Problem Set 7

Graduate Macro II, Spring 2013
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday April 23.

(1) Adjustment Costs: Consider a general equilibrium version of our model with capital adjustment costs. Assume that firms own the capital and make accumulation decisions. The household side of the model is therefore entirely standard:

$$\max_{C_t,N_t,B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi \frac{N_t^{1+\phi}}{1+\phi} \right)$$

s.t.

$$C_t + B_{t+1} \leq w_t N_t + \Pi_t + (1+r_t)B_t$$

(a) Find the first order conditions for the household problem.

Firm profits are given by:

$$\Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - I_t - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

The firm's objective is to maximize the present discounted value of profits, where it discounts by the stochastic discount factor of the household, subject to the law of motion for the capital stock. The discount factor and law of motion are, respectively:

$$\widetilde{M}_t = \beta^t \frac{u'(C_t)}{u'(C_0)}$$
$$K_{t+1} = I_t + (1-\delta)K_t$$

The firm problem is then:

$$\max_{N_t,I_t,K_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \widetilde{M}_t \Pi_t$$

s.t.

$$K_{t+1} = I_t + (1-\delta)K_t$$

Assume that the firm does not operate in debt markets.

(b) Find the first order conditions for the firm problem, using q_t as the multiplier on the constraint.

Assume that A_t follows a mean zero AR(1) process in the log (mean 1 in the level):

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t$$

(c) Write down the definition of a competitive equilibrium. Combine the optimality conditions for the firm and household to derive the aggregate market-clearing condition. (Assume that the household begins time with a zero stock of bonds).

(d) Write a Dynare .mod file to solve for the linearized policy functions of this model. Use the following parameter values: $\beta = 0.99$, $\psi = 2$, $\phi = 1$, $\delta = 0.025$, $\alpha = 0.33$, $\tau = 2$, $\rho = 0.95$, and set the standard deviation of the productivity shock to 0.01. Compute impulse responses to a one standard deviation productivity shock and show them here.

(e) Experiment with different values of τ: 4 and 8. How do the impulse responses (including that of q_t) to the productivity shock vary with τ? Do you have any intuition for this?

(f) One empirical failing of the baseline RBC model is that it predicts a real interest rate that is too procyclical. How does the inclusion of the adjustment cost affect the correlation between output and the real interest rate in the model?

A drawback of the model as written above is that it is not consistent with US National Income and Product Accounts – the adjustment cost shows up in the accounting identity relating output to consumption and investment. An alternative specification is to put the adjustment cost in the capital accumulation equation. That is, firm profit and capital accumulation are instead given by:

$$\Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - I_t$$

$$K_{t+1} = I_t + (1-\delta)K_t - \frac{\tau}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

(g) Describe the intuitive difference in these approaches to modeling the adjustment cost.

(h) Find the first order conditions characterizing the solution to the firm problem using this alternative specification. Again use q_t as the multiplier on the accumulation equation.

(i) Derive the market-clearing condition under this specification of the adjustment cost function.

(j) Repeat the exercises from (d)-(e) with this specification of adjustment costs, showing the impulse responses for different values of τ. Comment on any differences in the observed impulse responses.