1. Consumption:

(a) Changes in consumption ought to be larger the more persistent are changes in income. Mechanically, this is because the partial derivative of the consumption function with respect to future income is positive. If both $Y_t$ and $Y_{t+1}$ change, then consumption will increase by more than if just $Y_t$ changed holding $Y_{t+1}$ fixed (which is what the MPC tells you).

(b) The intuition for this is based on consumption smoothing. Households want to smooth their consumption relative to their income, and accomplish this by adjusting their saving behavior. In other words, when income increases in any period, households want to “spread” this around, increasing consumption in all periods. For a transitory change in income, the household must increase its saving when income increases. But the more persistent is a change in income, the household needs to adjust its saving by less since it will have more income in the future and consume out of that, rather than out of its savings.

(c) This has real world implications for things like tax policies which are designed to stimulate consumption expenditure. The more persistent a change in a household’s net income, the more it ought to want to increase its consumption.

(d) Wealth is another factor that ought to influence consumption behavior that is not modeled explicitly in the basic framework. Wealth could be things like the value of houses or stocks. Increases in wealth look isomorphic to increases in income and encourage greater consumption. Changes in wealth have had potentially important effects on recent boom and bust periods (the stock market increases of the late 1990s and ensuing decline in early 2000s and the housing price increase and decline in the mid-2000s).

(e) Uncertainty is another potential factor which can influence consumption. If your future income is uncertain, the Euler equation looks the same but there is an expectation operator on future marginal utility:

$$u'(C_t) = \beta (1 + r_t) E(u'(C_{t+1}))$$

The key insight is that, because $u'(C_{t+1})$ is itself a function, in general $E(u'(C_{t+1}) \neq u'(E(C_{t+1}))$. In other words, the expected marginal utility is not equal to the marginal utility of expected consumption. To see this concretely, suppose that future consumption can take on two values: 3 and 1, each with probability 1/2. Then expected future consumption is $E(C_{t+1}) = 0.5 \times 3 + 0.5 \times 1 = 2$. Suppose that the utility function is log, so that marginal utility is one over consumption. Then expected marginal utility is:

$$E\left(\frac{1}{C_{t+1}}\right) = 0.5 \times \frac{1}{3} + 0.5 \times \frac{1}{1} = 2/3.$$  
Marginal utility of expected consumption is $1/2$. Hence, for this utility function, expected marginal utility is greater than marginal utility of expected consumption.
If the utility function has a positive third derivative (which means that marginal utility is convex, in the sense that it has a positive second derivative), then an increase in uncertainty raises the expected marginal utility of future consumption. The log utility function satisfies this property. If expected marginal utility of future consumption goes up, then households will need to change their behavior to get the marginal utility of current consumption to increase in order to get the Euler equation to hold. This involves reducing current consumption (equivalently, increasing current saving). We call this precautionary saving.

Precautionary saving provides a different motive for saving than in our standard model where the future is known. In the standard model, the motivation for saving is to smooth consumption relative to income. Under precautionary saving, there is an insurance motive to saving: you save in the presence because you are more worried about bad future “states” (draws of income) than you are excited about good future “states” (provided the third derivative of the utility function is positive), and so save more to build up a bigger stock of savings, which makes the bad state not hurt you as much. High uncertainty has been cited as a potential explanation for low consumption expenditure in the wake of the recent recession.

Our two period model can be easily extended to account for more than two periods. Utility flows are always discounted relative to the previous period by $\beta$; this means that relative to period $t$, utility flow in period $t + j$ gets multiplied by $\beta^j$. For $j$ sufficiently big this weight gets small. The intertemporal budget constraint is conceptually similar, requiring that the present value of the flow of consumption expenditure equal the present value of the flow of income. For simplicity, we assume a constant real interest rate across time periods, $r$.

If there are $T + 1$ periods of life (the current period plus $T$ additional periods), then there will be $T$ Euler equations of the form:

$$u'(C_t) = \beta(1 + r)u'(C_{t+1})$$

$$u'(C_{t+1}) = \beta(1 + r)u'(C_{t+2})$$

$$\vdots$$

$$u'(C_{t+T-1}) = \beta(1 + r)u'(C_{t+T})$$

The intuitive interpretation of these Euler equations is the same as before. If $\beta(1+r) > 1$, then we will have $C_{t+1} > C_t$. In other words, consumption will be expected to grow over time. This is because there real interest rate is relatively high and the discount factor, $\beta$, is also relatively high, which means you are relatively patient. If $\beta(1+r) < 1$, then we will have $C_{t+1} < C_t$. In other words, consumption will be expected to decline over time. If $\beta(1+r) = 1$, then consumption will be expected to remain constant over time.

