1. **The Equity Premium: Microfoundations.** Suppose that there exists a household who lives for two periods: \( t \) (the present) and \( t+1 \) (the future). After \( t+1 \) the household dies. The household has access to two savings vehicles – a risk-free one period bond, \( B_t \), and a risky stock, \( S_t \). The household can purchase bonds at a price of \( P_t^B \). These are discount bonds – purchasing \( B_t \) units in period \( t \) generates \( B_t \) units of income in period \( t+1 \). The household can purchase stock at a price of \( P_t^S \). If the household owns \( S_t \) units of stock, in period \( t+1 \) it gets \( D_t+1 S_t \) in income, where \( D_t+1 \) is the dividend per share of stock. The dividend payout is unknown in period \( t \) – with probability \( p \) the dividend payout will be \( D_t+1^h \), and with probability \( 1-p \) it will be \( D_t+1^l \), where \( D_t+1^h \geq D_t+1^l \). The household earns exogenous income of \( Y_t \) in period \( t \), which is known. Future income, \( Y_{t+1} \), is unknown. When the dividend payout is \( D_t+1^l \), income is \( Y_t+1^l \). When the dividend payout is \( D_t+1^h \), income is \( Y_t+1^2 \). \( Y_t+1^2 \) could be the same, higher, or lower than \( Y_t+1^l \).

The budget constraint in period \( t \) is:

\[
C_t + P_t^B B_t + P_t^S S_t = Y_t
\]

Consumption plus purchases of bonds plus purchases of stock must equal income. The budget constraint in \( t+1 \) is:

\[
C_{t+1} = Y_{t+1} + B_t + D_{t+1} S_t
\]

Consumption in \( t+1 \) must equal income plus the payout from the bond, \( B_t \), plus the payout from the holding of stock, \( D_{t+1} S_t \). The budget constraint must hold in either “state of the world” – i.e. it must hold if \( D_{t+1} = D_t^l \) and \( Y_{t+1} = Y_t^l \) and it must hold when \( D_{t+1} = D_t^h \) and \( Y_{t+1} = Y_t^2 \). Since the household dies after \( t+1 \), it will not choose to buy any more bonds or stock in \( t+1 \).

The household wants to maximize its expected utility, which is given by:

\[
U = \ln C_t + \beta E[\ln C_{t+1}]
\]

Because the future dividend and future income are unknown, future consumption is unknown in period \( t \), and hence there is an expectations operator outside of utility from \( t+1 \) consumption. There is no uncertainty over period \( t \) consumption and hence no expectation operator there. The household’s objective is to pick \( C_t \), \( C_{t+1} \), \( B_t \), and \( S_t \) to maximize lifetime utility subject to the two budget constraints:
\[
\max_{C_t, C_{t+1}, B_t, S_t} \ln C_t + \beta E[\ln C_{t+1}]
\]

s.t.
\[
C_t + P_t^B B_t + P_t^S S_t = Y_t
\]
\[
C_{t+1} = Y_{t+1} + B_t + D_{t+1} S_t
\]

(a) Transform this into an unconstrained maximization problem by substituting out \(C_t\) and \(C_{t+1}\) from the constraints.

(b) Derive the first order optimality conditions. Provide some economic intuition for them.

(c) Suppose that, in equilibrium, the supply of both bonds and stocks are zero – i.e. \(B_t = 0\) and \(S_t = 0\). This then means that \(C_t = Y_t\) and \(C_{t+1} = Y_{t+1}\), but we can still price each asset. Suppose that \(D_{t+1}^h = D_{t+1}^l = 1.1\). Suppose that \(p = 0.5\). Suppose that \(Y_t = 1\), \(Y_{t+1}^1 = 1\), and \(Y_{t+1}^2 = 1.1\). Suppose that \(\beta = 0.95\). Use your optimality conditions from part (b) to solve for the price of the bond and the price of the stock.

(d) From the previous part, calculate the expected returns (i.e. discount rates) for the bond and for the stock using the following formulas:
\[
1 + \dot{r}_t^B = \frac{R_{t+1}^B}{P_t^B}
\]
\[
1 + \kappa_t^e = \frac{R_{t+1}^e}{P_t^S}
\]

Use your answers to calculate the equity premium, \(\psi_t = \kappa_t^e - \dot{r}_t^B\). Provide some intuition for why you find the equity premium you do.

(e) Re-do parts (c)-(d), this time assuming that \(D_{t+1}^h = D_{t+1}^l = 1.1\), but \(Y_{t+1}^1 = 0.95\) and \(Y_{t+1}^2 = 1.05\). \(p\), \(Y_t\), and \(\beta\) are the same. Provide some intuition for why your answers are (or are not) different from you found in (c)-(d).

