Problem Set 3

Graduate Macro II, Spring 2013
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Thursday, February 21.

(1) Comparing Solution Techniques: Deterministic Growth Model: In this problem you will compare policy functions from a deterministic neoclassical growth model using (a) value function iteration and (b) log-linearization.

The equilibrium of the economy can be described as the solution to the following social planner's problem:

$$\max_{c_t, k_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

s.t.

$$k_{t+1} = k_t^{\alpha} - c_t + (1 - \delta)k_t$$
$$k_0 \text{ given}$$

(a) Use L'Hopital's rule to prove that, as $\sigma \to 1$, the within period utility function goes to $\ln c_t$.

(b) Set this problem up as a dynamic programming problem. What is the state variable? What is the control variable? Write down the Bellman equation. Find the first order condition necessary for an optimal solution.

(c) Find an expression for the steady state capital stock and the steady state value of consumption.

(d) Suppose that the parameter values are as follows: $\beta = 0.99$, $\alpha = 0.36$, $\sigma = 2$, and $\delta = 0.025$. What are the numerical values of the steady state capital stock and consumption for these parameters? *Write your own code* to numerically solve for the value and policy functions. To do so, create a grid of the capital stock, with the minimum value 0.25 of the steady state capital stock and the maximum value 1.75 times the steady state capital stock, with 300 grid points between. Use linear interpolation to evaluate points off the grid. Show a graph of both the final value function and the policy function.

(e) Now set up the problem using a Lagrangian. Write out the first order conditions, including the transversality condition. Provide a verbal explanation for the intuition behind the transversality condition.

(f) Log-linearize the first order conditions about the steady state. Form a VAR(1) of the form:

$$\mathbf{X}_{t+1} = \mathbf{M}\mathbf{X}_t$$

Where $\mathbf{X}_t$ contains the variables expressed as percentage deviations about the steady state. Write out an expression for $\mathbf{M}$.

(g) Solve for the linear policy function mapping the state variable into the jump variable. Write out the numerical policy function here.

(h) Show a plot of the linearized policy function and the policy function from the value function iteration procedure obtained above together. Be sure to transform the linearized policy function, which is expressed as a percentage deviation about the steady state, into actual levels so as to make the comparison appropriate. Comment on the quality of the linear approximation.

(i) Repeat the exercise in (h) for the following different values of σ: 5 and 10. How does the quality of the linear approximation vary with σ? Why does this make sense?

(2) Linearizing the Stochastic Growth Model: Consider a similar problem as above, but now assume that there are stochastic shocks to the production function. The social planner's problem can be written:

$$\max_{c_t, k_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

$$\text{s.t.}$$

$$k_{t+1} = a_t k_t^\alpha - c_t + (1 - \delta) k_t$$
$$k_0 \text{ given}$$

Assume that the technology shifter, a_t, follows an AR(1) process in the log:

$$\ln a_t = \rho \ln a_{t-1} + e_t$$

The shock is e_t is drawn from a white noise process. Assume that $0 < \rho < 1$.

(a) What is the unconditional mean of log technology? What, therefore, is the mean of the level of log technology? What is the unconditional variance of log technology if the variance of the white noise process e_t is σ_e^2?

(b) Write down the first order conditions necessary for the solution to this problem.

(c) Log-linearize the first order conditions about the steady state. Form a VAR(1) representation of the form:

$$E_t \mathbf{X}_{t+1} = \mathbf{M} \mathbf{X}_t$$

Write out the elements of $\mathbf{M}$ as a function of the deep parameters of the model.

(d) Solve for the linearized policy function mapping the states into jump variables. Assume the same parameter values as in (1), with $\rho = 0.9$. Assume the benchmark value $\sigma = 2$. Find log-linear policy functions for the "static" variables output and investment as well. Use the fact that output is given by $y_t = a_t k_t^\alpha$ and investment by $i_t = y_t - c_t$.

(e) Compute the impulse responses to a one unit shock to $\ln a_t$ at time 0. Show graphs of the responses of consumption, the capital stock, technology, output, and investment to the shock for 20 periods.

(f) Simulate a data set with $T = 1000$ observations. Begin your simulation at the steady state, and draw technology shocks from a standard normal distribution with standard deviation of 0.1. HP filter the simulated data with smoothing parameter $\lambda = 1600$. Calculate the standard deviations (volatilities) of HP detrended consumption, output, and investment.