If we assume that $\beta(1+r) = 1$, then consumption is constant and we can combine this implication with the IBC to get a consumption function. In this consumption function, consumption is proportional to the present discounted value of the stream of income. For this reason, this theory of consumption is sometimes called the “permanent income hypothesis” – consumption depends on the present value of the stream of income (which is sometimes called “permanent income”), not income in any one period. With multiple periods the distinction between “permanent” and “transitory” changes in income on consumption is even starker than in the two period model – permanent changes in
income increase consumption one-for-one and transitory changes in income have small
effects on consumption (close to zero if the household lives long enough).
A simple way to see this point is to suppose that \( \beta = 1 \) and \( r = 0 \). This implies that
consumption is constant across time. In addition, because of a zero interest rate, the
IBC simply says that the sum of consumption across life must equal the sum of income.
This means that the consumption function is to set consumption in each period of life
equal to average lifetime income. A persistent or permanent change in income has a
relatively big effect on average lifetime income, and therefore has a relatively big effect
on consumption. A transitory change in income has a small effect on average lifetime
income, particularly if the household lives for a long time (\( T \) is big). Permanent changes
in income ought to be mostly all spent (with little or no adjustment of saving) while
transitory changes in income ought to be mostly saved (with little or no adjustment of
consumption).

(g) Our theory of consumption is often called the “lifecycle / permanent income hypothesis.”
The theory has the following testable implications:

i. Consumption ought to be forward-looking, incorporating information about future
income, not just current income

ii. Consumption ought to react a lot to permanent changes in income, and not much
to transitory changes in income

iii. Consumption ought not to react to changes in income that were anticipated in the
past – this is a corollary of the first point, which is that consumption today ought
to incorporate information about future income.

We looked at several pieces of empirical evidence to test the theory. We see that in some
cases the theory does well, in other cases it fails. In particular, there seem to be several
instances in which consumption appears to react to anticipated changes in income. This
suggests some kind of failure of forward-looking behavior on the part of households.

(h) One potential resolution of these issues is to assume that some households are inhibited
in their ability to borrow and save. We call such constraints “liquidity constraints” (or
sometimes “borrowing constraints”). Liquidity constraints seem pretty realistic for a
large fraction of the population. A simple form of a liquidity constraint is to assume
that households cannot borrow (in a two period framework), requiring \( S_T \geq 0 \). This
introduces a kink into the budget constraint occurring at the endowment point. We say
that a liquidity constraint is “binding” if the household would like to borrow but cannot.
If a household wants to save in the absence of the liquidity constraint then the constraint
is non-binding and therefore irrelevant for the household. An indifference curve budget
line diagram with a binding liquidity constraint is shown below:
A binding liquidity constraint has the following implications. First, the household will spend all of any additional income in period $t$ (i.e. its MPC will be 1), provided the change in current income is not so large as to make the liquidity constraint non-binding. This has implications for tax and transfer policies designed to stimulate spending, and suggests that these transfers should be targeted toward low income people who are most likely to be liquidity constrained. Second, a household will not react in period $t$ to an anticipated change in income in period $t+1$ (because this would cause the household to want to increase its borrowing in period $t$, which it cannot). This means that consumption will go up in period $t+1$ (with no change in period $t$ consumption), when the anticipated change in income takes place. This provides a potential resolution of some of the empirical failures of the theory we highlighted in class.

2. Equilibrium:

(a) We begin by studying equilibrium in an endowment economy. By “endowment economy” I mean that we take $Y_t$ and $Y_{t+1}$ to be exogenous

(b) The generic definition of a competitive equilibrium is a set of prices and allocations such that (i) all agents are behaving optimally, taking prices as given, and (ii) all markets are simultaneously clearing. In an endowment economy, the price is the real interest rate, $r_t$, the intertemporal price of consumption. Agents behaving optimally means that they behave according to the optimal decision rule from their problem, or in this case the consumption function, $C_t = C(Y_t, Y_{t+1}, r_t)$ (in a world with no government spending). Markets clearing requires that $Y_t = C_t$. So we can think about a competitive equilibrium in an endowment economy as the real interest rate adjusting endogenously to make $C_t = Y_t$ be consistent with the consumption function.
(c) We analyze the competitive equilibrium graphically. The $Y^s$ curve is defined as the set of $(r_t, Y_t)$ pairs consistent with the production function and agents behaving optimally. In an endowment economy we do not model production, so this is just a vertical line in a graph with $r_t$ on the vertical axis and $Y_t$ on the horizontal axis at the exogenous level of the endowment. The $Y^d$ curve is the set of $(r_t, Y_t)$ pairs (i) consistent with agent optimization and (ii) income being equal to expenditure. In equilibrium we must have income equal to expenditure which is equal to production. The $Y^s$ curve summarizes the production side of the economy (which is here not very interesting) and the $Y^d$ curve summarizes the income equaling expenditure side of the economy.

