Money Demand
ECON 40364: Monetary Theory & Policy

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Readings

- Mishkin Ch. 19
Classical Monetary Theory

- We have now defined what money is and how the supply of money is set.
- What determines the demand for money?
- How do the demand and supply of money determine the price level, interest rates, and inflation?
- We will focus on a framework in which money is neutral and the classical dichotomy holds: real variables (such as output and the real interest rate) are determined independently of nominal variables like money.
- We can think of such a world as characterizing the “medium” or “long” runs (periods of time measured in several years).
- We will soon discuss the “short run” when money is not neutral.
Velocity and the Equation of Exchange

- Let $Y_t$ denote real output in period $t$, which we can take to be exogenous with respect to the money supply.

- $P_t$ is the dollar price of output, so $P_t Y_t$ is the dollar value of output (i.e. nominal GDP).

- $\frac{1}{P_t}$ is the “price” of money measured in terms of goods.

- Define velocity as the average number of times per year that the typical unit of money, $M_t$, is spent on goods and serves. Denote by $V_t$.

- The “equation of exchange” or “quantity equation” is:

  $$M_t V_t = P_t Y_t$$

- This equation is an identity and defines velocity as the ratio of nominal GDP to the money supply.
From Equation of Exchange to Quantity Theory

- The quantity equation can be interpreted as a theory of money demand by making assumptions about velocity
- Can write:

\[ M_t = \frac{1}{V_t} P_t Y_t \]

- Monetarists: velocity is determined primarily by payments technology (e.g. credit cards, ATMs, etc) and is therefore close to constant (or at least changes are low frequency and therefore predictable)
- Let \( \kappa = V_t^{-1} \) and treat it as constant. Since money demand, \( M_t^d \), equals money supply, \( M_t \), our money demand function is:

\[ M_t^d = \kappa P_t Y_t \]

- Money demand proportional to nominal income; \( \kappa \) does not depend on things like interest rates
- This is called the quantity theory of money
Money and Prices

- Take natural logs of equation of exchange:

\[
\ln M_t + \ln V_t = \ln P_t + \ln Y_t
\]

- If \( V_t \) is constant and \( Y_t \) is exogenous with respect to \( M_t \), then:

\[
d \ln M_t = d \ln P_t
\]

- In other words, a change in the money supply results in a proportional change in the price level (i.e. if the money supply increases by 5 percent, the price level increases by 5 percent)
Money and Inflation

- Since the quantity equation holds in all periods, we can first difference it across time:

\[
\left( \ln M_t - \ln M_{t-1} \right) + \left( \ln V_t - \ln V_{t-1} \right) = \left( \ln P_t - \ln P_{t-1} \right) + \left( \ln Y_t - \ln Y_{t-1} \right)
\]

- The first difference of logs across time is approximately the growth rate

- Inflation, \( \pi_t \), is the growth rate of the price level

- Constant velocity implies:

\[
\pi_t = g^M_t - g^Y_t
\]

- Inflation is the difference between the growth rate of money and the growth rate of output

- If output growth is independent of the money supply, then inflation and money growth ought to be perfectly correlated
(a) U.S. Inflation and Money Growth Rates by Decade, 1870s–2000s

Decades with high money growth rates have high inflation rates.
(b) International Comparison of Average Inflation and Money Growth (2003–2013)

Countries with high money growth rates, such as Turkey, Ukraine, and Zambia, have high inflation rates.
Nominal and Real Interest Rates

- The nominal interest rate tells you what percentage of your nominal principal you get back (or have to pay back, in the case of borrowing) in exchange for saving your money. Denote by $i_t$

- There are many interest rates, differing by time to maturity and risk. Ignore this for now. Think about one period (riskless) interest rates – i.e. between $t$ and $t + 1$

- The real interest rate tells you what percentage of a good you get back (or have to pay back, in the case of borrowing) in exchange for saving a good. Denote by $r_t$

