Intermediate Macroeconomics:
The Real Business Cycle Model

Eric Sims
University of Notre Dame
Fall 2012

1 Introduction

Having developed an operational model of the economy, we want to ask ourselves the following two questions. First, can the model explain actual business cycle data? Second, to the extent to which it can fit the data, what are the implications for economic policy?

To proceed we first have make clear what is meant by “business cycle” data. This requires briefly looking at some US macroeconomic data. It is typical to summarize business cycle data by looking at “second moments” of de-trended natural logs of aggregate data. Second moments refer to variances (measures of volatility) and correlations (measures of co-movement). We will focus on correlations. Characterizing the qualitative co-movements in the data, we will then ask how well the equilibrium business cycle model we have developed can account for these co-movements. We will argue that the model can qualitatively match these co-movements if it is predominantly driven by exogenous changes in \( A_t \), which are sometimes labeled productivity shocks.

The model we have developed predominantly driven by changes in \( A_t \) is often referred to as the “real business cycle model.” It is “real” in the sense that there are no frictions which would give rise to monetary non-neutrality – the only source of movements in output and its components are “real shocks” like productivity and government spending. The model has the implication that fluctuations in output driven by these real shocks are “efficient” in the sense that there is no room for welfare improvement due to activist economic policies. Therefore, to the extent to which one believes the model, economic policies should not attempt to try to smooth out business cycle fluctuations.

We will conclude with a critique of real business cycle theory, and will then move on to models with frictions that give rise to more interesting policy implications.

2 Business Cycle Facts

We begin with a quick discussion of business cycle data. Our model makes predictions for the following economic series: GDP/output, consumption, investment, the real interest rate, hours
As noted early on in the course, many economic time series trend up. For looking at the business cycle, we want to focus on deviations about the trend (whereas growth theory studies the behavior of the trend itself). There are different ways of de-trending series. Perhaps the most obvious way to do so is with a linear trend: basically fit a straight line to a series that is trending up, and then calculate the de-trended or filtered series as the deviation of the actual series about the trend.

I do something that is slightly more sophisticated than straight linear de-trending. In particular, I de-trend the natural logs of series using the Hodrick-Prescott (HP) filter. The HP filter calculates a trend by essentially calculating a two-sided moving average of the series. Below is a plot of log real GDP, the HP trend, and the deviations of actual GDP from the HP trend. The left scale is for the deviations from trend (the de-trended or filtered series), and has as units percent deviations from trend. The right scale is a log scale.

One thing that immediately pops out is that the trend and the actual series are pretty close to one another; put differently, deviations from the trend are quite small.

I also took logs of the other series I mentioned and detrended them using the HP filter. The tradition within business cycle analysis is to focus on “second moments” of the data, or variances and co-variances (correlations are re-scaled covariances bound between -1 and 1). Correlations are

---

1 The series for GDP, consumption, and investment are real series taken from the National Income and Product Accounts accounts and are the broadest definitions of consumption and investment. The price level series is the GDP price deflator, also available from the NIPA accounts. For the real interest rate I take the three month Treasury Bill rate minus one-period ahead inflation; this is an approximation to the fact that the real interest rate is based on expected inflation. Total hours worked are hours worked in the non-farm business sector divided by the population aged 16 and over. The real wage series is real hourly compensation in the non-farm business sector. The hours and wage series are available from the Bureau of Labor Statistics; the data on GDP, consumption, investment, and the price level are the Bureau of Economic Analysis; I downloaded the three month Treasury Bill rate from the St. Louis Fed FRED data base, and computed the real interest rate calculating inflation from the GDP price deflator.

