1 Introduction

This handout provides a brief, rough, and incomplete review of what we’ve done this semester. I start by listing and defining variables, then parameters, then key equations, and then finally show a couple of graphs. Reviewing this sheet is not a substitute for going back through the course material.

A quick word. The timing notation in this course is that period $t$ is the “current” period and time runs discretely forward from that. With the exception of the Solow model and a couple of places in the two period consumption model, we just focus on two periods ($t + 1$ is a stand-in for the “future”).

2 Variables

1. $Y_t$: output (also equal to income and expenditure)
2. $Y^d_t$: expenditure (which must equal output in equilibrium)
3. $C_t$: consumption
4. $I_t$: investment
5. $N_t$: labor input (total hours, or if you prefer, total employment . . . we do not model the extensive and intensive margins separately, so using any term is fine)
6. $G_t$: government spending
7. $M_t$: money supply
8. $r_t$: real interest rate (how many goods you receive in period $t + 1$ for saving one good in period $t$)
9. $w_t$: real wage (how many goods a firm pays a worker in exchange for one unit of the worker’s labor effort)

10. $R_t$: real rental rate on capital in the Solow model; how many goods a firm must give up to lease one unit of capital for a period

11. $i_t$: nominal interest rate

12. $P_t$: price of goods in terms of money (price of money in terms of goods is therefore $\frac{1}{P_t}$)

13. $\pi_{t+1}^e$: expected one period ahead inflation rate

14. $P_t^e$: expected within period price level

15. $A_t$: total factor productivity

16. $Z_t$: variable governing trend productivity growth in the Solow model

17. $q$: investment-specific productivity (may also be interpreted as measuring health of financial sector)

18. $Y_t^f$: the flexible price level of output; the equilibrium level of output that would emerge in the RBC model. Sometimes also called the “natural rate.”

19. $k_t$: capital stock per worker in the Solow model (more generally, any lower case variable in the Solow model is per capita)

20. $\hat{k}_t$: capital stock per effective worker in the Solow model (more generally, any lower case variable with a “hat” in the Solow model is per effective/efficiency worker)

3 Parameters

1. $\beta$: utility discount factor for households

2. $\delta$: depreciation rate on capital

3. $s$: saving rate in the Solow model

4. $\alpha$: production function parameter for Cobb-Douglas production function; exponent on capital, corresponds to fraction of income paid out to capital

5. $\gamma$: parameter governing the “slope” of the Phillips Curve. A measure of price stickiness; the closer $\gamma$ is to zero, the stickier are prices; as $\gamma \to \infty$, prices are flexible
4 Equations

1. \( K_{t+1} = I_t + (1 - \delta)K_t \): capital accumulation equation. Capital is predetermined within period. In RBC and NK models we augment this equation with \( K_{t+1} = qI_t + (1 - \delta)K_t \), where \( q \) measures efficiency with which investment is transformed into capital.

2. \( Y_t = AF(K_t, Z_tN_t) \): production function in the Solow model. \( Z_tN_t \) is efficiency units of labor; we permit both \( Z_t \) and \( N_t \) to grow over time via \( Z_t = (1 + g_z)^t \) and \( N_t = (1 + g_n)^t \), where \( g_z \) and \( g_n \) are exogenous and fixed growth rates. In the Solow model we assume that \( A \) is fixed across time, and use that to think about variations in the level of income across countries. Example production function: \( Y_t = AK_t^\alpha Z_t^\gamma N_t^{1-\alpha} \)

3. \( Y_t = A_tF(K_t, N_t) \): production function in the business cycle models. It has the same properties as above: it is increase and concave in each argument, and constant returns to scale. We abstract from trend growth (think about setting \( g_z = 0 \), which above fixes \( Z_t = 1 \) for all periods. Here we permit \( A_t \) to be time-varying (hence, the \( t \) subscript).