(d) Graphically, the $Y^d$ curve can be derived as follows:

\[
\begin{align*}
Y^d_t &= Y_t \\
Y^d_t &= C(Y_t, Y_{t+1}, r_t^1) \\
Y^d_t &= C(Y_t, Y_{t+1}, r_t^0) \\
Y^d_t &= C(Y_t, Y_{t+1}, r_t^2)
\end{align*}
\]

In the upper graph we plot desired expenditure, which is the sum of desired expenditure by each type of agent, against income, not necessarily imposing that the two are the same. Total desired expenditure is just given by the consumption function, $Y^d_t = C(Y_t, Y_{t+1}, r_t)$. This is upward-sloping with slope equal to the MPC, which is less than one. We assume that there is positive desired expenditure even if $Y_t = 0$. The point where the expenditure line cross a 45 degree line showing all points where $Y^d_t = Y_t$ is the level of $Y_t$ consistent with agent optimization and income being equal to expenditure. The position of the expenditure line depends on the real interest rate. A higher real interest rate means that households want to consume less for every level of $Y_t$, so the expenditure line shifts down and therefore intersects the 45 degree line at a lower level of $Y_t$. The reverse is true for a lower real interest rate. Connecting the dots we get a downward-sloping curve which we call the $Y^d$ (or “output demand”) curve.

(e) Equilibrium requires being on both the $Y^d$ and $Y^s$ curves. Graphically:
(f) The exogenous variables in an endowment economy are the current endowment, $Y_t$, and the future endowment, $Y_{t+1}$. The endogenous variables are $r_t$ and $C_t$ and $C_{t+1}$ ($r_t$ is the price and $C_t$ and $C_{t+1}$ are the allocations). We can examine the equilibrium effects of changes in $Y_t$ and $Y_{t+1}$ as follows. An increase in the current endowment shifts the $Y^s$ curve out, results in a lower real interest rate, and has consumption increasing by the amount of the increase in $Y_t$: 
An anticipated change in future income results in the $Y^d$ curve shifting right, an increase in $r_t$, and no change in $C_t$ (with $C_{t+1}$ going up by the amount of the increase in $Y_{t+1}$):
In equilibrium, the real interest rate ends up being a measure of the expected plenitude of the future relative to the present. If the future looks better relative to the present, the real interest rate rises. If the present looks more plentiful relative to the future, the real interest rate must fall. This happens because, in essence, the real interest rate adjusts to undo the desired consumption smoothing behavior by households. When the future looks better, households want to increase their consumption in the present, which requires reducing their saving. But there is no means for them to reduce their saving (or increase their borrowing): since everyone is the same and there is no means by which to transfer resources intertemporally (i.e. no capital), saving must be zero in equilibrium (i.e. \( Y_t = C_t \) so \( S_t = Y_t - C_t = 0 \)). Hence, the real interest rate must rise to dissuade any reduction in saving / increase in borrowing. When the present becomes better than the future, households would like to increase their saving (or reduce their borrowing). This again cannot happen, so the real interest rate must fall to offset this, so that having \( C_t = Y_t \) is consistent with the optimal decision rule, i.e. the consumption function.

We then add a government to the model. We do not model why the government does any spending, we just assume that it does so and take it to be exogenous. In particular, let \( G_t \) and \( G_{t+1} \) be exogenous. The government finances this spending with lump sum taxes, \( T_t \) and \( T_{t+1} \). Lump sum taxes mean that these taxes are additive – the household has to pay them regardless of decisions that the household makes. The government has two within period budget constraints given by:

\[
G_t + S_t^G = T_t \\
G_{t+1} = T_{t+1} + (1 + r_t)S_t^G
\]
$S^G_t$ denotes government saving. I have imposed a terminal condition that the government does not die in debt (so $S_{t+1}$ cannot be negative) and does not die with any positive assets (so that $S^G_{t+1}$ cannot be positive); hence, $S^G_{t+1} = 0$ and is therefore omitted in the $t+1$ constraint. These constraints can be combined to yield the intertemporal budget constraint for the government, which in words says that the present value of the stream of government expenditure must equal the present value of the stream of government revenue:

$$G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t}$$

The household’s two within period budget constraints are now:

$$C_t + S_t = Y_t - T_t$$

$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

These can be combined to yield the intertemporal budget constraint:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t}$$