(f) Re-do parts (c)-(d), this time assuming that \(D_{t+1}^h = 1.2\) and \(D_{t+1}^l = 1\), but \(Y_{t+1}^1 = Y_{t+1}^2 = 1.\) \(p\), \(Y_t\), and \(\beta\) are the same. Provide some intuition for why your answers are (or are not) different from you found in (c)-(d).

(g) Re-do parts (c)-(d), this time assuming that \(D_{t+1}^h = 1.2\) and \(D_{t+1}^l = 1\) with \(Y_{t+1}^1 = 0.95\) and \(Y_{t+1}^2 = 1.05\). \(p\), \(Y_t\), and \(\beta\) are the same. Provide some intuition for why your answers are (or are not) different from you found in (c)-(d).

(h) Re-do part (g), but this time assume that \(Y_{t+1}^1 = 1.05\) and \(Y_{t+1}^2 = 0.95\). Provide some intuition for why your answers are (or are not) different from you found in (g).

(i) Which of the correlational structure among dividends and future income – part (g) or (h) – do you think is more likely to characterize reality? As such, can our simple microfounded model make sense of the fact that on average there is a positive equity premium in the data?

2. **The Gordon Growth Model**: Suppose that there is a stock which currently pays a dividend of 1 (\(D_t = 1\)). It is expected that dividends will grow into the future at a constant rate of \(g = 0.02\) and will do so forever (i.e. \(D_{t+h} = (1 + g)^h D_t\) for \(h \geq 0\). The discount rate for equity is constant \(\kappa^e = 0.05\).
(a) Imposing a no-bubble condition, solve for the price of the stock in period $t$.

(b) What is the expected price of the stock in $t+1$? What is the expected return? Decompose the expected return into dividend and capital gain.

(c) For the general case (i.e. use symbols, not actual numbers), derive an expression for the dividend component of the expected return (as a function of $\kappa^e$ and $g$) and the capital gain component of the expected return (as a function of $\kappa^e$ and $g$). As $g$ gets bigger, which term – dividends or capital gains – drives a bigger component of the total expected return?

3. Bubbles: Suppose that you have a stock that currently pays a dividend of $D_t = 1$. Future dividends are discounted at a constant rate of $\kappa^e = 0.07$, and dividends are expected to grow at a constant rate forever of $g = 0.02$. The current price of the stock is $P_t = 25$.

(a) What is the magnitude of the bubble term for this stock?

(b) Given your answer on (a), what would you expect the price of the stock to be in $t+1$?

(c) Given your answers on (a)-(b), what is your expected return from holding this stock from $t$ to $t+1$?

(d) Suppose that the bubble bursts in period $t+1$. What is your realized return (not your expected return) from holding this stock from $t$ to $t+1$?

4. Adverse Selection and Collateral: Suppose that there is one bank and two types firms that seek funding of $\$1$ for a project. The two types of firms are safe and risky. If a safe firm undertakes a project, it will succeed and earn $\$2$ with probability $4/5$; with probability $1/5$ it will earn nothing. If a risky firm undertakes a project, it will earn $\$3$ if the project succeeds, but the project only succeeds with probability $1/6$; with probability $5/6$ the project fails and the firm earns nothing.

Assume that there is a bank that has funds. It can lend these funds to either kind of firm at a (gross) interest rate of $R$. The bank’s opportunity cost of funds is $\$1$ – it will not make a loan unless it expects to get back at least $\$1$. There is limited liability – if a project fails a firm owes the bank nothing, whereas if the project succeeds a firm owes the bank $R$.

(a) Suppose that the bank could tell safe firms apart from risky firms and charge them different interest rates. Suppose that the market structure is such that if the bank makes a loan, it just breaks even (i.e. earns $\$1$ in expectation, its opportunity cost). Will both types of firms be able to secure funding? If not, which type of firm will be able to secure funding?

(b) Now suppose that the bank cannot tell safe firms apart from risky firms. It only knows that 50 percent of firms are safe, and 50 percent are risky. It can only post one loan contract. Please plot the lender’s expected payout (vertical axis) against $R$. What will happen in this market? Will both firms be able to secure funding? If not, which one type of firm be able to secure funding? Explain briefly.

(c) Now suppose that the lender can require a firm to post collateral in the amount of $C = \frac{1}{2}$ on a loan contract – should a project be undertaken and not succeed, the lender can get back the collateral. If the bank must just break even, will either firm (or both) be able to secure funding? If so, which one? Justify your answer.