2 An exception here is that I do not take logs of the interest rate, since its units are already in percentage units.
therefore a measure of how series co-move. Positive correlations mean that one series being above
trend typically coincides with the other series being above trend, and vice-versa. Since we will not
be doing any serious quantitative analysis, we will focus on correlations and not volatilities. Below
is a table with the correlations of the HP filtered series with HP filtered GDP:

<table>
<thead>
<tr>
<th>Series</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.78</td>
</tr>
<tr>
<td>Investment</td>
<td>0.85</td>
</tr>
<tr>
<td>Hours</td>
<td>0.87</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.14</td>
</tr>
<tr>
<td>Real Interest Rate</td>
<td>-0.05</td>
</tr>
<tr>
<td>Price Level</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

We can see that GDP, its components, and total hours worked are very strongly correlated with
one another – the pairwise correlations between these series are all greater than 0.75. Movements
in GDP are positively correlated with movements in the real wage, though this correlation is much
weaker. We say that these series are all procyclical. A series is said to be procyclical if it is positively
correlated with GDP – in words, when output is above trend, the series is above trend. The price
level and the real interest rate are countercyclical in the sense that they are negatively correlated
with GDP, though the correlation between the real interest rate and GDP is pretty close to zero.

3 Can Our Model Explain These Facts?

In the previous section we saw that output, hours, consumption, and investment all tend to go
up and down together in the data. The real wage is also procyclical, but less so than these other
series. The real interest rate and the price level are countercyclical. In this section we want to ask
whether or not our equilibrium model of the business cycle can account for these co-movements.

In addressing this question, it is important to go back to a point we made in the previous set
of notes. That point was the following: in our equilibrium model under standard assumptions on
preferences, the only way that consumption and hours worked can move in the same direction is
if \( A_t \) changes. Changes in the three other exogenous variables – \( A_{t+1} \), \( G_t \), and \( G_{t+1} \) – generate
negative co-movement between consumption and hours, and hence negative co-movement between
consumption and investment and consumption and output. We can see this by combining the first
order condition for labor supply from the household side with the first order condition for labor
demand from the firm:

\[
\nu'(1 - N_t) = \psi'(C_t)A_t F_N(K_t, N_t)
\]
Suppose that $A_t$ and $K_t$ do not change, and suppose that $N_t$ goes up. This makes $F_N(K_t, N_t)$ smaller and $u'(1 - N_t)$ bigger. The only way for the condition to hold is if $u'(C_t)$ gets bigger, which means that $C_t$ must fall. Put differently, if neither $A_t$ nor $K_t$ change, then $C_t$ and $N_t$ must move in opposite directions. A corollary of this is that, if neither $A_t$ nor $K_t$ change, then $C_t$ and $Y_t$ must move in opposite directions too, which is flatly at odds with the data.

In the table above we see that consumption and output are positively correlated. Consumption is also positively correlated with hours worked (correlation of 0.65) and total investment (correlation of 0.72). This means that changes in $A_{t+1}$, $G_t$, or $G_{t+1}$ cannot be the main driving force behind the business cycle, at least in the context of this model. The only exogenous variable in our model that could possibly account for these correlations is $A_t$.

Suppose that there is an exogenous increase in $A_t$. We can work through the implications of this change as we have done before. Starting in the labor market, we see that the labor demand curve shifts out. This, in conjunction with the production function itself shifting, leads to a rightward shift in the $Y^*$ curve. There is no shift in the $Y^d$ curve. This means that $Y_t$ increases and $r_t$ falls. $r_t$ falling with $Y_t$ rising means that $I_t$ will rise and $C_t$ will rise. The rise in $C_t$ works to shift the labor supply curve back in such that it intersects the labor demand curve at a level of employment consistent with the new equilibrium level of output. This means that the real wage is definitely higher, while the effect on employment is ambiguous. I have drawn the picture where total hours worked goes up. Moving down to the money market, the real interest rate (and hence the nominal rate) being lower and consumption being higher means the money demand curve pivots to the right. Along a stable money supply curve, this means that the price level must fall.
Based on the diagram and discussion above, in response to an increase/decrease in $A_t$, the model produces movements in the endogenous variables that are broadly consistent with what we observe in the data. Output, consumption, investment, and the real wage all move in the same direction, while the real interest rate and the price level move in the opposite direction of output. The movement in hours is technically ambiguous but for plausible specifications of preferences hours should rise if $A_t$ increases, so that hours will be positively correlated with output. Hence, qualitatively at least, all of these movements are consistent with what we observe in the data. At least as a first pass, we can see that the model does a reasonable job of “fitting the data.”

