Problem Set 3
Intermediate Macroeconomics, Fall 2012
The University of Notre Dame
Professor Sims

Instructions You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member’s name on the first page of your solutions. This problem set is due in class on Thursday, September 20.

(1) Differences in Standards of Living: Suppose that we have two countries that behave according to a Solow model. Call these countries 1 and 2. There is no population or technology growth in either country, and the countries do not interact with one another. The central equations in each country are, respectively:

\[ K_{1,t+1} = s_1 A_1 K_{1,t}^\alpha + (1 - \delta) K_{1,t} \]
\[ K_{2,t+1} = s_2 A_2 K_{2,t}^\alpha + (1 - \delta) K_{2,t} \]

The countries have the same \( \alpha \) and \( \delta \), but potentially different saving rates and levels of \( A \).

(a) Solve for analytic expressions for the steady state capital stocks in each country.

(b) Solve for analytic expressions for the steady state levels of output in each country, where output is given by \( Y_{1,t} = A_1 K_{1,t}^\alpha \) for country 1 and \( Y_{2,t} = A_2 K_{2,t}^\alpha \) for country 2.

(c) Let \( \alpha = \frac{1}{3} \) and \( \delta = 0.1 \) for both countries. Suppose that country 1 has \( A_1 = 1 \) and \( s_1 = 0.2 \). Suppose that country 2 has \( A_2 = 1 \) as well, but that country 2 has 50 percent of the output that country 1 has (e.g. \( \frac{Y_{2,t}}{Y_{1,t}} = 0.5 \)). What must country 2’s saving rate be?

(d) Continue to assume that country 1 has \( A_1 = 1 \) and \( s_1 = 0.2 \). Suppose instead that country 2 has \( s_2 = 0.2 \), the same as country 1. Continue to assume that country 2 has 50 percent of the output of country 1 (e.g. \( \frac{Y_{2,t}}{Y_{1,t}} = 0.5 \)). What must country 2’s \( A_2 \) be?

(2) Basic Consumer Theory: Suppose that a consumer lives for two periods and has lifetime utility given by:

\[ U = \ln C_t + \beta \ln C_{t+1} \]

The household faces a sequence of budget constraints, where \( Y_t \) and \( Y_{t+1} \) are exogenously given and the household takes the real interest rate, \( r_t \), as given:

\[ C_t + S_t = Y_t \]
\[ C_{t+1} = Y_{t+1} + (1 + r_t) S_t \]
(a) Explain in words why $S_{t+1} = 0$, and therefore does not appear in the second period budget constraint.

(b) What are the units of the real interest rate?

(c) Combine the two period budget constraints into one unified budget constraint.

(d) Derive an optimality condition that characterizes optimal behavior (i.e., the Euler equation).

(e) Graphically characterize the optimality condition in an indifference curve-budget line diagram.

(f) Suppose there is an increase in current income, $Y_t$. Show how this affects the indifference curve-budget line diagram. What should happen to current saving, $S_t$, in response to the increase in $Y_t$?

(g) Suppose instead that there is a known increase in future income, $Y_{t+1}$. Show how this affects the indifference curve-budget line diagram. What should happen to current saving, $S_t$, in response to the increase in $Y_{t+1}$?