Because these taxes enter the household’s budget constraint additively (i.e. they are lump sum), the IBC for the household can be written:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t} - \left( T_t + \frac{T_{t+1}}{1 + r_t} \right)$$

Now, because the household knows that the government’s IBC must hold, this can be written:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t} - \left( G_t + \frac{G_{t+1}}{1 + r_t} \right)$$

Re-arranging:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - G_t + \frac{Y_{t+1} - G_{t+1}}{1 + r_t}$$

This gives rise to the Ricardian Equivalence result: households behave as though the government balances its budget each period, with $T_t = G_t$ and $T_{t+1} = G_{t+1}$, whether the government does so or not. Combining the Euler equation (which is standard) with the intertemporal budget constraint so written then implies the consumption function:

$$C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$

Again, the basic gist here is that the household behaves as though the government balances its budget each period. The essential intuition here is that the household only cares about the present value of the stream of its net income; since the the government’s intertemporal budget constraint must hold, the present value of the stream of the household’s net income must equal the present value of the stream of its income minus the present value of the stream of government spending. A change in current taxes, $T_t$, not met by a change in $G_t$ or $G_{t+1}$ has no effect on the present value of the stream of the household’s tax obligations, and therefore does not affect household behavior. A change in $G_t$ (or $G_{t+1}$) has the same effect on the household whether it is financed with taxes (so that $T_t$ goes up by the increase in $G_t$) or debt (where there is no change in $T_t$, with $S^G_t$ going down). The latter effect results because $T_{t+1}$ must go up by an amount equal
in present value to the increase in $G_t$. So the “equivalence” part of Ricardian Equivalence means that the manner of government finance (taxes or debt) is irrelevant for what happens to household behavior and hence what happens in equilibrium when there is a change in $G_t$ (or $G_{t+1}$) – taxes and debt are equivalent, because debt is in essence future taxes. Ricardian Equivalence does not mean that changes in government spending have no effect on variables in equilibrium; it means that the manner of government finance is irrelevant.

Ricardian Equivalence requires the following to hold:

i. Taxes must be lump sum
ii. Agents must be forward-looking
iii. Agents cannot be liquidity constraints
iv. There is no heterogeneity among households
v. Households live the same amount of time as the government

None of these are particularly realistic features. For example, if the government “outlasts” households, then it can tax future generations to pay for current spending, which current generations would prefer relative to paying taxes now. Similarly, if the government taxes the rich and gives that to the poor (i.e. there is some heterogeneity to work with), Ricardian Equivalence will break down. And, in the real world, some households are likely liquidity constrained, not everyone is forward-looking and will anticipate that the government’s IBC must hold, and taxes aren’t really lump sum, but are rather functions of things over which people have at least some control (e.g. how much income to earn). So Ricardian Equivalence probably doesn’t exactly hold in the real world, but it nevertheless provides us with some useful insights.

(h) In equilibrium we must have $S_t = -S^G_t$ – household saving must equal government borrowing. Using this fact with the household’s budget constraint gives rise to the aggregate resource constraint, $Y_t = C_t + G_t$. This just in essence sums up expenditure by the two types of agent in the model. Plugging in the optimal consumption function assuming Ricardian Equivalence, we get that $Y_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + G_t$. Taxes do not show up (a consequence of Ricardian Equivalence), and therefore do not affect the equilibrium.

(i) An increase in $G_t$ results in the $Y^d$ curve shifting out and $r_t$ rises. Consumption gets completely “crowded out” by the increase in $G_t$, with consumption falling by the full amount of the increase in $G_t$. We can see this graphically:
An increase in $G_{t+1}$ causes the $Y^d$ curve to shift in. There is a reduction in $r_t$ and no change in consumption. We can see this graphically:
(j) The government spending multiplier is defined as the change in output divided by a change in current government spending. Mathematically, it is $\frac{dY}{dG_t}$. We can see in the endowment economy the multiplier is 0. This is not a statement about the real world necessarily and is driven by the assumption that current output is exogenous, and hence by definition cannot react endogenously to an increase in $G_t$.

