Problem Set 3 Solutions

Intermediate Macroeconomics, Fall 2013
The University of Notre Dame
Professor Sims

Instructions You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member’s name on the first page of your solutions. This problem set is due in class on Thursday, September 19.

(1) A Quantitative Solow Model Exercise: Consider a Solow model with both population and technological growth. Assume that the production function is Cobb-Douglas, as in (1), and let lowercase variables with a “hat” denote per efficiency units of labor variables, e.g. \( \hat{k}_t = \frac{K_t}{Z_tN_t} \). The central Solow model equation can be written:

\[
(1 + g_z)(1 + g_n)\hat{k}_{t+1} = sA\hat{k}_t^\alpha + (1 - \delta)\hat{k}_t
\]

(a) Analytically solve for expression for the steady state capital stock per effective workers, \( \hat{k}^* \), output per effective worker, \( \hat{y}^* \), and consumption per effective worker, \( \hat{c}^* \).

(b) In the steady state, what will be the growth rate of per capita capital, output, and consumption?

(c) Suppose that the values of the parameters in the model are as follows: \( g_n = 0.01 \), \( g_z = 0.02 \), \( \delta = 0.1 \), \( \alpha = 0.33 \), \( s = 0.2 \), \( A = 1 \). Create an Excel spreadsheet, with rows corresponding to time periods (e.g. 1, 2, etc.) and columns corresponding to variables. Have \( \hat{k} \), \( \hat{y} \), and \( \hat{c} \) sit in their steady states for periods 1-9. In period 10, suppose that the saving rate increases to \( s = 0.25 \). Use your Excel spreadsheet to compute the time paths of \( \hat{k} \), \( \hat{y} \), and \( \hat{c} \) following this change (go up to period 100). Plot these time paths out in separate plots (for periods 1-100).

(d) Approximately how many periods does it take \( \hat{k} \) to get one-half of the way to its new steady state?

(e) Calculate the growth rate of \( \hat{y}_t \) for each period of the simulation (you don’t have an observed growth rate in the first period) using the log first difference approximation. Plot the simulated time path of this growth rate.

(f) Re-do exercises (c)-(e), but this time assume that \( \alpha = 0.67 \). Describe any differences, paying particular attention to the exercises in (d) and (e).

(g) Given your answers on (f), speculate intelligently on how the desirability of trying to encourage a higher saving rate depends on the value of \( \alpha \).

(2) Differences in Standards of Living: Suppose that we have two countries that behave according to a Solow model. Call these countries 1 and 2. There is no population or technology growth in either country, and the countries do not interact with one another. The central equations in each country are, respectively:
\[ K_{1,t+1} = s_1 A_1 K_{1,t}^\alpha + (1 - \delta) K_{1,t} \]
\[ K_{2,t+1} = s_2 A_2 K_{2,t}^\alpha + (1 - \delta) K_{2,t} \]

The countries have the same \( \alpha \) and \( \delta \), but potentially different saving rates and levels of \( A \).

(a) Solve for analytic expressions for the steady state capital stocks in each country.

(b) Solve for analytic expressions for the steady state levels of output in each country, where output is given by \( Y_{1,t} = A_1 K_{1,t}^\alpha \) for country 1 and \( Y_{2,t} = A_2 K_{2,t}^\alpha \) for country 2.

(c) Let \( \alpha = \frac{1}{3} \) and \( \delta = 0.1 \) for both countries. Suppose that country 1 has \( A_1 = 1 \) and \( s_1 = 0.2 \). Suppose that country 2 has \( A_2 = 1 \) as well, but that country 2 has 50 percent of the output that country 1 has (e.g. \( \frac{Y_{2,t}}{Y_{1,t}} = 0.5 \)). What must country 2’s saving rate be?

(d) Continue to assume that country 1 has \( A_1 = 1 \) and \( s_1 = 0.2 \). Suppose instead that country 2 has \( s_2 = 0.2 \), the same as country 1. Continue to assume that country 2 has 50 percent of the output of country 1 (e.g. \( \frac{Y_{2,t}}{Y_{1,t}} = 0.5 \)). What must country 2’s \( A_2 \) be?