4 Productivity Shocks in the Data

We see that our “real business cycle model” can do a pretty good job of producing co-movements among variables that are qualitatively in line with what we observe in the data if it is driven by
exogenous changes in $A_t$. Before declaring the model a “success,” we’d like to know if there is any external validity to this story. Are there frequent changes in $A_t$ in the data, and do they correspond with what we see happening to output?

While we cannot get a direct measure of $A_t$, we can get an indirect measure if we are willing to make some assumptions about the economy’s production function. Suppose that the production function is Cobb-Douglas:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}$$

Take logs:

$$\ln Y_t = \ln A_t + \alpha \ln K_t + (1 - \alpha) \ln N_t$$

“Solve” for $\ln A_t$:

$$\ln A_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln N_t$$

In principle, $Y_t$, $N_t$, and $K_t$ are objects that we can measure. $\alpha$ is an other object that can be backed out of the data, since optimization by firms implies that $(1 - \alpha) = \frac{w_t N_t}{Y_t}$, or labor’s share of total income, which is about two-thirds. Hence, $\alpha$ is about 1/3. This means that we can construct a measure of $\ln A_t$ by subtracting $\alpha \ln K_t$ and $(1 - \alpha) \ln N_t$ from measured output. Put differently, we can measure $\ln A_t$ as output less share-weighted inputs, where $\alpha$ and $(1 - \alpha)$ are the “shares.” Sometimes we call this a “residual,” specifically sometimes the “Solow residual” as this kind of exercise is often attributed to Robert Solow (the same Solow of growth model fame). Alternatively we call this empirical measure “total factor productivity.”

Below is a time series plot of detrended total factor productivity and detrended real GDP:

![TFP and Output](image)

We can see that these two series look very similar. All post-war US recessions are associated with
drops in measured $A_t$ that line up nicely with drops in $Y_t$. In other words, empirically there is some evidence that there are in fact large and frequent changes in $A_t$ that line up well with movements in $Y_t$. Indeed, the correlation between de-trended TFP and de-trended output is 0.79. In other words, not only is the model capable of qualitatively matching the data when driven by changes in $A_t$, there is some relatively “model-free” evidence that there are changes in $A_t$ that line up well with observed changes in $Y_t$.

5 The Planner’s Problem

We have thus far established that (i) our equilibrium “real business cycle model” is capable of qualitatively matching actual business cycle data when driven by changes in $A_t$, and (ii) there is some evidence consistent with there actually being changes in $A_t$ that line up well with the actual business cycle. This means that there is some empirical evidence to suggest that the model is a pretty good model. Given that we think the model fits the data fairly well, we want to ask what kinds of policy implications the model has.

We want to analyze the problem of a fictitious social planner who acts to benevolently maximize household utility subject to the resource constraints that the economy as a whole faces. There will be no explicit prices (e.g. the real interest rate and the real wage) in the planner’s problem, since prices serve to coordinate actions in a decentralized equilibrium. Since the presence of money does not affect the real allocations we abstract from money altogether here. Think about the planner coordinating actions by choosing quantities. Our objective is to find the solution to the social planner’s problem and then to compare it to the competitive equilibrium outcome. To the extent to which the two solutions do not coincide there may be welfare gains to be had from government intervention of one sort or the other.

The planner faces the resource constraint that consumption plus investment plus government spending (which is taken as an exogenous value) be equal to total production. There is one resource constraint for each period:

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = A_tF(K_t, N_t)$$
$$C_{t+1} - (1 - \delta)K_{t+1} + G_{t+1} = A_{t+1}F(K_{t+1}, N_{t+1})$$

In writing these constraints I have eliminated investment using the capital accumulation equation, so that $I_t = K_{t+1} - (1 - \delta)K_t$, and have imposed the previously discussed terminal condition that $I_{t+1} = -(1 - \delta)K_{t+1}$. The planner’s objective is to maximize household lifetime utility subject to these two constraints:

$$\max_{C_t, C_{t+1}, N_t, N_{t+1}, K_{t+1}} \ U = u(C_t) + v(1 - N_t) + \beta u(C_{t+1}) + \beta v(1 - N_{t+1})$$

s.t.
\[ C_t + K_{t+1} - (1 - \delta)K_t + G_t = A_t F(K_t, N_t) \]
\[ C_{t+1} - (1 - \delta)K_{t+1} + G_{t+1} = A_{t+1} F(K_{t+1}, N_{t+1}) \]