4. \( U = u(C_t) + v(1 - N_t) + \phi \left( \frac{M_t}{P_t} \right) + \beta (u(C_{t+1}) + v(1 - N_{t+1})) \): lifetime utility function. Households get flow utility from both consumption and leisure (leisure is \( 1 - N_t \), where I have normalized the time endowment to unity). They also get utility from real money balances; real money balances are held between periods, and we assume utility is received in period \( t \). Utility is increasing and concave in consumption, leisure, and real money balances. \( \beta \) is the discount factor. In models where we don’t have labor as a choice, just drop the \( v(1 - N_t) \) terms; in models without money, just drop the \( \phi \left( \frac{M_t}{P_t} \right) \) term.

5. \( C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_{t+1}} \): the basic intertemporal budget constraint in the two period consumption-saving model.

6. \( Y_t = C_t + I_t + G_t \): aggregate resource constraint. This reflects that, in equilibrium total output/income (the left hand side) must equal total expenditure (the right hand side). In models without investment or government spending, just set these terms equal to zero.

7. \( 1 + r_t = (1 + i_t) \frac{P_{t+1}}{P_t} \): Fisher relationship relating nominal and real interest rates. Can be written in approximate forms by taking logs as \( r_t = i_t - \pi_{t+1} \), where \( \pi_{t+1} = \ln P_{t+1} - \ln P_t \) is the expected inflation rate between \( t \) and \( t+1 \), which we take as exogenously given.

8. \( u'(C_t) = \beta u'(C_{t+1})(1 + r_t) \): Euler equation reflecting the optimal tradeoff between current and future consumption; implicitly defines consumption function.

9. \( v'(1 - N_t) = u'(C_t)w_t \): first order condition for optimal labor supply; implicitly defines labor supply curve.

10. \( w_t = A_tF_N(K_t, N_t) \): first order condition for choice of labor by firms; says to hire labor up until point where wage equals marginal product of labor; implicitly defines labor demand curve.
11. $\phi'(\frac{M_t}{P_t}) = \frac{i_t}{1+r_t}u'(C_t)$: optimality condition for choice of money holdings; implicitly defines a money demand curve

12. $1 = \frac{1}{1+r_t}(qA_{t+1}F_{K_t}(K_{t+1}, N_{t+1} + (1-\delta))$: first order condition for optimal choice of future capital; implicitly defines investment demand

13. $C_t = C(Y_t-G_t, Y_{t+1}-G_{t+1}, r_t)$: consumption function; says consumption increasing function of current “perceived” net income, future “perceived” net income, and the real interest rate. Consumption is increasing in the first two arguments and decreasing in the real interest rate; the partial derivative with respect to the first argument, or the MPC, is between 0 and 1 because of households have a desire to smooth consumption. I write “perceived” with quotation marks because we assume that Ricardian equivalence holds, which has the implication that households behave as though the government balances its budget every period, whether it does so or not.

14. $N_t = N^s(w_t, r_t)$: labor supply curve, says that labor supply is increasing in the real wage (substitution effect dominates income effect) and increasing in the real interest rate (households want to save more when $r_t$ goes up, so they both consume less and work more).

15. $N_t = N^d(w_t, A_t, K_t)$: labor demand curve. Labor demand is decreasing in the real wage, increasing in $A_t$, and increasing in $K_t$.

16. $I_t = I^d(r_t, A_{t+1}, q, K_t)$: investment demand curve. Says investment is decreasing in the real interest rate, increasing in future productivity, $A_{t+1}$, increasing in $q$, and decreasing in $K_t$.

17. $\frac{M_t}{P_t} = M^d(i_t, Y_t)$: money demand curve. Says demand for real balances is decreasing in the nominal interest rate and increasing in real output. Can be re-written using the approximate Fisher relationship as: $\frac{M_t}{P_t} = M^d(r_t + \pi_{t+1}^e, Y_t)$. Can also be written by multiplying both sides by $P_t$ to isolate $M_t$ on the left hand side.