- Putting one good “in the bank” $\Rightarrow P_t$ dollars in bank $\Rightarrow (1 + i_t)P_t$ dollars tomorrow $\Rightarrow$ purchases $(1 + i_t)\frac{P_t}{P_{t+1}}$ goods tomorrow
The Fisher Relationship

- The relationship between the real and nominal interest rate is then:
  \[ 1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \]

- Since the inverse of the ratio of prices across time is the expected gross inflation rate, we have:
  \[ 1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}} \]

- Here \( \pi^e_{t+1} \) is expected inflation between \( t \) and \( t + 1 \)
- Approximately:
  \[ r_t = i_t - \pi^e_{t+1} \]
The Natural Rate of Interest

▶ Over the medium to long run, the real interest rate is an equilibrium construct which balances the supply and demand for savings and investment
▶ We sometimes refer to this as the “natural rate of interest” after Knut Wicksell
▶ Simple theory based on the consumption Euler equation with log utility:

\[ \frac{C_{t+1}}{C_t} = \beta(1 + r_t^P) \]

▶ \( r_t^P \) is the natural rate of interest, or the real interest rate consistent with “potential output.” Take logs, approximate, and treat consumption growth as equal to output growth:

\[ r_t^P = g_{t+1} Y_t - \ln \beta \]

▶ Intuition based on supply and demand for savings and investment
Money, Inflation, and Interest Rates

▶ Over the medium to long run, the natural rate of interest just depends on output growth and attitudes about saving, captured by $\beta$. Independent of monetary factors. Think of this as constant.

▶ Over the medium to long run, we should also expect expected inflation to equal realized inflation, $\pi_{t+1}^e = \pi_t$

▶ From the Fisher relationship, this means that nominal interest rates and inflation ought to move together
The theoretical quantity theory in which the classical dichotomy holds (real output, real output growth, and the real interest rate independent of nominal things) makes a number of stark predictions:

1. The level of the money supply and the price level are closely linked.
2. The growth rate of the money supply and the inflation rate are closely linked.
3. The inflation rate and the nominal interest rate are closely linked.
Problems with the Quantity Theory

- The quantity theory seems to provide a pretty good theory of inflation and interest rates over the medium to long run as well as in a cross section of countries.
- What about the short run?
- Problems with the quantity theory:
  - The shorter term relationships between money growth and both inflation and nominal interest rates are weak.
  - Velocity is not constant and has become harder to predict, particularly since the early 1980s.
Moving Beyond the Quantity Theory

- The key assumption in the quantity theory is that the demand for money (i.e. velocity) is stable (or at least predictable) – you hold money to buy stuff, and how much money you need is proportional to how much you buy.

- Liquidity preference theory of money demand: money competes with other assets as a store of value. Money is more liquid (can be used in exchange), but how much you want to hold depends on return on other assets.

- Demand for real money balances, \( m_t = \frac{M_t}{P_t} \), is an increasing function of output, \( Y_t \), but a decreasing function of the nominal interest rate, \( i_t \):

\[
\frac{M_t}{P_t} = L(i_t, Y_t)
\]

- But then velocity:

\[
V_t = \frac{P_t Y_t}{M_t} = \frac{Y_t}{L(i_t, Y_t)}
\]
Suppose that there is a representative household who receives utility from consuming goods and holding real money balances, \( m_t = \frac{M_t}{P_t} \). Flow utility:

\[
U \left( C_t, \frac{M_t}{P_t} \right) = \ln C_t + \psi \ln \left( \frac{M_t}{P_t} \right)
\]

Flow budget constraint:

\[
P_t C_t + B_t - B_{t-1} + M_t - M_{t-1} \leq P_t Y_t - P_t T_t + i_{t-1} B_{t-1}
\]

\( B_{t-1} \) and \( M_{t-1} \): stocks of bonds and money household enters \( t \) with

Both enter as stores of value. Difference being that bonds pay interest

Household discounts future utility flows by \( \beta \in [0, 1) \)
Optimality Conditions

