Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, February 3.

(1) The Lucas Critique and Estimating the MPC: Suppose that the economy is populated by a single type of agent. This agent lives forever and discounts the future by $\beta < 1$. The agent receives an exogenous and potentially stochastic income flow each period. This agent receives flow utility from consumption; the utility function is quadratic. The agent can borrow or save at a constant real interest rate, $r$. We can collapse the sequence of flow budget constraints into one intertemporal budget constraint, which says that the present discounted value of consumption must equal the present discounted value of income. The constant real interest rate is such that $\beta (1+r) = 1$. We can write the agent’s problem as picking a stream of consumption to maximize the expected present discounted value of utility subject to the intertemporal budget constraint holding:

$$\max_{C_t, C_{t+1}, C_{t+2}, \ldots} E_t \sum_{j=0}^{\infty} \beta^j \left( C_{t+j} - bC_{t+j}^2 \right)$$

s.t.

$$\sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j} \leq \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j}$$

For this problem, assume that the value of the parameter $b$ is always such that the marginal utility of consumption is always positive (quadratic utility functions have a so-called satiation point, which we assume is never reached).

(a) Find the first order necessary conditions for this problem and derive a relationship between $C_t$ and $E_t C_{t+j}$ for all $j > 0$. In what respect is the assumption of quadratic utility important in deriving this expression? Explain.

(b) Plug this condition into the intertemporal budget constraint to derive an expression for $C_t$ as a function of current and expected future income flows. In doing so, note that the intertemporal budget constraint must hold both ex-post as well as in expectation. Note also that $\sum_{j=0}^{\infty} S^j = \frac{1}{1-S}$ for $0 \leq S < 1$.

(c) Use this consumption function derived in part (b) to get an analytic expression for the marginal propensity to consume (MPC), the partial derivative of $C_t$ with respect to $Y_t$. If $\beta \approx 1$, what is true about the value of the MPC?

(d) Suppose that income follows an AR(1) process with unconditional mean of 1: $Y_t = (1-\rho) + \rho Y_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma^2)$ and $0 \leq \rho < 1$. Use this process to simplify your expression for the consumption function from $C_t$ as a function of only $Y_t$ on the right hand side.
(e) Take your process found in part (d). If an econometrician were interested in estimating the MPC and regressed $C_t$ on $Y_t$, would he recover the true MPC you derived in (c)? How would this depend on the value of $\rho$? Explain.

(2) Equilibrium in Endowment Economies with Two Types of Agents Suppose that there are two agents who are identical in terms of their preferences, but who have potentially different endowments. The endowments potentially differ from one another but are otherwise deterministic. Agents have access to a one period bond that costs $q_t$ units of consumption in period $t$ and pays out one unit of income in period $t+1$. There are $N_i$ types of each agent, and $N_i$ is sufficiently big that both types of agents behave as price-takers. The problem of agent of type $i = 1, 2$ is:

$$\max_{C_{i,t}, B_{i,t}, C_{i,t+1}} \ln C_{i,t} + \beta \ln C_{i,t+1}$$

s.t.

$$C_{i,t} + q_t B_{i,t} \leq Y_{i,t}$$
$$C_{i,t+1} \leq Y_{i,t+1} + B_{i,t}$$

(a) Set up a Lagrangian to derive an Euler equation describing an optimal consumption plan for agent of type $i$. Provide some brief intuition for the Euler equation.

(b) Write down the generic definition of a competitive equilibrium. What is the aggregate market-clearing condition in this context?

(c) Suppose that the endowment pattern for type 1 agents is $(Y_{1,t}, Y_{1,t+1}) = (1, 0)$ and the pattern for type 2 agents is $(Y_{2,t}, Y_{2,t+1}) = (0, 1)$. Derive analytic expressions for $q_t$, $C_{1,t}$, $C_{2,t}$, $C_{1,t+1}$, $C_{2,t+1}$, $B_{1,t}$, and $B_{2,t}$ in the competitive equilibrium allocation. What is true about agent of type $i$’s consumption as a function of his/her relative endowment in a period (e.g. is consumption low in the period with an endowment of 0 relative to the period where the endowment is 1)?

(d) Suppose that there is an increase in the number of type 1 agents, $N_1$. Given your answers on the previous part, how would this affect the equilibrium price of bonds, $q_t$? What is the intuition for this? How would the welfare of type 1 and type 2 agents be impacted by the increase in $N_1$ (by “welfare” I mean present discounted value of utility, e.g. $U_i = \ln C_{i,t} + \beta \ln C_{i,t+1}$).

Consider next the problem of a social planner. The social planner wants to maximize the weighted sum of discounted utilities of both agents, subject to the economy-wide resource constraints. The planner attaches weights $\mu$ and $1 - \mu$ to type 1 and 2 agents, respectively. The planner’s problem can be written:

$$\max_{C_{1,t}, C_{2,t}, C_{1,t+1}, C_{2,t+1}} \mu N_1 (\ln C_{1,t} + \beta \ln C_{1,t+1}) + (1 - \mu) N_2 (\ln C_{2,t} + \beta \ln C_{2,t+1})$$

s.t.

$$N_1 C_{1,t} + N_2 C_{2,t} \leq N_1$$
$$N_1 C_{1,t+1} + N_2 C_{2,t+1} \leq N_2$$

(e) Solve for the first order conditions of the planner’s problem. What would $\mu$ have to be for the competitive equilibrium allocation you found in (c) to be the solution to the planner’s problem?
What would have to be true about $N_1$ and $N_2$ for $\mu = \frac{1}{2}$?

(f) Suppose that the planner had $\mu = \frac{1}{2}$ (this is sometimes called a utilitarian social welfare function). What would be true about the allocations of consumption in the planner’s solution for these welfare weights?