Problem Set 6

Intermediate Macroeconomics, Fall 2013
The University of Notre Dame
Professor Sims

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member’s name on the first page of your solutions. This problem set is due in class on Thursday, October 17.

(1) General Equilibrium with Government Spending: Suppose that we have an endowment economy model. There is a representative household who seeks to maximize:

\[ U = \ln C_t + \beta \ln C_{t+1} \]

Subject to:

\[ C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t} \]

The household takes each as given.

(a) Derive the Euler equation characterizing an optimal consumption plan. How does the presence of the government affect the Euler equation?

(b) Use the Euler equation in consumption with the intertemporal budget constraint to derive the consumption function. How does the presence of a government affect the consumption function?

(c) What is the “marginal propensity” to consume out of current income, MPC, for this consumption function?

The government chooses its spending, \( G_t \) and \( G_{t+1} \), exogenously, and must raise taxes, \( T_t \) and \( T_{t+1} \), to satisfy:

\[ G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} \]

(d) Write down the definition of a competitive equilibrium for this economy.

(e) Assuming that the household knows (and anticipates) that the government budget constraint will bind, you can combine that with the consumption function. Do this. What happens to taxes in the consumption function? What is the intuition for this?

(f) Define the \( Y^d \) curve and qualitatively graphically derive it as we did in class and in the notes.

(g) As noted above, output is exogenous in this economy, with \( Y^s = Y_t \). Use the supply-demand diagram to qualitatively discuss how an increases in \( G_t \) or \( G_{t+1} \) (considered separately, one at a
time) ought to affect $r_t$.

(h) How will changes in $T_t$ or $T_{t+1}$ affect the equilibrium of the economy? What is the intuition for this?

(i) Now, use the consumption function you derived in (b), and the market-clearing condition from the definition of the competitive equilibrium in (d), to derive an algebraic expression for the $Y^d$ curve.

(j) Find an expression for $\frac{\partial Y^d_t}{\partial G_t}$: the change in the quantity of goods demand for a change in government spending, holding the real interest rate fixed.

(k) Does it matter how the increase in $G_t$ is financed for how much the quantity demanded changes when $G_t$ changes? In other words, does it matter if it is “tax financed” ($dT_t = dG_t$) or “debt-financed” ($dT_t = 0$)? What is the intuition for your answer?

(l) Find an expression for $\frac{\partial Y_t}{\partial G_t}$: the change in equilibrium output for a change in government spending. Why does this differ from what you found in (j), and what is the explanation for why it differs?

“Old School” models did not feature forward-looping behavior on the part of households. Rather, it was typically assumed that consumption was just a function of current net income, not future income as well. Suppose that we postulate the existence of the following consumption function:

$$C_t = \gamma_0 + \text{MPC} (Y_t - T_t) - \gamma_1 r_t$$

Above, $\gamma_0$ and $\gamma_1$ are both positive, and MPC is the marginal propensity to consume.

(m) The definition of a competitive equilibrium is the same with this kind of consumption function. Derive an algebraic expression for the $Y^d$ curve using this consumption function instead of the one you derived via optimization above.

(n) For this new demand function, derive an expression for $\frac{\partial Y^d_t}{\partial G_t}$ assuming that the spending increase is “debt-financed” (so that $T_t$ does not change, or $dT_t = 0$).

(o) Repeat this exercise, but instead assume that the spending change is tax-financed, so that $dT_t = dG_t$.

(p) Compare your answers from (n) and (o) to your answers from (j) and (k). What accounts for the difference? If we had a model where supply were endogenous (so that $Y^*$ would not be vertical), comment on how the nature of the behavior of households would impact the efficacy of government spending changes as a tool to affect $Y_t$. 

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