- Plugging constraints in and taking derivatives yields:

\[
\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1 + i_t) \frac{P_t}{P_{t+1}}
\]

\[
\psi \frac{P_t}{M_t} = \frac{1}{C_t} - \beta E_t \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}
\]

- Government’s budget constraint with \( G_t = 0 \) (ignore distinction between base and money supply):

\[
P_t T_t = (1 + i_t) B_{G,t-1} - B_{G,t} - (M_t - M_{t-1})
\]

- Market-clearing: \( B_{G,t} = B_t \), so \( C_t = Y_t \)
Money Demand Function

- Making use of market-clearing and combining the FOC yields:

\[ \psi m_t^{-1} = \frac{1}{Y_t} \frac{i_t}{1 + i_t} \]

- Re-arranging:

\[ m_t = \psi Y_t \frac{1 + i_t}{i_t} \]

- Demand for real balances: (i) increasing in \( Y_t \), (ii) decreasing in \( i_t \)

- Zero lower bound: must have \( i_t \geq 0 \) to get non-negative real balances. At \( i_t \to 0 \), demand for real balances goes to infinity
Baumol-Tobin

- You need to spend $Y$ over the course of a period (say, a year)
- You keep wealth “in the bank” earning nominal interest $i_t$ (say a savings account, not demand deposit)
- You need to determine how many trips you take to bank
- Each trip incurs a cost (“shoeleather cost”) of $K$
- Let $m_t$ denote average real balances holdings over the period. Opportunity cost of holding money is $i_t m_t$
- Each time you withdraw money, you withdraw $2m_t$ dollars. Total trips to bank is $\frac{Y}{2m_t}$
- Objective is to pick $m_t$ to minimize:

$$\min_{m_t} \quad i_t m_t + K \frac{Y}{2m_t}$$
Money Demand Function

- Use calculus to get first order condition:
  \[ m_t = \sqrt{\frac{KY_t}{2i_t}} \]

- Or re-arranging:
  \[ m_t = \left( \frac{KY_t}{2} \right)^{\frac{1}{2}} i_t^{-\frac{1}{2}} \]

- Demand for real balances again increasing in \( Y_t \) and decreasing in \( i_t \)

- There is again a zero lower bound: \( i_t \geq 0 \) for demand for real balances to be positive
Friedman Rule

- **Milton Friedman** argued that optimal monetary policy in the medium to long run would target a nominal interest rate of zero.
- With a positive natural rate of interest, this would require deflation.
- Basic intuition: a positive nominal interest rate dissuades people from holding money by increasing the opportunity cost of liquidity relative to bonds, whereas the marginal cost of producing (fiat) money is essentially zero.
- At a social optimum, want to equate private cost of holding money (interest rate) to the public cost of producing money (zero).
- Holds in both the MIU model \(i = 0\) maximizes utility and the B-T model \(i = 0\) minimizes the cost of holding money.
- Why don’t central banks follow Friedman rule? Because of the zero lower bound and short run stabilization policy.
- Does help us understand desire for low interest rates, however.
Optimality of $i = 0$
Instability of Velocity: Movement Away from Focusing on Monetary Aggregates

- **Paul Volcker** and the Fed experimented with targeting monetary aggregates in the early 1980s
- This brought inflation down from the 1970s, but led to high and variable interest rates
- Most monetary economists concluded that the demand for money is not in fact stable, i.e. a rejection of monetarism
- If the money supply is not closely and predictably connected to aggregate spending, targeting the money supply probably not a good policy
- This has led most monetary economists to instead favoring focusing on short term interest rates as the target of monetary policy, as we saw with a discussion of the Taylor rule and the Fed controlling the Fed Funds Rate (FFR)
Money and Inflation: The Case of Hyperinflations

- Milton Friedman famously said that “inflation is everywhere and always a monetary phenomenon”
- Simple logic based on the quantity equation. Works pretty well in the medium to long run
- What about extreme situations of inflation, or what are called “hyperinflations”?
- Monetary phenomena triggered by fiscal problems
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<th>Country</th>
<th>Year</th>
<th>Highest Inflation per Month %</th>
<th>Country</th>
<th>Year</th>
<th>Highest Inflation per Month %</th>
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<td>Hungary</td>
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Hyperinflations Usually a Fiscal Phenomenon

