what the average growth rate of prices is (i.e. the average rate of inflation). In other words, this expression holds remarkably well in the aggregate US data: over long periods of time, inflation is determined by the difference between the growth rate of money and the growth rate of output. This kind of expression led Milton Friedman to famously conclude that “inflation is everywhere and always a monetary phenomenon.”

The figure above plots the HP trend levels of money growth (growth in M2) and inflation (growth in the GDP deflator) across time. I label these as “smoothed” inflation and money growth, because the HP trend is essentially a two-sided moving average. Plotting this way makes the figure much easier to see relative to a plot of the raw series, where there would be lots of high frequency gyrations. We can observe that there is a strong correlation between these two series – money growth and inflation are closely linked.

\[\text{http://en.wikipedia.org/wiki/Money_supply}\]
growth and inflation seem to move in lock step with one another. The actual correlation between
the series in the data is 0.67. Among other things, we see that the end of the high inflation of the
1970s coincided with a slowdown in the growth rate of the money supply. There is some divergence
in the behavior of the series starting about 1990. In particular, the growth rate of the money supply
ticked up without any pick up in inflation. Very recently we’ve also seen an up-tick in the growth of
the money supply without any inflation materializing (yet). To reiterate: in the long run, inflation
is equal to the excess growth of money over output.

In our short run analysis of the economy we treated expected inflation as an exogenous constant.
This allowed us to interpret movements in the real interest rate and the nominal interest rate as one
in the same. Over long periods of time, it is unlikely that expected inflation is simply an exogenous
constant. Rather, we would expect people to observe what actual inflation is and update their
expectations accordingly. Put differently, if people expect 2 percent inflation but keep seeing 4
percent inflation, it stands to reason that over time they would adjust up their expected inflation.
Hence, over long periods of time, we would expect people to be correct on average, so that expected
inflation should equal average inflation over long enough periods of time.

If growth in the money supply determines inflation over long time horizons, then inflation over
long time horizons determines nominal interest rates. This is because of the Fisher relationship,
which states that: \( r_t \approx i_t - \pi_t. \) If \( r_t \) is determined on the real side of the economy and isn’t
growing (recall the Solow stylized facts), then \( i_t = r_t + \pi_t. \) In the long run, the real interest
rate is determined from the household’s consumption Euler equation and depends on consumption
growth and \( \beta. \) It should be independent of inflation.

Since movements in inflation shouldn’t affect real interest rates over long horizons, this suggests
that movements in inflation ought to be correlated with movements in nominal interest rates. This
is in fact what we observe in the data. The figure below plots the HP trend of nominal interest
rates (the three month Treasury Bill rate) and the HP trend of inflation (growth rate of the GDP
deflator). We again see that these series move pretty well in lock step with one another. The actual
correlation is 0.71.
The conclusion here is that, in the long run, the growth rate of the money supply determines average inflation, and average inflation determines expected inflation, which then determines the level of nominal interest rates. We therefore expect (sufficiently long) periods of high money growth to be associated with high inflation and high nominal interest rates. This is in fact what we observe. We also see this in the cross-section: countries with high inflation rates tend to have high nominal interest rates.

5 Money and Output: the Short Run

The basic neoclassical model predicts that money is neutral – changes in the quantity of money have no real effects. That doesn’t seem to correspond with how most casual observers view the world. People certainly think that central banks have some control over the real economy, at least for a while.

What do the data say? I again measure the money supply by the natural log of M2. I measure output as the log of real GDP. The plot below shows the HP detrended time series of the money supply and output. We observe that the series appear to be positively correlated (in the sense of tending to move up and down together). This positive co-movement is particularly apparent in the early half of the sample, and is less obvious more recently. Most of the early recessions (indicated by gray bars in the graph) in the early part of the sample are clearly associated with declines in both the money supply and output relative to trend. This is somewhat less obvious later in the sample – though there is a decline in the money supply just prior to the 2001 recession, but we don’t really see that in the lead up to the most recent recession (the money supply increased a lot during the recession, before declining in its wake).
The table below formalizes some of this co-movement. In particular, I compute correlations between HP filtered \( \ln M_t \) and \( \ln Y_{t+j} \), for \( j \geq 0 \). When \( j = 0 \) this is just the contemporaneous correlation between the two series. For the full sample, we see that this correlation is positive at about 0.2. When \( j > 0 \) we are looking at the correlation between the current money supply and future output. This is a crude way to assess whether changes in the money supply cause changes in output. To the extent to which this correlation is significant, we would say that the money supply leads output in the sense that a high money supply predicts high future output. We do in fact see that money appears to lead output: the correlation of the money supply with output led three quarters is about 0.35, for example, and remains positive at a two year (eight quarter) lead. These correlations are not overwhelming but are definitely positive.

<table>
<thead>
<tr>
<th>Series</th>
<th>Correlation with ( \ln M_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln Y_t )</td>
<td>0.1915</td>
</tr>
<tr>
<td>( \ln Y_{t+1} )</td>
<td>0.2912</td>
</tr>
<tr>
<td>( \ln Y_{t+2} )</td>
<td>0.3410</td>
</tr>
<tr>
<td>( \ln Y_{t+3} )</td>
<td>0.3352</td>
</tr>
<tr>
<td>( \ln Y_{t+4} )</td>
<td>0.3055</td>
</tr>
<tr>
<td>( \ln Y_{t+5} )</td>
<td>0.2577</td>
</tr>
<tr>
<td>( \ln Y_{t+6} )</td>
<td>0.2009</td>
</tr>
<tr>
<td>( \ln Y_{t+7} )</td>
<td>0.1285</td>
</tr>
<tr>
<td>( \ln Y_{t+8} )</td>
<td>0.0503</td>
</tr>
</tbody>
</table>

The table below shows correlations between the growth rates of the money supply and output,
with output led several quarters (the growth rates are just log first differences, denoted with the $\Delta$ operator). We see a similar picture here – money growth leads output positively in the sense that high money growth predicts higher output growth, both in the present and in the future.

<table>
<thead>
<tr>
<th>Series</th>
<th>Correlation with $\Delta \ln M_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \ln Y_t$</td>
<td>0.0805</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+1}$</td>
<td>0.1952</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+2}$</td>
<td>0.2119</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+3}$</td>
<td>0.1334</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+4}$</td>
<td>0.1025</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+5}$</td>
<td>0.0646</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+6}$</td>
<td>0.0799</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+7}$</td>
<td>0.0454</td>
</tr>
<tr>
<td>$\Delta \ln Y_{t+8}$</td>
<td>0.0182</td>
</tr>
</tbody>
</table>

Does that fact that money seems to lead output in the data imply that changes in the money supply causes real output to move? Not necessarily. There could be reverse causality at play – central banks could anticipate higher future output (say, because they think that future productivity will be high) and increase the money supply in anticipation of that. That said, there are a number of more sophisticated ways to look at this question which try to address the reverse causality issue. Most of these studies do indicate that changes in the money supply do in fact result in movements in output and other real variables.\footnote{See, for example, Christiano, Lawrence, Martin Eichenbaum, and Charles Evans. 1999. “Monetary Policy Shocks: What Have We Learned and to What End?” Handbook of Macroeconomics, vol. 1A, edited by John B. Taylor and Michael Woodford.} In other words, empirically money does not seem to be neutral as the neoclassical model would suggest. To get monetary non-neutrality the model needs to be modified in some way. The most obvious way is to introduce some kind of sluggishness in nominal prices or wages. This is what we will be doing next when we study Keynesian models.

References