Problem Set 8
Graduate Macro II, Spring 2010
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due in class on Tuesday, April 27.

(1) Output Gap Targeting: Suppose that we have a conventional New Keynesian model. The Phillips and IS curves (written in terms of the output gap, $\bar{x}_t \equiv \bar{y}_t - \bar{y}_t^f$) are:

\[
\bar{\pi}_t = \gamma \bar{x}_t + \beta E_t \bar{\pi}_{t+1} \\
E_t \bar{x}_{t+1} = \bar{x}_t + \frac{1}{\sigma} \left( \bar{r}_t - \bar{r}_t^f \right)
\]

Suppose that the central bank desires to keep the output gap equal to zero at all times; i.e. the objective is to set $\bar{x}_t = 0 \ \forall \ t$.

(a) Derive a nominal interest rate rule of the following form which will achieve this objective:

\[
\tilde{i}_t = \tilde{i}_t^* + \theta_\pi \bar{\pi}_t + \theta_x \bar{x}_t
\]

Derive expressions for $\tilde{i}_t^*$, $\theta_\pi$, and $\theta_x$. The Taylor principle requires that $\theta_\pi + \left( \frac{1-\beta}{\gamma} \right) \theta_x > 1$ for there to exist a unique rational expectations equilibrium. Is the Taylor principle satisfied for the parameters of your rule?

(b) Instead suppose that there is a cost-push shock in the Phillips Curve:

\[
\bar{\pi}_t = \gamma \bar{x}_t + \beta E_t \bar{\pi}_{t+1} + \bar{u}_t \\
\bar{u}_t = \rho \bar{u}_{t-1} + \epsilon_t \quad 0 < \rho < 1
\]

Derive a rule similarly to above under this specification. What are $\tilde{i}_t^*$, $\theta_\pi$, and $\theta_x$? Will the Taylor principle be satisfied for this rule?

(c) Proceed similarly to (b), but allow the central bank to respond directly to the cost-push shock in its rule:

\[
\tilde{i}_t = \tilde{i}_t^* + \theta_\pi \bar{\pi}_t + \theta_x \bar{x}_t + \theta_u \bar{u}_t
\]

Derive expressions for this rule consistent with the central bank hitting its target of no output gap. Will the Taylor principle be satisfied here?
(d) Suppose that the central bank is constrained to a nominal interest rate rule of the form:

\[ \tilde{\pi}_t = \theta_x \tilde{\pi}_t + \theta_x \tilde{x}_t \]

The only difference from above is that the intercept is constant (so that \( \tilde{\pi}^c_t = 0 \)). Suppose that \( \theta_x = 1.5 \). Make an argument for a value of \( \theta_x \) that will make this rule numerically equivalent to the rule you found in part (c).

(2) Taxes and Spending: Suppose that households receive utility from consuming goods and taking leisure. Their expected discounted lifetime utility is:

\[ E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(l_t)) \]

The functions \( u(\cdot) \) and \( v(\cdot) \) are increasing and concave. Leisure is constrained to \( 1 - n_t \), with one being the time endowment and \( n_t \) denoting employment. Households own the capital stock and supply labor. They receive wage rate \( w_t \) for supply labor and rental rate on capital (net of depreciation) \( r^k_t \). They can save through either government bonds, \( b_t \), or by accumulating more capital, \( k_t \). Bonds pay a riskless interest rate, \( r^b_t \). Households face tax rates on consumption, wage earnings, and capital income, and also pay/receive lump sum taxes/transfers, \( T_t \), from the government. The household flow budget constraint is:

\[ (1 + \tau^c_t)c_t + k_{t+1} + b_{t+1} \leq (1 - \tau^w_t)w_t n_t + (1 + (1 - \tau^k_t)r^k_t)k_t + (1 + r^b_t)b_t \]

(a) Write down the first order conditions characterizing an interior solution to the household’s problem.

(b) Suppose that firms are competitive, with a constant returns to scale production technology, \( f(k_t, n_t) \). The production function is increasing and concave, with a positive cross partial between capital and labor. The implicit rental rate on capital is \( r^k_t + \delta \) and the wage rate on labor is \( w_t \). Write down the firm’s problem and associated first order conditions.

(c) The government budget constraint is:

\[ g_t - b_{t+1} \leq \tau^w_t w_t n_t + \tau^k_t r^k_t k_t + \tau^c_t c_t - (1 + r^b_t)b_t + T_t \]

Thus, the government can finance its expenditure through distortionary taxation, lump sum taxation, or by issuing more debt. The government can choose \( b_{t+1}, g_t, T_t \), and the various tax rates at each point in time. If the government is benevolent and seeks to maximize household welfare subject to its budget constraint and the aggregate resource constraint, characterize the time paths of spending, lump sum taxation, and the various tax rates. Note that government expenditures cannot be negative.

(d) Instead suppose that the time path of spending is exogenously fixed by the political system. The government’s problem is then to choose lump sum taxes and the tax rates to maximize household welfare, subject to its budget constraint and the aggregate resource constraint. Characterize the optimal solution.
(e) Now suppose that lump sum taxes/transfers are unavailable to the government; that is, $T_t = 0 \ \forall \ t$. Continue to assume that the time path of spending is exogenously fixed and that the government is benevolent. Show that the optimal tax rate on capital in the long run must be zero.