Problem Set 8

Graduate Macro II, Spring 2013
The University of Notre Dame
Professor Sims

Instructions: You may consult with other members of the class, but please make sure to turn in your own work. Where applicable, please print out figures and codes. This problem set is due by "end of business" on Thursday, May 2.

(1) Solving a New Keynesian Model by Hand: Consider the baseline New Keynesian model with no capital. Rather than explicitly modeling money in the utility function, I follow much of the literature and assume that the economy is "cashless." Rather than setting the money supply, the central bank instead sets the nominal interest rate according to a simple rule. Implicitly, there is a demand for money generated via money in the utility function; the central bank prints the amount of money necessary to make the money market clear at its target interest rate. Hence, we can avoid money altogether.

There is a representative household who solves the following problem, where P_t is the nominal price of goods, W_t is the nominal wage, i_{t-1} the nominal interest rate on bonds carried over from period $t-1$ into period t (note the mild difference in the timing convention relative to how I usually write it), and Π_t^n is nominal profits distributed from firms (which the household takes as given):

$$\max_{C_t, N_t, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \frac{N_t^{1+\phi}}{1+\phi} \right)$$

s.t.

$$P_t C_t + B_{t+1} \leq W_t N_t + \Pi_t^n + (1+i_{t-1}) B_t$$

(a) Find the first order conditions for an interior solution to the household problem.

Production is split up into two sectors. There is a competitive, representative final goods firm which produces final output, Y_t, which is a CES aggregate of a continuum of intermediate goods, $Y_{j,t}$, $j \in (0,1)$.

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

(b) Write down the profit maximization problem of the final good firm in nominal terms (nominal output is $P_t Y_t$) and derive the demand curve for each intermediate good.

(c) Define nominal output as $P_t Y_t = \int_0^1 P_{j,t} Y_{j,t} dj$. Use your answer from (b) to derive an expression for the aggregate price level.

Intermediate goods producers produce output using a linear production technology in labor input:

$$Y_{j,t} = A_t N_{j,t}$$

(d) Intermediate goods producers are price-takers in input markets, taking W_t as given. Set up the cost-minimizing problem of a generic intermediate producer, subject to the constraint that it produce as much as is demanded at a given price. Find the first order condition and provide an economic interpretation of the Lagrange multiplier.

Intermediate goods producers are not freely able to adjust their prices period-by-period. Each period they face a probability of $1-\theta$, $\theta \in (0,1)$, of being able to adjust their price. The probability of being able to adjust is independent of when they last updated their price. Firms discount future profit flows by the stochastic discount factor and the probability of being stuck with their currently chosen price.

(e) Write down the (dynamic) pricing problem of the firm, and derive the optimal "reset price" for a firm able to update its price in a given period.

Assume that the central bank sets the nominal interest rate according to a simple Taylor rule of the form:

$$i_t = i^* + \phi(\pi_t - \pi^*), \quad \phi > 1$$

Here i^* is the steady state nominal interest rate and π^* is the target inflation rate, which will also be equal to the inflation rate in the non-stochastic steady state. ϕ is a parameter restricted to be greater than 1. Assume also that A_t follows a mean-zero stationary AR(1) in the log:

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

(f) Write down the definition of an equilibrium for this economy. Assume that households do not begin time with any debt, and that firms and the government also do not participate in debt markets. Write down the market-clearing conditions.

(g) Use the properties of Calvo pricing (in particular, that firms are randomly selected to update their price and that $1-\theta$ is not only the probability of being able to adjust price but also the fraction of all firms who get to update their price) to derive an expression for the aggregate price level which depends on the optimal reset price and the lagged aggregate price level.

(h) Define inflation as $1+\pi_t = \frac{P_t}{P_{t-1}}$ and reset price inflation as $1+\pi_t^{\#} = \frac{P_t^{\#}}{P_{t-1}}$. Re-write your expression for the evolution of the aggregate price level above in terms of inflation and reset price inflation.

(i) Derive an expression for the aggregate production function in terms of A_t, N_t, and a term related to price dispersion. Write the term related to price dispersion recursively in terms of inflation and reset price inflation in a way that does not depend on j.

(j) Assume that inflation is zero in steady state, so $\pi^* = 0$. Solve for the non-stochastic steady state values of $\pi_t^{\#}$, i_t, Y_t, N_t, w_t (the real wage), and mc_t (real marginal cost).

(k) Log-linearize the first order conditions about the zero inflation steady state. You should be able to derive a Phillips Curve expressing current inflation as a function of current real marginal cost and expected future inflation.

(l) If prices were flexible, e.g. $\theta = 0$, then one can show that real marginal cost would be constant at its steady state value. Use this fact to solve for the (log-linearized) flexible price level of output, $\widetilde{Y}_t^f$, as a function of $\widetilde{A}_t$. Use this in conjunction with the household labor supply condition to write marginal cost as a function of the output gap, $\widetilde{X}_t = \widetilde{Y}_t - \widetilde{Y}_t^f$.

(m) Define the natural rate of interest, r_t^f, as the real interest rate that would obtain if prices were flexible. Use your answer from (l) to solve for an expression for r_t^f. Hint: take the Euler equation evaluated at the flexible price level of output as a function of $\widetilde{A}_t$. Solve for r_t^f as a function of $\widetilde{A}_t$ and $E_t\widetilde{A}_{t+1}$. Then you should be able to write it as though r_t^f follows an AR(1) process, replacing $\widetilde{A}_t$ from the analysis.

(n) Use your results from previous parts. You should be able to reduce the system to an Euler equation, a Phillips Curve, an effectively exogenous process for r_t^f, and the policy rule. You can eliminate i_t using the policy rule. This leaves three log-linearized variables: $\widetilde{X}_t$, π_t, and r_t^f. There are two endogenous variables, $\widetilde{X}_t$ and π_t, and one effectively exogenous variable, r_t^f. Guess that there exist linear policy functions of the form $\widetilde{X}_t = \psi_1 r_t^f$ and $\pi_t = \psi_2 r_t^f$. Plug these guesses into the three equation system and solve for the ψ_1 and ψ_2 that make all three equations simultaneously hold. This is called the "method of undetermined coefficients."

(o) How do the responses of inflation and the output gap vary with movements in r_t^f? How does the magnitude of these movements depend on the policy rule parameter, ϕ? To the extent to which a central bank dislikes movements in both inflation and the gap, would higher or lower values of ϕ be preferred?