Problem Set 7
Graduate Macro II, Spring 2014
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Wednesday, April 23.

(1) Evaluating Different Monetary Policy Rules: Consider a highly stylized New Keynesian model. The non-policy block of the model can be characterized by an IS curve and a Phillips Curve (all variables are deviations (either percentage or absolute) from steady state; for simplicity I do not use “tilde” notation):

\[
x_t = E_t x_{t+1} - (i_t - E_t \pi_{t+1} - r^f_t)
\]

(1)

\[
\pi_t = \gamma x_t + \beta E_t \pi_{t+1} + u_t
\]

(2)

\(x_t\) is the output gap and \(\pi_t\) is inflation. \(\gamma\) is the slope coefficient on the Phillips Curve, which depends on the amount of price rigidity, the discount factor, the elasticity of intertemporal substitution, and the Frisch labor supply elasticity. \(r^f_t\) is the natural rate of interest and \(u_t\) is a cost-push shock. The exogenous processes for these two variables are:

\[
r^f_t = \rho r^f_{t-1} + s_r e_{r,t}, \quad e_{r,t} \sim N(0,1)
\]

(3)

\[
u_t = \rho u_{t-1} + s_u e_{u,t}, \quad e_{u,t} \sim N(0,1)
\]

(4)

Both shocks follow standard normal distributions, with \(s_r\) and \(s_u\) the standard deviations of the shocks. The central bank loss function depends on inflation and the output gap as follows:

\[
L = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \omega x_t^2)
\]

(5)

(a) Suppose that the central bank acts with discretion. In particular, it minimizes its loss function period-by-period by choosing inflation and the output gap subject to the constraint of the Phillips Curve holding. Find the first order condition for the central bank problem.

(b) Now suppose that the central bank acts with commitment. It minimizes the entire loss function subject to the Phillips Curve, taking into account that its choices today will affect the future. Find the first order condition for the central bank problem. Comment on how it differs from the case of discretion.

Now we’re going to quantitatively analyze the model in Dynare. Use the following parameter values. Set \(\gamma = 0.25\) and \(\beta = 0.99\). The weight on the output gap in the loss function is given by \(\omega = \frac{\epsilon}{\gamma^2}\), where \(\epsilon\) is the price elasticity of demand. A good value for this parameter consistent with data on markups is \(\epsilon = 10\), which implies that \(\omega = 0.025\). Set \(\rho_r = \rho_u = 0.95\), and set \(s_r = 0.01\) and \(s_u = 0.002\). You can calculate the welfare loss in the model as being proportional to the weighted sum of the variances of inflation and the output gap (Dynare stores information on the theoretical variances of the variables in the diagonal elements of “oo_.var”). In particular, you should report welfare losses as \(-100 \times (\text{var}(\pi_t) + \omega \text{var}(x_t))\).
(c) Solve the model under the optimal policy under discretion. To do this, simply include the first order condition from (a) as the fifth equation in the model. Report the welfare loss.

(d) Repeat this exercise, but use the optimal policy under commitment. To do this, you may need to introduce another variable (the price level) as a variable in your code. Report the welfare loss under the optimal policy under commitment. Comment on how this loss compares to the one you reported in (c).

(e) Instead of using either of these optimal policies, consider, separately, inflation or gap targeting. That is, include either $\pi_t = 0$ or $x_t = 0$ as the fifth equation in your model in Dynare. Compute the welfare losses under these policies, and comment on how they differ relative to the losses under optimal discretion or commitment from (c) and (d).

(f) Instead of any of these policies, consider a simple Taylor rule of the form: $i_t = \phi_\pi \pi_t + \phi_x x_t$ (abstract from both interest rate smoothing and a monetary policy shock). Use values of $\phi_\pi = 1.5$ and $\phi_x = 0.5$. Calculate the welfare loss under this policy rule, and compare it to your previous answers.

(g) Separately consider Taylor rules which respond more strongly either to the output gap or inflation. In particular, consider one rule with $\phi_\pi = 3$ and $\phi_x = 0.5$, and another rule with $\phi_\pi = 1.5$ and $\phi_x = 1$. How do these different parameterizations of the policy rule affect welfare?

(h) Repeat all of the previous exercises, but this time set the standard deviation of the cost-push shock to zero, $s_u = 0$.

(i) Repeat all of the previous exercises, but this time set the standard deviation of the cost-push shock to double its initial value, $s_u = 0.004$.

(j) Using your answers to previous parts, briefly summarize how the magnitude of cost-push shocks impacts the relative merits of inflation and gap stabilization, both in a strict sense as well as in terms of the parameters on these variables in a Taylor rule formulation.