We can derive an expression for the “fixed interest rate multiplier” in our model by totally differentiating the expression for desired expenditure holding the real interest rate fixed. Here we see that the fixed interest rate multiplier is 1 – this means that the horizontal shift in the $Y^d$ curve when $G_t$ goes up is equal to the change in $G_t$. To get a the multiplier expression from a principles class, in addition to a fixed real interest rate one needs to assume away Ricardian Equivalence and assume that the spending increase is not financed by current taxes. Then you would get a fixed interest rate multiplier of $\frac{1}{1 - MPC} > 1$ since $0 < MPC < 1$. The equilibrium multiplier would still be 0 in an endowment economy. Since Ricardian Equivalence is unlikely to hold exactly and there may be other features which allow output to rise after an increase in $G_t$, the real world multiplier ought to be somewhere between 0 and $\frac{1}{1 - MPC}$. In other words, the endowment economy multiplier of 0 is a lower bound and the principles multiplier of $\frac{1}{1 - MPC}$ is an upper bound.

3. Neoclassical business cycle model: the neoclassical business cycle model combines some elements from the Solow model related to production with some of our study of consumption. It will produce an “operation model” of the business cycle which can be “taken to the data.”

(a) Firms produce output using the production function $Y_t = A_tF(K_t, N_t)$. This function has the same properties as in the Solow model, except we abstract from long run trend
growth and put a $t$ subscript on $A$ to allow $A_t$ and $A_{t+1}$ to potentially differ.

(b) Differently from the Solow model, here we model the firm as owning its own capital stock and making capital accumulation decisions. Firm profit is given by

$$\Pi_t = Y_t - w_t N_t - I_t.$$  

Profits are distributed to owners via dividends each period, so I will use the terms “profit” and “dividend” interchangeably. You can think about investment, $I_t$, as foregoing current profit in exchange for more profit opportunities in the future. The value of the firm is the present discounted value of the stream of profits:

$$V_t = \Pi_t + \frac{\Pi_{t+1}}{1+r_t}$$

(c) Capital accumulates according to the following law of motion:

$$K_{t+1} = qI_t + (1 - \delta)K_t$$

This looks the same as the capital accumulation equation in the Solow model, except for the presence of the variable $q$. We call $q$ an “investment shock.” It measures the efficiency with which investment is transformed into new physical capital, $K_{t+1}$. Implicitly we had normalized it to 1 in the Solow model. We can think about $q$ as measuring the health of the financial system – the higher is $q$, the better is the financial system.

(d) The firm problem is to pick $N_t$, $N_{t+1}$, $I_t$, and $I_{t+1}$ to maximize its value subject to the capital accumulation equation. Since the firm ceases operations after period $t+1$, it will want $K_{t+2} = 0$. This implies a terminal condition on $I_{t+1} = -\frac{(1-\delta)K_{t+1}}{q}$. We can think of this as the liquidation value of the firm’s capital. After producing in $t+1$ it will have $(1 - \delta)K_{t+1}$ units of capital left over, which it can transform back into profit at rate $\frac{1}{q}$. We can then write the problem as unconstrained and as choosing $K_{t+1}$ (instead of $I_t$) by solving for $I_t$ from the accumulation equation as $I_t = \frac{K_{t+1}}{q} - \frac{(1-\delta)K_t}{q}$. The first order optimality conditions for the firm problem are:

$$w_t = A_t F_N(K_t, N_t)$$

$$w_{t+1} = A_{t+1} F_N(K_{t+1}, N_{t+1})$$

$$1 = \frac{1}{1+r_t} [qA_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta)]$$

The first two first order conditions are the same as in the Solow model. The fact that they are the same in both periods $t$ and $t+1$, and that they only depend on current period stuff, means that we sometimes call these “static” conditions. They have the interpretation of hiring labor up until the point at which the marginal cost of an additional unit of labor (the real wage) equals the marginal benefit of an additional unit of labor (the extra output you get from more labor, the marginal product of labor). The final condition has the same marginal benefit equals marginal cost interpretation. The marginal cost of an additional unit of $I_t$ is 1 fewer unit of current profit. The marginal benefit of more investment is $q$ more units of $K_{t+1}$. This increases future output by the future marginal product of capital times $q$ (the extra capital). You are also left over with $(1 - \delta)q$ extra units of capital to liquidate at $\frac{1}{q}$. So the term inside the brackets above is the extra profit/dividend in $t+1$ from more investment in $t$. Because the benefit is received in the future, it is discounted by $\frac{1}{1+r_t}$. 