18. $P_t = P_t^e + \gamma (Y_t - Y_t^f)$: Phillips Curve/aggregate supply relationship. Says that if $P_t > P_t^e$ (prices higher than expected), then $Y_t > Y_t^f$ (as long as $\gamma$ isn’t $\infty$). Reflects the underlying idea that if prices are higher than expected, some firms end up with relative prices that are lower than desired, which means they have to produce more; hence aggregate output ends up higher.

5 Graphs

The subsections below show the main graphs from some different models with which we have worked. These are not complete and I do not re-do derivations.
5.1 Solow Model

This plots capital per effective worker in period $t + 1$ against capital per effective worker in period $t$. The curve starts in the origin, is upward-sloping, and concave. Given our assumptions, it must cross a 45 degree line showing points where $\hat{k}_{t+1} = \hat{k}_t$ exactly once. This is what we call the steady state, which we denote with a * superscript. In version of the model without growth, just set $g_z = g_n = 0$ and re-interpret $\hat{k}_t$ as $K_t$.

5.2 Two Period Consumption Model

This figure plots an indifference curve associated with a particular level of lifetime utility, $U_0$. The slope of the indifference curves if the ratio of the marginal utilities, $-\frac{u'(C_t)}{\beta u'(C_{t+1})} = l + r_t$. Given the assumed concavity of the utility function, this starts out steep and ends up flat. The straight line is a graphical representation of the intertemporal budget constraint which we call a budget line. The slope of the budget line is $-(1 + r_t)$. At an optimum you are on the highest possible indifference curve, a necessary condition for which is that the indifference curve and the budget line have the same slope.
In an endowment economy total output/production/income is exogenous, hence the vertical $Y^s$ curve. The $Y^d$ curve shows $(r_t, Y_t)$ pairs where (i) households behave optimally (according to their consumption function) and (ii) income equals expenditure. The intersection of the two curves determines the equilibrium real interest rate.
The above picture shows the graphs used to describe equilibrium in the RBC model. The $Y^d$ curve is functionally the same as in the endowment economy (and its derivation is similar); we just include investment in it now. The $Y^s$ curve shows the set of $(r_t, Y_t)$ pairs (i) consistent with household and firm optimization, (ii) the labor market clearing, and (iii) the production function. Households and firms optimizing means that they are on their labor supply and demand curves, respectively (upper left graph). The labor market clearing means that these curves intersect. The production function is shown in the lower left graph; it is increasing in $N_t$ (holding $A_t$ and $K_t$ fixed), but at a decreasing rate (the production function is concave). The 45 degree line in the lower rate is just a graphical tool to reflect the vertical axis onto a horizontal axis. The $Y^s$ curve is upward-sloping because higher $r_t$ leads households to supply more labor; this results in more $N_t$. 

5.4 Real Business Cycle Model
when the labor market clears, which from the production function means more output.

\[
M_t^d = P_t M_d^d(r_t + \pi_{t+1}, Y_t)
\]

In the RBC model with flexible prices, we can determine nominal variables “after” real variables (e.g. the classical dichotomy holds). Money demand is upward-sloping in \( P_t \) (recall the price of money in terms of goods is \( \frac{1}{Y_t} \), so it being upward-sloping in this graph is not “odd”); the money supply is exogenous. The intersection determines the price level.

### 5.5 New Keynesian Model

The IS curve is exactly the same as the \( Y^d \) curve above. The LM curve shows the set of \((r_t, Y_t)\) pairs where the money market is in equilibrium, taking \( M_t \) and \( P_t \) as given. The AD curve is the set of \((P_t, Y_t)\) pairs where we’re on both the IS and the LM curves. The PC curve is just the graphical
representation of the equation given above. The intersection of these curves determines $P_t$ and $Y_t$; the intersection of IS-LM determines $r_t$.

![Graph showing labor supply and demand](image)

Labor supply is the same as before. Labor demand is now implicitly determined by the production function; since $Y_t$ is determined in the graphs above, there is only one value of $N_t$ consistent with the level of output, taking the exogenous variables $A_t$ and $K_t$ as given. The intersection determines $w_t$. 
