Problem Set 6
Intermediate Macroeconomics, Fall 2012
The University of Notre Dame
Professor Sims

Instructions You may work on this problem set in groups of up to four people. Should you choose to do so, please make sure to legibly write each group member’s name on the first page of your solutions. This problem set is due in class on Thursday, October 25.

(1) An Endowment Economy with a Government: Suppose that we have an economy populated by many, identical households. Since the households are identical, they will behave as price-takers. We can normalize the number of households to be one, and thereby study the problem of the representative household. This household faces the following problem:

\[
\max_{C_t, C_{t+1}, S_t} U = \ln C_t + \beta \ln C_{t+1}
\]

s.t.

\[
C_t + S_t = Y_t - T_t
\]

\[
C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t
\]

The household’s income is exogenous, and it takes taxes, \( T_t \) and \( T_{t+1} \), as given.

(a) Combine the two period constraints into one intertemporal budget constraint.

(b) Write down/derive the Euler equation characterizing an optimal consumption plan.

The government chooses a sequence of expenditure/consumption, \( G_t \) and \( G_{t+1} \), exogenously. It faces two period budget constraints:

\[
G_t + S_t^G = T_t
\]

\[
G_{t+1} = T_{t+1} + (1 + r_t)S_t^G
\]

(c) Combine these two period constraints into one intertemporal budget constraint.

(d) What will happen if the household knows that the government intertemporal budget constraint must bind? Combine this with the household intertemporal budget constraint and the household Euler equation to derive an algebraic expression for the consumption function.

(e) In equilibrium, what must be true of \( S_t \) and \( S_t^G \)?

(f) Total desired expenditure in the economy, \( Y_t^d \), is equal to consumption plus government expenditure. Use your consumption function to derive \( Y_t^d \) as a function of \( Y_t, Y_{t+1}, G_t, G_{t+1}, \) and \( r_t \) (i.e.
for now, treat $Y_t^d$ and $Y_t$ as distinct).

(g) What must be true of $Y_t$ and $Y_t^d$ in any equilibrium? Impose this condition to find an expression for $Y_t^d$ as a function of $Y_{t+1}$, $G_t$, $G_{t+1}$, and $r_t$.

(h) In an endowment economy supply is exogenous, with $Y_t^s = Y_t$. Combine this with your answer on (g) to algebraically solve for the equilibrium real interest rate, $r_t$.

(i) What happens to $r_t$ following a surprise increase in $G_t$? Provide some intuition for your answer.

(j) What happens to $r_t$ following a surprise increase in $G_{t+1}$? Provide some intuition for your answer.

(k) What would happen to $r_t$ following a surprise decrease in $T_t$? What would happen to $S_t$ and $S_t^{G}$? Explain your answer.

(2) Heterogeneity: Suppose that we have an endowment economy, but with two different types of agents. There are sufficiently many of each type of agent that they all behave as price-takers. The agents differ in their endowment streams – type 1 agents have endowment pattern $(Y^1_t, Y^1_{t+1}) = (1, 0)$, while type 2 agents have endowment pattern $(Y^2_t, Y^2_{t+1}) = (0, 1)$. In words, type 1 agents have income today but none in the future, while type 2 agents have no income today but one unit in the future. Let there be $N^1$ of type 1 agents and $N^2$ of type 2 agents. These agents can save, borrow or borrow at the common real interest rate, $r_t$.

(a) Before doing any math, do you think that saving/borrowing will take place in equilibrium in this economy? If so, what type of agents will be saving? Which type will be borrowing? Why?

The problem of the type 1 agents is:

$$\max_{C^1_t, S^1_t, C^1_{t+1}} \quad U = \ln C^1_t + \beta \ln C^1_{t+1}$$

s.t.

$$C^1_t + S^1_t = 1$$
$$C^1_{t+1} = (1 + r_t)S^1_t$$

(b) Combine the two within period constraints into one intertemporal budget constraint and derive the Euler equation characterizing an optimal consumption plan for type 1 agents.

(c) Derive the consumption function for type 1 agents.

(d) Use your consumption function from (b) to derive a saving function for type 1 agents (e.g. $S^1_t = Y^1_t - C^1_t$, so just plug in your consumption function).

The problem of type 2 agents is:

$$\max_{C^2_t, S^2_t, C^2_{t+1}} \quad U = \ln C^2_t + \beta \ln C^2_{t+1}$$
s.t.

\[ C^2_t + S^2_t = 0 \]
\[ C^2_{t+1} = 1 + (1 + r_t)S^2_t \]

(e) Combine the two within period constraints into one intertemporal budget constraint and derive the Euler equation characterizing an optimal consumption plan for type 2 agents.

(f) Derive the consumption function for type 2 agents.

(g) Use your consumption function from (b) to derive a saving function for type 2 agents (e.g. \( S^2_t = Y^2_t - C^2_t \), so just plug in your consumption function).

(h) In equilibrium, what must be true about \( N^1 S^1_t \) (aggregate saving of type 1 agents) and \( N^2 S^2_t \) (aggregate saving of type 2 agents)?

(i) Use the equilibrium condition from (h) to solve for the equilibrium real interest rate, as well as the equilibrium consumption allocations of each type of agent (e.g. \( C^1_t \) and \( C^2_t \)).

(j) Suppose that there is an increase in the number of type 2 agents (e.g. \( N^2 \) increases). How will this affect the equilibrium real interest rate and the consumption allocations? Will type 1 agents be better or worse off following the increase in the population of type 2 agents?