Problem Set 4
Intermediate Macroeconomics, Fall 2012
The University of Notre Dame
Professor Sims

Instructions  You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member’s name on the first page of your solutions. This problem set is due in class on Thursday, October 1.

(1) Life Cycle / Permanent Income Consumption Model  Suppose that we have a household that lives for $T + 1$ periods, from period 0 to period $T$. Its lifetime utility is:

$$ U = u(C_0) + \beta u(C_1) + \beta^2 u(C_2) + \cdots + \beta^T u(C_T) $$

$$ U = \sum_{t=0}^{T} \beta^t u(C_t) $$

The household has a sequence of income, $Y_0, Y_1, \ldots, Y_T$, which it takes as given. The household can borrow or lend at constant real interest rate $r$, with $r > 0$. The household faces a sequence of period budget constraints:

$$ C_0 + S_0 = Y_0 $$
$$ C_1 + S_1 = Y_1 + (1 + r)S_0 $$
$$ C_2 + S_2 = Y_2 + (1 + r)S_1 $$
$$ \vdots $$
$$ C_T = Y_T + (1 + r)S_{T-1} $$

Here $S_t$, $t = 0, 1, \ldots, T$ is the stock of savings that the household takes from period $t$ to period $t + 1$. The flow, saving, is defined as the change in the stock, or $S_t - S_{t-1}$ (hence, in period 0, the flow and the stock are the same thing). The sequence of budget constraints can be combined into the intertemporal budget constraint:

$$ C_0 + \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} + \cdots + \frac{C_T}{(1 + r)^T} = Y_0 + \frac{Y_1}{1 + r} + \frac{Y_2}{(1 + r)^2} + \cdots + \frac{Y_T}{(1 + r)^T} $$

$$ \sum_{t=0}^{T} \frac{C_t}{(1 + r)^t} = \sum_{t=0}^{T} \frac{Y_t}{(1 + r)^t} $$

Once can show that there are $T$ different optimality conditions, satisfying:

$$ u'(C_t) = \beta(1 + r)u'(C_{t+1}) \text{ for } t = 0, 1, \ldots, T - 1 $$

(a) Provide some intuition for this sequence of optimality conditions.

(b) Assume that $\beta(1 + r) = 1$. What does this imply about consumption across time? Explain.
(c) Assume that \( r = 0.05 \). What must \( \beta \) be for the restriction in (b) to be satisfied?

(d) Using your answer from (b), solve for an analytic expression for consumption as a function of \( r \) and the stream of income.

(e) Now create an Excel file to numerically analyze this problem. Suppose that income grows over time. In particular, let \( Y_t = (1 + g_y)tY_0 \) for \( t = 0, 1, \ldots, T \). Suppose that \( g_y = 0.02 \) and that \( Y_0 = 10 \). Assume that \( T = 50 \). Use this, in conjunction with the value of \( r \) from (c), to numerically solve for the time path of consumption. Create a graph plotting consumption and income against time.

(f) Given your time series of consumption and income, create a time series of savings (stock) and saving (flow). In period \( t, t = 0, 1, \ldots, T \), your savings should be the stock of savings that the household leaves that period with (they enter period 0 with nothing, but leave with something, either positive or negative). Create a graph plotting the time series of savings. What is true about the stock of savings that the household leaves over after period \( T \)?

(g) Are there periods in which your flow saving variable is negative/positive but consumption is less than/greater than income? If so, what accounts for this? Explain.

(h) Now modify the basic problem such that the household retires at date \( R < T \). In particular, assume that the income process is the same as before, but goes to zero at date \( R+1 \): \( Y_t = (1 + g_y)tY_0 \) for \( t = 0, 1, \ldots, R \). Re-do the Excel exercise assuming that \( R = 39 \), so that income goes to 0 in period 40. Show the plot of consumption and income against time, and also plot the time series behavior of the stock of savings. Comment on how the life cycle of savings is affected by retirement.

(i) One popular proposal floating around right now is to raise the retirement age in the hope of making Social Security solvent. Suppose that the retirement age were increased by five years, from \( R = 39 \) to \( R = 44 \). What effect would this have on consumption? Other things being equal, do you think this change would be good or bad for the economy in the short run?