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From these first order conditions we can derive the following qualitative decision rules:

\[ N_t = N^d(w_t, A_t, K_t) \]
\[ I_t = I^d(r_t, A_{t+1}, q, K_t) \]

Labor demand is a decreasing function of the real wage and is an increasing function of \( A_t \) and \( K_t \) (i.e. the labor demand curve would shift right if either of these exogenous variables were to increase). Investment demand is a decreasing function of \( r_t \), an increasing function of \( A_{t+1} \), and a decreasing function of \( K_t \). We assume that investment demand is increasing in \( q \), which loosely relies on an assumption that something like a substitution effect dominates something akin to an “income effect.”

(e) The household problem needs to be amended to take account of variable labor supply. We assume that lifetime utility is:

\[ U = u(C_t) + v(1 - N_t) + \beta [u(C_{t+1}) + v(1 - N_{t+1})] \]

\( u(\cdot) \) and \( v(\cdot) \) are increasing and concave functions of their arguments, consumption and leisure. Leisure is \( 1 - N_t \), where the total time endowment is normalized to one. \( N_t \) is labor supplied. \( \beta \) is a discount factor. The household’s intertemporal budget constraint says that the present value of the stream of consumption must equal the present value of the stream of income, but not there is an endogenous component to income owing to variable labor supply:

\[ C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t - T_t + \Pi_t + \frac{1}{1 + r_t} [w_{t+1} N_{t+1} - T_{t+1} + \Pi_{t+1}] \]

The household takes taxes, \( T_t \) and \( T_{t+1} \), and dividends received, \( \Pi_t \) and \( \Pi_{t+1} \), as given. The household knows that the government’s intertemporal budget constraint must hold. Since households are not liquidity constrained and taxes are lump sum, Ricardian Equivalence is assumed to hold, so the household acts as if the government balances its budget each period.

We can solve for \( C_{t+1} \) from the constraint and plug it into the lifetime utility function to turn the household’s optimization problem into an unconstrained problem. The first order conditions can be written:

\[ u'(C_t) = \beta (1 + r_t) u'(C_{t+1}) \]
\[ v'(1 - N_t) = u'(C_t) w_t \]
\[ v'(1 - N_{t+1}) = u'(C_{t+1}) w_{t+1} \]

The first expression is the same consumption Euler equation we’ve seen before and has the same interpretation. The latter two conditions implicitly define optimal labor supply and look the same in both \( t \) and \( t + 1 \) (and are hence sometimes said to be “static” optimality conditions). These have the interpretation as marginal benefit equals marginal cost. If you take an additional unit of leisure, you increase your current utility by the marginal utility of leisure, \( v'(1 - N_t) \). What is the cost of this? An additional unit of leisure means working one less unit of time, which lowers your income by \( w_t \). You value this in utils by \( u'(C_t) \) (which measures the foregone utility from foregoing \( w_t \) units of income). At an optimum, the marginal benefit of taking more leisure must equal the marginal cost.
We can use these first order conditions to derive optimal decision rules. The consumption function looks the same as in the simpler frameworks, where we impose that Ricardian Equivalence holds:

\[ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]

The labor supply curve assumes that the substitution effect of a higher real wage dominates the income effect, so that labor supply is increasing in the real wage. We abstract from other things which would impact optimal labor supply (mechanically, anything which affects \( C_t \) other than \( w_t \) would do this) and fold them into the exogenous variable \( H_t \). We define it such that an increase in \( H_t \) makes households want to supply more labor:

\[ N_t = N^s(w_t, H_t) \]

(f) The government is the same as in the endowment economy. It chooses spending exogenously and finances this spending with lump sum taxes. We assume that its intertemporal budget constraint must hold at all times.

(g) The definition of a competitive equilibrium is the same as before. It is a set of prices and allocations such that all agents are behaving optimally, taking prices as given, and all markets simultaneously clear. The following equations must hold in a competitive equilibrium:

\[ N_t = N^s(w_t, H_t) \]
\[ N_t = N^d(w_t, A_t, K_t) \]
\[ C_t = C(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]
\[ I_t = I^d(r_t, A_{t+1}, q, K_t) \]
\[ Y_t = A_t F(K_t, N_t) \]
\[ Y_t = C_t + I_t + G_t \]

This is six equations in six endogenous time \( t \) variables – \( Y_t, C_t, I_t, N_t, w_t, \) and \( r_t \). Loosely, we can think about there being two markets. \( w_t \) adjusts to clear the labor market and \( r_t \) adjusts to clear the goods market. The exogenous variables are \( A_t, A_{t+1}, q, G_t, G_{t+1}, K_t, \) and \( H_t \).

