Production and Labor Supply

- We continue working with a two period, optimizing, equilibrium model of the economy.
- We augment the model with which we have been working along the following two dimensions:
  1. We model production and capital accumulation.
  2. Model endogenous labor supply.
There exists a representative firm. The firm produces output using capital, $K_t$, and labor, $N_t$, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

$A_t$ is exogenous productivity variable. Abstract from trend growth.

$F(\cdot)$ has the same properties as assumed in the Solow model – increasing in both arguments, concave in both arguments, both inputs necessary. For example:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}, \quad 0 < \alpha < 1$$
Capital Accumulation

- Differently than the Solow model, we assume that the firm owns its capital and makes the capital accumulation decisions.
- Actually doesn’t matter whether household or firm makes capital accumulation decisions.
- Current capital, $K_t$, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = q_t l_t + (1 - \delta) K_t$$

- $q_t$: exogenous variable which measures efficiency of transforming investment into capital.
- Will call “investment shock” for short. Reduced form way to model health of financial sector. Negative financial shock $\rightarrow \downarrow q_t$
Dividends and Firm Valuation

- The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend.

- Dividend is output less payments to labor less expenditure on new capital, which is investment:

\[
D_t = Y_t - w_t N_t - I_t \\
D_{t+1} = Y_{t+1} - w_{t+1} N_{t+1} - I_{t+1}
\]

- Value of the firm is present discounted value of stream of dividends:

\[
V_t = D_t + \frac{D_{t+1}}{1 + r_t}
\]
Firm Problem

- Pick $N_t$, $N_{t+1}$, $l_t$, and $l_{t+1}$ to maximize $V_t$, subject to capital accumulation equation in both periods:

\[
\begin{align*}
\max_{N_t, N_{t+1}, l_t, l_{t+1}} & \quad V_t = D_t + \frac{D_{t+1}}{1 + r_t} \\
\text{s.t.} & \quad K_{t+1} = q_t l_t + (1 - \delta) K_t \\
& \quad K_{t+2} = q_{t+1} l_{t+1} + (1 - \delta) K_{t+1}
\end{align*}
\]

- Terminal condition: want $K_{t+2} = 0$, implies:

\[
l_{t+1} = -\frac{(1 - \delta) K_{t+1}}{q_{t+1}}
\]

- Could call this the “liquidation value” of the firm
First Order Conditions

- Easiest to (i) impose terminal condition and (ii) re-write problem in terms of choosing $K_{t+1}$ instead of $I_t$ by substituting in the period $t$ capital accumulation equation.
- First order conditions are:

$$w_t = A_t F_N(K_t, N_t)$$

$$w_{t+1} = A_{t+1} F_N(K_{t+1}, N_{t+1})$$

$$1 = \frac{1}{1 + r_t} \left[ q_t A_{t+1} F_K(K_{t+1}, N_{t+1}) + \frac{(1 - \delta) q_t}{q_{t+1}} \right]$$

- Discussion and intuition
First two conditions are “static” (same in each period) and implicitly characterize a downward-sloping labor demand curve:

\[ N_t = N^d(w_t, A_t, K_t) \]
Investment Demand

- Second first order condition implicitly defines a demand for $K_{t+1}$, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I^d(r_t, A_{t+1}, q_t, K_t)$$

$r_t$ $I_t$ $I_t$ $I_t = I^d(r_t, A_{t+1}, q_t, K_t)$

↑ $A_{t+1}$ or ↑ $q_t$ or ↓ $K_t$
There exists a representative household. Households get utility from consumption and leisure, where leisure is $L_t = 1 - N_t$, with $N_t$ labor and available time normalized to 1.

Lifetime utility:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

Example flow utility functions:

$$u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$$

$$u(C_t, 1 - N_t) = \ln [C_t + \theta_t \ln(1 - N_t)]$$

Here, $\theta_t$ is an exogenous “labor supply shock” governing utility form leisure (equivalently, disutility from labor).