This is a constrained problem with two constraints. To deal with the two constraints I am going to eliminate both \( C_t \) and \( C_{t+1} \) as choice variables by solving for them from the constraints and substituting back into the objective function. The re-written problem is:

\[
\max_{N_t, N_{t+1}, K_{t+1}} \quad U = u(A_t F(K_t, N_t) - K_{t+1} + (1 - \delta)K_t - G_t) + v(1 - N_t) + \ldots \\
\ldots + \beta u(A_{t+1} F(K_{t+1}, N_{t+1}) + (1 - \delta)K_{t+1} - G_{t+1}) + \beta v(1 - N_{t+1})
\]

This is now an unconstrained problem. To characterize the solution take the derivatives with respect to the choice variables and set them equal to zero:

\[
\frac{\partial U}{\partial N_t} = 0 \Leftrightarrow u'(A_t F(K_t, N_t) - K_{t+1} + (1 - \delta)K_t - G_t) A_t F_N(K_t, N_t) - u'(1 - N_t) = 0 \\
\frac{\partial U}{\partial N_{t+1}} = 0 \Leftrightarrow \beta u'(A_{t+1} F(K_{t+1}, N_{t+1}) + (1 - \delta)K_{t+1} - G_{t+1}) A_{t+1} F_N(K_{t+1}, N_{t+1}) - \beta u'(1 - N_{t+1}) = 0 \\
\frac{\partial U}{\partial K_{t+1}} = 0 \Leftrightarrow -u'(A_t F(K_t, N_t) - K_{t+1} + (1 - \delta)K_t - G_t) + \ldots \\
\ldots + \beta u'(A_{t+1} F(K_{t+1}, N_{t+1}) + (1 - \delta)K_{t+1} - G_{t+1})(A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta)) = 0
\]

These can be re-arranged and re-written by substituting \( C_t \) and \( C_{t+1} \) back in to obtain:

\[
v'(1 - N_t) = u'(C_t) A_t F_N(K_t, N_t) \\
v'(1 - N_{t+1}) = u'(C_{t+1}) A_{t+1} F_N(K_{t+1}, N_{t+1}) \\
u'(C_t) = \beta u'(C_{t+1})(A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta))
\]

These three conditions characterize the solution to the social planner’s problem.

Now, let’s take a step back and recall what were the equilibrium conditions of the competitive equilibrium. On the household side, we had:

\[
v'(1 - N_t) = u'(C_t) w_t \\
v'(1 - N_{t+1}) = u'(C_{t+1}) w_{t+1} \\
u'(C_t) = \beta (1 + r_t) u'(C_{t+1})
\]

On the firm side, we had:
\begin{align*}
  w_t &= A_t F_N(K_t, N_t) \\
  w_{t+1} &= A_{t+1} F_N(K_{t+1}, N_{t+1}) \\
  1 + r_t &= (A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta))
\end{align*}

If you combine the firm first order conditions with the household first order conditions, you get:

\begin{align*}
  v'(1 - N_t) &= u'(C_t) A_t F_N(K_t, N_t) \\
  v'(1 - N_{t+1}) &= u'(C_{t+1}) A_{t+1} F_N(K_{t+1}, N_{t+1}) \\
  u'(C_t) &= \beta u'(C_{t+1}) (A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta))
\end{align*}

These are exactly the same first order conditions that obtain in the planner’s solution. Put differently – the competitive equilibrium outcome and the solution to the planner’s problem are one in the same. This means that a planner can do no better than the private economy left to its own devices. In some sense this is a formalization of Adam Smith’s laissez faire idea – a private economy left to its own devices achieves a Pareto optimal allocation, by which I mean that it would not be possible to improve upon the allocations. Sometimes we say that the equilibrium is “efficient.” In modern economics this result is formalized by the first fundamental welfare theorem, which states that, under some conditions, a competitive equilibrium is optimal from the planner’s perspective. Those conditions include things like price-taking behavior and no distortionary taxation and are satisfied in the model we have written down to this point.