(h) We again use the \( Y^d \) and \( Y^s \) curves to graphically analyze the equilibrium of the economy. The \( Y^s \) curve is defined as the set of \((r_t, Y_t)\) pairs consistent with the production function and the labor market clearing. Hence, it graphically summarizes labor demand, labor supply, and the production function. The \( Y^d \) curve is the set of \((r_t, Y_t)\) pairs consistent with agent optimization where income equals expenditure. It graphically summarizes the consumption function, the investment demand function, and the resource constraint. Graphically, equilibrium is given by:
(i) Increases in $A_t$ or $H_t$ will shift the $Y^s$ curve to the right; a decrease in $K_t$ would shift the $Y^s$ curve to the left. We call changes in these exogenous variables “supply shocks” since they shift the supply curve. Increases in $A_{t+1}$, $q$, and $G_t$, and decreases in $G_{t+1}$ and $K_t$, shift the $Y^d$ curve right. We therefore call these “demand shocks.” Only changes in $K_t$ affect both the output demand and supply curves.

(j) When analyzing the equilibrium effects of changes in exogenous variables, we ignore the effect that changes in $I_t$ will have on $K_{t+1}$ and hence $Y_{t+1}$. This is to simplify matters but is also reasonable given that the stock of capital is large relative to the flow of investment.

(k) To graphically analyze the equilibrium effects of changes in an exogenous variable, obey the following steps:

i. Start in the labor market and see if the labor demand or supply curves shift
ii. Then see if the production function shifts in the sense of getting more (or less) $Y_t$ for a given $N_t$
iii. Combine the previous two steps to determine if the $Y^s$ curve shifts and in which direction
iv. See if the $Y^d$ curve shifts
v. Combine the $Y^d$ and $Y^s$ shifts to determine new values of $r_t$ and $Y_t$
vi. Look at the decision rules for consumption and investment, along with the equilibrium changes in $r_t$ and $Y_t$ and the given exogenous change, to determine how $C_t$ and $I_t$ react.

(l) Graphically, the effects of an increase in $A_t$ are shown below:

(m) Graphically, the effects of an increase in $H_t$ are shown below:
(n) Graphically, the effects of an increase in $A_{t+1}$, an increase in $q$, an increase in $G_t$, or a decrease in $G_{t+1}$, are given below (for the purposes of this graph, it doesn’t matter which exogenous variable is changing; for figuring out $C_t$ and $I_t$ react it is relevant):
The table below summarizes the qualitative effects of changes in exogenous variables on the endogenous variables of the model:

<table>
<thead>
<tr>
<th>Variable</th>
<th>↑ $A_t$</th>
<th>↑ $A_{t+1}$</th>
<th>↑ $q$</th>
<th>↑ $G_t$</th>
<th>↑ $G_{t+1}$</th>
<th>↑ $H_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Hours</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Consumption</td>
<td>+</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Investment</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Real wage</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

(o) Can the neoclassical model provide a realistic account of real world economic fluctuations? The model is sometimes called the “real business cycle” model (RBC) because everything in the model is real. When referring to the “business cycle” we are thinking about the short run fluctuations in output about trend. In the data, we can think about the business cycle as being measured by the detrended fluctuations in output:
(p) If we define the business cycle in terms of movements in output, the only exogenous variables which can account for business cycle fluctuations are $A_t$ and $H_t$ because these are variables which shift the output supply curve. In this sense the neoclassical model emphasizes “supply shocks” as the source of business cycles. To determine which is a better candidate source of business cycles we need to dig deeper. In the data, we observe that output is positively correlated with consumption, investment, labor hours, and the real wage; it is negatively correlated with the real interest rate. We say that series positively correlated with output are procyclical and series negatively correlated with output are countercyclical. The table below summarizes the correlations among series we see in the data:

<table>
<thead>
<tr>
<th>Series</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.78</td>
</tr>
<tr>
<td>Investment</td>
<td>0.85</td>
</tr>
<tr>
<td>Hours</td>
<td>0.87</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.14</td>
</tr>
<tr>
<td>Real Interest Rate</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

The variables in this table are the same endogenous variables for which the model makes predictions. Changes in $H_t$ move $w_t$ and $Y_t$ in opposite directions in the model. Since the real wage is procyclical (and this number likely understates the true procyclical of the real wage because of the composition bias, which refers to the fact that job loss in a recession tends to be concentrated among lower wage workers, and removing these workers from the ranks for the employed tends to artificially make the average wage look high in a downturn), changes in $H_t$ are not a good candidate source of fluctuations in the model. Changes in $A_t$, however, produce qualitative co-movements among endogenous variables which are qualitatively consistent with what we observe in the data.