Notation: $u_C$ denotes marginal utility of consumption, $u_L$ marginal utility of leisure (marginal utility of labor is $-u_L$)
Budget Constraints

- Household faces two flow budget constraints, conceptually the same as before, but now income is partly endogenous:

\[
C_t + S_t \leq w_t N_t + D_t \\
C_{t+1} + S_{t+1} - S_t \leq w_{t+1} N_{t+1} + D_{t+1} + r_t S_t
\]

- Household takes \( D_t \) and \( D_{t+1} \) as given (ownership different than management)

- Terminal condition: \( S_{t+1} = 0 \). Gives rise to IBC:

\[
C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1}}{1 + r_t}
\]
First Order Conditions

Do the optimization in the usual way. The following first order conditions emerge:

\[ u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_{t+1}) \]

This is the usual Euler equation, only looks different to accommodate utility from leisure

\[ u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t) \]
\[ u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1}) \]

Discussion and intuition
Can go from first order conditions to optimal decision rules

Cutting a few corners, we get the same consumption function as before:

\[ C_t = C^d(Y_t, Y_{t+1}, r_t) \]

Or, if there were government spending, with Ricardian Equivalence we’d have:

\[ C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \]
Labor Supply

- First order condition for $N_t$ can be characterized by an indifference curve / budget line diagram similar to the two period consumption case:
Income and Substitution Effects of Higher Wage

\[ C_t = w_{1,t} + D_t \]

\[ w_{0,t} + D_t \]

\[ C_{0,t} \]

\[ C_{1,t} \]

\[ D_t \]

\[ L_{0,t} \]

\[ L_{1,t} \]

\[ L_{0,t} \]

\[ U_0 \]

\[ U_1 \]
Labor Supply Function

- We assume that the substitution effect of a higher wage dominates the income effect.
- This means that labor supply is increasing in $w_t$.
- In principle, labor supply would also be affected by non-wage income and the real interest rate (anything which would impact $C_t$).
- We will abstract from this. We assume that labor is an increasing function of $w_t$ and a decreasing function of $\theta_t$, an exogenous variable which we take to be a measure of preferences for leisure (or more generally anything other than $w_t$ which affects labor supply):

$$N_t = N^s(w_t, \theta_t)$$
Labor Supply Graphically

\[ N_t = N^s(w_t, \theta) \]
Market-Clearing

Market-Clearing Requires that $S_t = 0$. Why? No other entity operates in market for bonds.

Household budget constraint imposing this:

$$C_t = w_t N_t + D_t$$

Given definition of $D_t$, this is:

$$Y_t = C_t + I_t$$

Murky definition of saving here. Could write problem where firm finances capital by issuing debt (as opposed to equity, as presented here). Optimal decision rules would be the same – see. 11.1.1

Effectively, firm “saves” for household by doing investment and not paying a dividend. So even though there is no bond-holding in this economy in equilibrium, one can think about $S_t = Y_t - C_t$ being equal to $I_t$. 
Equilibrium Conditions

▶ Four optimal decision rules:
\[
\begin{align*}
C_t &= C^d(Y_t, Y_{t+1}, r_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
I_t &= I^d(r_t, A_{t+1}, q_t, K_t)
\end{align*}
\]

▶ Market-clearing condition plus production function:
\[
\begin{align*}
Y_t &= C_t + I_t \\
Y_t &= A_t F(K_t, N_t)
\end{align*}
\]

▶ Endogenous variables: \(Y_t, N_t, C_t, I_t\) (quantities), \(w_t\) and \(r_t\) (prices)
▶ Exogenous variables: \(A_t, A_{t+1}, K_t, \theta_t, q_t\)
Adding a Government

- Doesn’t change much. Ricardian Equivalence still holds:

\[
\begin{align*}
C_t &= C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
I_t &= I^d(r_t, A_{t+1}, q_t, K_t) \\
Y_t &= C_t + I_t + G_t \\
Y_t &= A_t F(K_t, N_t)
\end{align*}
\]

- Now $G_t$ and $G_{t+1}$ are exogenous, and $T_t$ and $T_{t+1}$ are irrelevant