Problem Set 2

Graduate Macro II, Spring 2013
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Thursday, February 7.

(1) The Hodrick-Prescott Filter: In this problem you will derive the HP filter and create your own code for implementing it. Suppose you have a sequence of data, y_t with $t = 1, ..., T$ (i.e. there are T total observations). Our objective is to find a trend, $\{\tau_t\}_{t=1}^{T}$ to minimize the following objective function:

$$\min_{\tau_t} \quad \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$$

The parameter $\lambda \geq 0$.

(a) Provide a verbal interpretation of this objective function and what exactly one is trying to get at when choosing a trend, $\{\tau_t\}_{t=1}^{T}$.

(b) Prove that, if $\lambda = 0$, then the solution is for $\tau_t = y_t \; \forall \; t$. In other words, the trend and the actual series would be identical.

(c) Prove that, as $\lambda \to \infty$, then the solution is for for the trend to be a linear time trend; i.e. for $\tau_t = \alpha t$ for some α.

(d) For the more general case in which $0 < \lambda < \infty$, derive analytical conditions which implicitly define the trend. To do this, take the derivative with respect to τ_t for each $t = 1, ..., T$ and set them equal to zero, yielding T first order conditions.

(e) Write a Matlab code to find the trend taking the actual data series, y_t, and parameter, λ, as inputs. To do this, express the first order conditions in (d) in matrix form as follows:

$$\mathbf{\Lambda T} = \mathbf{Y}$$

$$\mathbf{T} = \mathbf{\Lambda}^{-1}\mathbf{Y}$$

Where $\mathbf{\Lambda}$ is a $T \times T$ matrix whose elements are functions of λ; $\mathbf{T}$ is a $T \times 1$ vector equal to $[\tau_1 \; \tau_2 \; . . . \; \tau_T]'$ and $\mathbf{Y}$ is a $T \times 1$ vector equal to $[y_1 \; y_2 \; . . . \; y_T]'$.

(f) Download quarterly, seasonally adjusted data on US real GDP from 1947 quarter 1 to 2010 quarter 3 (this should be available, for example, on the St. Louis Fed FRED data website). Take natural logs of the data, and then use your code to compute the HP trend using $\lambda = 1600$. Show a plot of the trend. Get the "cycle" series by subtracting the trend from the actual series. Show a plot of the cyclical component. What is the standard deviation of the HP detrended GDP?

(2) The Neoclassical Growth Model: Decentralized vs. Planner's Problem: Consider the standard neoclassical growth model from class. There is a single representative agent. Time begins in period t and lasts forever. The initial capital stock, K_t is given, and total factor productivity, A_t, is exogenous in each period. There is no exogenous growth and labor is supplied inelastically, and can be normalized to one each period if you like (e.g. $N_{t+j} = 1$). Lifetime utility of the representative agent is:

$$U = E_t \sum_{t=0}^{\infty} \beta^j u(C_{t+j})$$

The aggregate resource constraint is:

$$K_{t+j+1} = A_{t+j} F(K_{t+j}, N_{t+j}) - C_{t+j} + (1 - \delta) K_{t+j}$$

The function $F(K_{t+j}, N_{t+j})$ is increasing and concave in both arguments and homogeneous of degree one.

(a) What are the choice variables for the planner? What are the state variables?

(b) Find the first order conditions for an interior solution to the planner's problem.

Instead suppose that we consider a decentralized version of the economy. In particular, the household supplies capital and labor to a representative firm at competitive prices R_{t+j} and w_{t+j}, respectively. Both the household and the firm take these prices as given. The household accumulates capital and leases it on a period-by-period basis to the firm. The firm faces a static problem and want to maximize profit, where per-period profit is given by:

$$\Pi_{t+j} = A_{t+j} F(K_{t+j}, N_{t+j}) - R_{t+j} K_{t+j} - w_{t+j} N_{t+j}$$

(c) Find the first order conditions for profit maximization by the firm.

The household owns the capital stock. It receives income from labor, from leasing capital, and distributed profit (it takes the latter as given). Its lifetime utility is as given above. Its resource constraint is:

$$C_{t+j} + K_{t+j+1} - (1 - \delta) K_{t+j} = w_{t+j} N_{t+j} + R_{t+j} K_{t+j} + \Pi_{t+j}$$

(d) What are the choice variables for the household (recall that labor is inelastically supplied, and so isn't really a choice). What are the state variables?

(e) Find the first order conditions consistent with the household problem.

(f) A competitive equilibrium can be defined as a set of prices and allocations such that all agents are behaving according to optimal decision rules and all markets are clearing. Use this definition to combine the first order conditions of the household and firm problems. Argue that the resulting allocations from the competitive equilibrium will be identical to the allocations from the planner's solution.