Problem Set 7
Graduate Macro II, Spring 2010
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Friday, April 16.

(1) Interest Rate Rules, the Taylor Principle, and Determinacy: Suppose that we have a New Keynesian model with steady state inflation equal to zero. The log-linearized Phillips Curve and Euler equations are:

\[ \tilde{\pi}_t = \gamma \left( \tilde{y}_t - \tilde{y}_t^f \right) + \beta E_t \tilde{\pi}_{t+1} \]
\[ -\tilde{y}_t = \frac{1}{\sigma} \tilde{z}_t - E_t \tilde{y}_{t+1} \]
\[ \tilde{y}_t^f = \rho \tilde{y}_{t-1} + \epsilon_t \]

Here, \( \gamma = \frac{(1-\phi)(1-\phi \beta)}{\phi} (\sigma + \xi \frac{n}{1-n}) \). \( \sigma \) is the inverse elasticity of intertemporal substitution in consumption, and \( \xi \frac{n}{1-n} \) is the inverse of the Frisch labor supply elasticity. \( \tilde{y}_t^f \) is the flexible price level of output – that level of output which would obtain in the absence of price stickiness (which is governed by the Calvo parameter, \( \phi \)). For simplicity, we model the evolution of the flexible price equilibrium level of output as a stationary AR(1).

Instead of assuming that agents get utility from real money balances and that the central bank controls the supply of money, instead we assume that the central bank follows a nominal interest rate targeting rule. Indeed we need not make mention of money at all. Suppose that the central bank follows a rule of keeping the nominal interest rate constant, so that:

\[ \tilde{i}_t = 0 \]

(a) Use the Fisher relationship \( \tilde{r}_t = \tilde{i}_t - E_t \tilde{\pi}_{t+1} \) and the nominal interest rate rule to eliminate \( \tilde{r}_t \) as a variable. Reduce the system to three variables, and write it in the form:

\[ E_t \begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{y}_{t+1} \\ \tilde{y}_t^f \end{bmatrix} = M \begin{bmatrix} \tilde{\pi}_t \\ \tilde{y}_t \\ \tilde{y}_t^f \end{bmatrix} \]

What are the jump variables? What are the predetermined variables? Produce an analytical expression for \( M \).

(b) What does it mean for a dynamic system to be saddle point stable? How many unstable eigenvalues must \( M \) have for this system to be saddle point stable?
(c) Show that there are an insufficient number of unstable eigenvalues for any permissible parameterization of this system. In your proof, ignore the possibility of complex roots. Will this lead to a multiplicity of stable solutions, or a lack of a stable solution? Hint: you will not be able to derive an analytical expression for all of the eigenvalues, but should be able to say something about their stability.

(d) Instead suppose that the central bank obeys the following rule:

\[
\tilde{i}_t = \zeta_\pi \tilde{\pi}_t + \zeta_y \left( \tilde{y}_t - \tilde{y}_t^f \right)
\]

Rules of this form are sometimes called “Taylor Rules”, after John Taylor (1993). Assume that \( \zeta_\pi \geq 0 \) and \( \zeta_y \geq 0 \). Repeat (a)-(c) for this rule, and provide a necessary condition for there to be the “right” number of unstable eigenvalues for saddle point stability to obtain. Once again, you may ignore the possibility of complex roots.

(2) Technology Shocks and New Keynesian Models: Suppose that we have a simple New Keynesian model, characterized by the following log-linearized relationships. All variables as defined as percentage (or absolute in the case of things like interest rates and inflation) deviations from steady state:

\[
\tilde{\pi}_t = \left(1 - \phi \right) \left(1 - \phi \beta \right) \tilde{\pi}_{t-1} + \beta E_t \tilde{\pi}_{t+1}
\]

(1)

\[
\tilde{m}_t = \tilde{w}_t - \tilde{\pi}_t
\]

(2)

\[
\xi n^* \tilde{n}_t = -\sigma \tilde{y}_t + \tilde{w}_t
\]

(3)

\[
-\tilde{y}_t = \frac{1}{\sigma} \tilde{r}_t - E_t \tilde{y}_{t+1}
\]

(4)

\[-v \tilde{m}_t = -\sigma \tilde{y}_t + \left( \frac{1}{\tilde{y}_t} - \frac{1}{1 + \tilde{y}_t} \right) \tilde{i}_t
\]

(5)

\[
\Delta \tilde{m}_t + \tilde{\pi}_t = \rho_m \Delta \tilde{m}_{t-1} + \rho_n \tilde{\pi}_{t-1} + u_t
\]

(6)

\[
\Delta \tilde{m}_t = \tilde{m}_t - \tilde{m}_{t-1}
\]

(7)

\[
\tilde{r}_t = \tilde{i}_t - E_t \tilde{\pi}_{t+1}
\]

(8)

\[
\tilde{y}_t = \tilde{a}_t + \tilde{\pi}_t
\]

(9)

\[
\tilde{a}_t = \rho \tilde{a}_{t-1} + e_t
\]

(10)
(a) Give a brief description of where each of these ten equations come from, what they mean, and what the parameters in them are.

(b) Suppose that you receive incontrovertible evidence that firms change their prices once every six quarters on average. Given this information, what would be an appropriate value for $\phi$?

(c) In the flexible price equilibrium, what is true of $\overline{mC_t}$? Use this fact to derive expressions for the flexible price equilibrium levels of output, $\overline{y}_t^f$, and the real interest rate, $\overline{r}_t^f$ as functions of $\overline{a}_t$.

(d) Create a Dynare .mod file to solve this model and produce impulse responses to both the technology and money growth shocks. You can enter the variables of the model as they are above. Just note that the model will be interpreting the variables you give it as deviations from steady state, which means that the steady state values should all be zero. Use the value of $\phi$ from part (b). Calibrate the remaining parameters as follows: $\beta = 0.99$, $\sigma = 1$, $n^* = 0.2$, $\xi = 1$, $\rho = 0.9$, $\upsilon = 1$, $\rho_m = 0.5$, $\sigma_e = 0.01$, and $\sigma_u = 0.005$. Show the impulse responses to both shocks. Show also the responses of the price level and nominal money supply (which must be created outside of the .mod file as in the previous problem set).

(e) Provide some intuition for the impulse response of hours to a technology shock.

(f) Replace the money growth rule with a Taylor type nominal interest rate rule. That is, replace equation (6) with the following, and drop equation (7) altogether:

$$\tilde{\pi}_t = \zeta_{\pi} \pi_t + \zeta_y \left( \tilde{y}_t - \tilde{y}_t^f \right) + u_t$$

Set $\zeta_{\pi} = 1.5$ and $\zeta_y = 0.5$. Set the standard deviation of the policy shock, $\sigma_u = 0.001$. Now compute the impulse responses to a technology shock. Discuss how they are different under this rule than under the money growth rule. In particular, what happens to the nominal money supply in response to a technology shock here as opposed to in (d)?