Problem Set 6

Graduate Macro II, Spring 2013
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on April 11.

(1) Government Spending Shocks in a RBC Model: Consider a decentralized equilibrium model in which there is government spending. Households take government spending as exogenously given, though they receive utility from government spending in an additively seperable way. The preferences of the representative household are given by:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi \frac{N_t^{1+\phi}}{1+\phi} + \theta \ln G_t \right)$$

The household owns the capital stock. It can save via capital or real bonds, and pays lump sum taxes (or receives lump sum transfers) from the government. The period budget constraint is:

$$C_t + K_{t+1} - (1-\delta)K_t + b_{t+1} \leq w_t N_t + R_t K_t - T_t + \Pi_t + (1+r_t)b_t$$

(a) Find the first order conditions characterizing the solution to the household problem.

The firm picks capital and labor to maximize period profit from a Cobb-Douglas production function:

$$\Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t K_t$$

(b) Find the first order conditions of the firm problem.

Government spending is exogenous and can be financed via lump-sum taxes, T_t, or debt-issuance, d_t (as defined, $d_t < 0$ is issuing debt, whereas $d_t > 0$ is saving). The period government budget constraint is:

$$G_t + d_{t+1} \leq T_t + (1+r_t)d_t$$

(c) Assume that neither the household nor the government begin life with any net bond holdings, e.g. $b_0 = d_0 = 0$, though the economy does begin with an endowment of capital. Write down the definition of a competitive equilibrium. In the competitive equilibrium, what must be true about d_t and b_t? Use this to derive an aggregate law of motion for the capital stock.

Suppose that G_t and A_t obey the following stochastic processes:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \;\; \varepsilon_{a,t} \sim N(0, 0.01^2)$$
$$\ln G_t = (1-\rho_g)\ln G^* + \rho_g \ln G_{t-1} + \varepsilon_{g,t}, \;\; \varepsilon_{g,t} \sim N(0, 0.01^2)$$

(d) Suppose that mean government spending, G^*, is set to equal a fraction, $\omega \in (0,1)$, of output. Solve for the non-stochastic steady state values of N^*, C^*, K^*, and Y^* as a function of ω and other parameters. How does ω affect the steady state? What is the intuition for this effect?

(e) Assume the following parameter values: $\beta = 0.99$, $\psi = 1$, $\omega = 0.2$, $\phi = 1$, $\alpha = 0.33$, $\delta = 0.025$, and $\rho_a = 0.95$, and $\rho_g = 0.95$. Write a Dynare file and compute impulse responses to both a one standard deviation technology shock and a one standard deviation government spending shock.

(f) Repeat the exercise in (e) for two different values of ρ_g, 0.99 and 0.50. How does the impulse response of investment ($I_t = Y_t - C_t - G_t$). How does the response of investment vary with ρ_g? What is the intuition for this effect?

(g) Now we're going to change the problem such that G_t is endogenous. In particular, write the problem as a planner's problem where the planner gets to choose G_t (in addition to the other endogenous choice variables). Find the first order condition characterizing the optimal choice of G_t.

(h) Solve for the non-stochastic steady state of the economy with endogenous G_t using your conditions from part (g). How is the steady state a function of θ?

(i) Write a new Dynare code in the endogenous government spending model. Use the parameters from the previous part, and assume that $\theta = 1/3$. Calculate impulse responses to a technology shock in this economy. How do they compare to the economy in which government spending was exogenous?