This result has a very important implication in the context of the empirical analysis we started these notes with. The business cycle model we’ve written down leads to an equilibrium which is efficient and cannot be improved upon by a planner. Driven by exogenous productivity shocks (exogenous movements in \( A_t \)), it can produce movements in output and its components that share qualitative features with actual data, and an empirical measure of shocks to \( A_t \) (total factor productivity or the Solow residual) are strongly correlated with movements in \( Y_t \). Put differently, there is some suggestive evidence that our business cycle model “fits” actual aggregate data pretty well, which means that it is plausible that a model like that generated the actual data we observe. If that is the case, then it means that business cycles are “efficient” and that there is no welfare justification for activist government policies to try and “smooth out” business cycles via countercyclical policies.

This was (and is) a controversial idea, and the basic real business cycle framework resulted in a Nobel Prize for its two main developers, Edward Prescott and Finn Kydland. Prior to their work in the late 1970s and early 1980s, economists essentially assumed that movements in output about trend were necessarily bad from a welfare perspective, and that policy (both fiscal and monetary) should try to prevent such movements. The real business cycle framework offers an alternative interpretation: fluctuations in output about trend may be the efficient response of the economy to
changes in $A_t$ (which the model does not seek to explain). This means that there is no justification for activist short run stabilization policies. Rather, policies should focus on raising the level of $A_t$ in the long run without regard to what happens in the short run (as in the Solow model).

6 Criticisms of Real Business Cycle Theory

Real business cycle has met with substantial criticism since its inception. Some of this criticism is empirical, questioning the idea that what show up as changes in $A_t$ in the data really represent changes in $A_t$ in the model. Some other criticisms are more theoretical, arguing that the model is not realistic because it fails to capture several aspects of the real world that the criticizers feel are relevant.

A non-exhaustive list of criticisms of the real business cycle model are below:

1. In the data output and hours worked are about as volatile as one another. The model does not generate large enough movements in hours worked relative to the data. Qualitatively you can see this by noting that the change in hours worked given a change in $A_t$ is technically ambiguous, whereas $Y_t$ definitely moves in the same direction as $A_t$. This means that $Y_t$ moves more than $N_t$ in the model, which is not true in the data.

2. The model has no unemployment – the labor market is always in equilibrium. Unemployment is a real world phenomenon and is costly from a welfare perspective.

3. The model has no role for active monetary policy and features monetary non-neutrality. In the real world people certainly think that the Fed matters.

4. If $A_t$ is a measure of productivity, what does it mean for productivity to decline? Did we forget how to produce? Is it plausible that that $A_t$ moves around a lot over short horizons?

5. There have been lots of people that have questioned the measurement of $A_t$. In particular, we may not have good measures of $N_t$ and $K_t$. In particular, there could be unobserved utilization – for example, during recessions workers sit at work idle, so actual labor input is declining, but measured labor input does not decline by much, and so too much of the movement in output gets attributed to changes in $A_t$.

6. The model does not have any heterogeneity. Real world recessions are costly not because everyone suffers income that is slightly lower, but rather because some people lose all of their income whereas others suffer no income loss. This means that we want to be careful about taking the model’s policy and welfare implications too seriously.

Each of these criticisms has merit. I am least sympathetic to point (4); properly understood, $A_t$ is more than just “technology,” and so there is no compelling reason to insist that it can’t decline or that it can’t move around a lot over short horizons. One thing that could potentially manifest itself as $A_t$ is the health of the financial system – if the financial system collapses, then there will
be a poor allocation of resources across the economy, which may manifest itself as low $A_t$, much like what we observed during the recent recession.

My own view is that real business cycle theory is a decent first pass at understanding economic fluctuations, though it is admittedly too simple and one probably needs to be weary about taking its policy and welfare implications too seriously. Nevertheless, I think it’s a good benchmark. What was (and is) revolutionary about the real business cycle theory is that it suggests that there may be no justification for activist policies because fluctuations in economic aggregates may be the efficient reaction to changing conditions like $A_t$. This does not mean that in all circumstances there is no need for activist economic policy; what it does mean is that proponents of activist policies need to articulate the friction or market failure that moves us beyond the real business cycle world and into a world in which policy can improve outcomes by smoothing out the cycle.