Problem Set 5
Graduate Macro II, Spring 2015
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Thursday, March 26, 2015.

(1) Preference Shocks and Variable Capital Utilization: Suppose we have a model with variable capital utilization and a time-varying disutility of labor. A representative household owns the capital stock, $K_t$, and makes capital accumulation decisions. This agent also gets to pick how intensively to utilize capital, $u_t$, leasing capital services (the product of utilization and physical capital), $K_t^\hat{=} = u_tK_t$, to a representative firm on a period-by-period basis. The cost of more intensive utilization of capital is faster depreciation.

The problem of the household can be written as follows:

$$
\max_{C_t, K_{t+1}, B_{t+1}, N_t, u_t} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \nu_t \theta \frac{N_t^{1+\chi}}{1+\chi} \right\}
$$

s.t.

$$
C_t + K_{t+1} - (1 - \delta(u_t))K_t + B_{t+1} \leq w_t N_t + R_t u_t K_t + \Pi_t + (1 + r_{t-1})B_t
$$

where

$$
\delta(u_t) = \delta_0 + \phi_1(u_t - 1) + \phi_2 (u_t - 1)^2
$$

$\nu_t$ is a time-varying shock to preferences over labor, obeying a stationary AR(1) process that is mean zero in the log:

$$
\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \varepsilon_{\nu,t}
$$

(a) Derive the first order conditions for the household problem.

The firm produces output using labor and capital services, $K_t^\hat{=} = u_tK_t$, and takes factor prices, $w_t$ and $R_t$, as given. Its problem is:

$$
\max_{N_t, K_t} \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t K_t
$$

(b) Derive the first order conditions for the firm problem.

Productivity obeys an AR(1) process in the log:

$$
\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}
$$

(c) Derive the aggregate resource constraint using the the definition of profit and the bond market-clearing condition.

(d) Provide a condition on the parameter $\phi_1$ to ensure that steady state capital utilization is 1.
(e) Derive a condition giving the value of \( \theta \) necessary to target steady state labor hours of \( N^* = 1/3 \).

(f) Write a Dynare code to solve the model and compute impulse responses to both productivity and preference shocks using a first order log-linear approximation. Use the following parameter values: \( \beta = 0.99 \), \( \delta = 0.02 \), \( \alpha = 1/3 \), \( \chi = 1 \), \( \rho_a = 0.97 \), \( \rho_\nu = 0.95 \), \( s_a = 0.01 \) (standard deviation of productivity shock), and \( s_\nu = 0.02 \) (standard deviation of the preference shock). Use the value of \( \theta \) consistent with steady state hours of 1/3 found in (e) above. Consider three different values of \( \phi_2 \): 100, 0.1, 0.01. Graphically show impulse responses of output, hours, consumption, investment, utilization, the real wage, and the real interest rate to both shocks. Comment on how the impulse responses to both shocks vary with the parameter \( \phi_2 \), and provide some intuition for your results.

(g) Suppose that you measure total factor productivity not taking into account variable utilization. That is, let \( \hat{A}_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln N_t \). Show impulse responses of measured TFP to both shocks for the three different values of \( \phi_2 \) from above. How does \( \phi_2 \) affect the response of measured TFP to both shocks?

(h) How does the inclusion of the preference shock affect the relative volatility of HP filtered hours (relative to GDP) in the model? To see this, compute the relative volatility of hours to output with both shocks “turned on” and again with the preference shock “turned off” (e.g. set the standard deviation of that shock to zero). Use a value of \( \phi_2 = 0.1 \) in doing this part.

(2) GHH vs. Traditional Preferences and the Effects of Government Spending and Productivity Shocks: Suppose you have a RBC model with two stochastic shocks: a shock to productivity and a shock to government spending. We will consider two different preference specifications: standard separable preferences and GHH preferences.

The household problem can be written:

\[
\max_{C_t, K_{t+1}, N_t, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\
\text{s.t.} \\
C_t + K_{t+1} - (1 - \delta)K_t + B_{t+1} \leq w_tN_t + R_tK_t + H_t - T_t + (1 + r_{t-1})B_t
\]

(a) For the arbitrary specification of household preferences, \( U(C_t, N_t) \), find the first order conditions for a solution to the problem.

The firm problem is the same in both setups:

\[
\max_{N_t, K_t} \quad H_t = A_tK_t^\theta N_t^{1-\alpha} - w_tN_t - R_tK_t
\]

(b) Find the first order conditions necessary for a solution to the firm problem.

(c) The government chooses its spending exogenously and balances its budget each period, \( G_t = T_t \). What then must be true about bond-holding by households in equilibrium? Write down the aggregate resource constraint.

Suppose that preferences are given by the standard separable form:

\[
U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1 + \chi}
\]
(d) Suppose that the non-stochastic steady state value of $A_t$ is $A = 1$, while the steady state value of government spending is $G = \omega Y$, where $Y$ is the steady state value of output and $0 < \omega < 1$. Using these preferences, derive an expression for the value of $\theta$ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of $Y$, $C$, $I$, $K$, $w$, and $R$ as a function of parameters.

Suppose that $G_t$ and $A_t$ follow stationary AR(1) processes in the log:

$$
\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}
$$

$$
\ln G_t = (1 - \rho_G) \ln G + \rho_G \ln G_{t-1} + \varepsilon_{G,t}
$$

(e) Solve the model using a first order log-linear approximation in Dynare using the following parameter values: $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, $\chi = 1$, $\rho_A = 0.97$, $\rho_G = 0.95$, $s_A = 0.01$ (standard deviation of productivity shock), $s_G = 0.01$ (standard deviation of government spending shock), $\omega = 0.20$, and the value of $\theta$ consistent with $N = 1/3$. Produce impulse response graphs of $Y_t$, $C_t$, $I_t$, $N_t$, $w_t$, and $r_t$ to each shock over a 20 period horizon. Calculate the "government spending multiplier," defined as the ratio of the impact response of the level of output to the impact response of the level of government spending following a government spending shock.

Now instead suppose that preferences are given by the GHH variety:

$$
U(C_t, N_t) = \ln \left( C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} \right)
$$

(f) Using these preferences, derive an expression for the value of $\theta$ consistent with steady state hours of $N = 1/3$, and provide expressions for the steady state values of $Y$, $C$, $I$, $K$, $w$, and $R$ as a function of parameters.

(g) Solve the model using a first order log-linear approximation using the same parameter values as above. Produce impulse response graphs of $Y_t$, $C_t$, $I_t$, $N_t$, $w_t$, and $r_t$ to each shock over a 20 period horizon. Calculate the "government spending multiplier," defined as the ratio of the impact response of the level of output to the impact response of the level of government spending following a government spending shock.

(h) Compare and contrast your impulse responses to the two shocks with the two different preference specifications. Provide some intuition for your findings.