(q) Is there any evidence that $A_t$ fluctuates in the data in a way consistent with what we see happening to output? It turns out that we can back out a measure of $A_t$ in the
data (which we sometimes call total factor productivity, or TFP) by assuming a Cobb-Douglas production function and measuring $A_t$ as the residual output which cannot be accounted for by observed capital and labor. The graph below plots detrended empirical TFP with detrended output:

These series are very positively correlated, with periods with output falling below trend corresponding with periods where empirical $A_t$ seems to be declining (and vice-versa). The correlation between measured TFP and output in the data is 0.79. Hence, not only can the model qualitatively account for business cycles if predominantly driven by changes in $A_t$, there seems to be some evidence that $A_t$ does indeed move around in ways that line up with observed fluctuations in output. We might interpret this as evidence in favor of the empirical validity of the model.

(r) To the extent to which we think the neoclassical / RBC model is a good empirical model of business cycles, what are the normative implications? Let us examine the problem of a fictitious “social planner” who chooses allocations of consumption, investment, and labor to maximize household well-being, subject to the constraints that the economy as a whole faces. The planner’s problem is:

$$\max_{C_t, C_{t+1}, N_t, N_{t+1}, K_{t+1}} \quad U = u(C_t) + v(1 - N_t) + \beta u(C_{t+1}) + \beta v(1 - N_{t+1})$$

s.t.

$$C_t + \frac{K_{t+1}}{q} - \frac{(1 - \delta)K_t}{q} + G_t = A_t F(K_t, N_t)$$

$$C_{t+1} - \frac{(1 - \delta)K_{t+1}}{q} + G_{t+1} = A_{t+1} F(K_{t+1}, N_{t+1})$$

There are no prices in the planner’s problem. Prices coordinate actions in a competitive equilibrium, but here the planner makes allocation decisions without regard to prices. The first order conditions of the planner’s problem are:
\[ v'(1 - N_t) = u'(C_t)A_tF_N(K_t, N_t) \]
\[ v'(1 - N_{t+1}) = u'(C_{t+1})A_{t+1}F_N(K_{t+1}, N_{t+1}) \]
\[ u'(C_t) = \beta u'(C_{t+1})(qA_{t+1}F_K(K_{t+1}, N_{t+1}) + (1 - \delta)) \]

These turn out to be the same as the first order conditions from the competitive equilibrium when we combine conditions from the household and firm side so as to eliminate prices. In other words, the competitive equilibrium allocations are the same as what the social planner would choose. We say that the competitive equilibrium is “efficient” in this sense. The competitive equilibrium allocations represent the best possible (welfare-maximizing) allocations, given the exogenous variables. We may not like it if \( A_t \) declines, for example, but the reaction of the economy is the best possible reaction to this.

This has the implication that there is no justification for government intervention in the business cycle. Short run fluctuations in output are the efficient reactions to changing exogenous variables.

(s) The neoclassical / RBC model and its normative implications have been the subject of much criticism. A non-exhaustive list of criticisms are shown below:

i. In the data output and hours worked are about as volatile as one another. The model does not generate large enough movements in hours worked relative to the data. A potential way for the model to generate more movements in hours is to also have volatile labor supply shocks through \( H_t \). This would also tend to lower the model’s correlation between \( w_t \) and \( Y_t \). However, it seems pretty unrealistic that there are large swings in preferences over work period-to-period.

ii. The model has no role for active monetary policy and features monetary non-neutrality. In the real world people certainly think that the Fed matters. Empirically, measures of the money supply are positively correlated with economic activity. This could represent reverse causality (a central bank adjusting the money supply in response to economic activity), but a lot of empirical research suggests that exogenous changes in the stance of monetary policy do impact economic activity.

iii. The model does not allow for demand shocks to impact output, which seems counter to what we observe in the data.

iv. If \( A_t \) is a measure of productivity, what does it mean for productivity to decline? Did we forget how to produce? Is it plausible that that \( A_t \) moves around a lot over short horizons?

v. There have been lots of people that have questioned the measurement of \( A_t \). In particular, we may not have good measures of \( N_t \) and \( K_t \). In particular, there could be unobserved utilization – for example, during recessions workers sit at work idle, so actual labor input is declining, but measured labor input does not decline by much, and so too much of the movement in output gets attributed to changes in \( A_t \).

vi. The model does not have any heterogeneity. Real world recessions are costly not because everyone suffers income that is slightly lower, but rather because some people lose all of their income whereas others suffer no income loss. This means that we want to be careful about taking the model’s policy and welfare implications too seriously. See the previous comments about inequality and some of our previous work on inequality.