- Most hyperinflations in history are associated with fiscal mischief.
- Government’s budget constraint:

\[ P_t G_t + i_{t-1} B_{G,t-1} = P_t T_t + M_t - M_{t-1} + B_{G,t} - B_{G,t-1} \]

- Here \( P_t \) is the nominal price of goods (i.e. the price level), \( B_{G,t-1} \) is the stock of debt with which a government enters period \( t \), \( B_{G,t} \) is the stock of debt the government takes from \( t \) to \( t + 1 \), \( i_{t-1} \) is the nominal interest rate on that debt, \( T_t \) is tax revenue (real), and \( M_t \) is the money supply.

- Deficit equals change in money supply plus change in debt:

\[ P_t G_t + i_{t-1} B_{G,t-1} - P_t T_t = M_t - M_{t-1} + B_{G,t} - B_{G,t-1} \]
Monetizing the Debt

- If tax revenue doesn’t cover expenditure (spending plus interest on debt), then government either has to issue more debt or “print more money”
- In some cases printing more money is explicit, in others implicit
- Monetizing the debt: fiscal authority issues debt to finance deficit, but monetary authority buys the debt by doing open market operations, which creates base money
Recall from the government’s budget constraint above when talking about hyperinflations that *nominal* revenue from printing money is simply: \( M_t - M_{t-1} \)

*Real* revenue from printing money is \( \frac{M_t - M_{t-1}}{P_t} \)

We call the real revenue from printing money **seigniorage**

This can be written:

\[
\text{Seigniorage} = \frac{M_t - M_{t-1}}{P_t}
\]

This can equivalently be written:

\[
\text{Seigniorage} = \frac{M_t}{M_{t-1}} \frac{M_{t-1}}{M_t} \frac{M_t}{P_t}
\]
More Seigniorage

- Define the growth rate of money as:

\[ g_t^M = \frac{M_t - M_{t-1}}{M_{t-1}} \]

- Then the expression for seigniorage can be written:

\[ \text{Seigniorage} = \frac{g_t^M}{1 + g_t^M} m_t \]

- This is approximately:

\[ \text{Seigniorage} = g_t^M m_t \]

- Seigniorage is tax revenue from printing more money – \( g_t^M \) is effectively the “tax rate” and \( m_t \) is the “tax base”
Seigniorage in the Medium to Long Run

- Suppose that the growth rate of money is constant in the medium to long run, \( g^M_t = g^M \)
- Suppose that output, \( Y_t \), is independent of the money growth rate and is constant, so \( Y_t = Y \)
- Suppose that the real interest rate equals the natural rate of interest, so the nominal rate is constant and is:

\[
i = r^P + \pi
\]

- Suppose that the inflation rate equals the money growth rate, so:

\[
i = r^P + g^M
\]

- If demand for real balances is generically given by: \( m_t = L(i_t, Y_t) \), then we can write demand for real balances as:

\[
m = L(r^P + g^M, Y)
\]
Suppose that a central bank wants to pick $g^M$ to maximize seigniorage. Problem is:

$$\max_{g^M} g^M L(r^P + g^M, Y)$$

Provided money demand is decreasing in nominal interest rate (i.e. $L_i(\cdot) < 0$), then two competing effects of higher $g^M$:

1. Tax rate: higher $g^M \Rightarrow$ higher tax rate
2. Base: higher $g^M \Rightarrow$ lower tax base

First order condition:

$$g^M = -\frac{L(r^P + g^M, Y)}{L_i(r^P + g^M, Y)}$$

Revenue-maximizing growth rate of money inversely related to interest sensitivity of money demand

If money demand interest insensitive (e.g. quantity theory), then revenue-maximizing $g^M = \infty$!

Desire for seigniorage another reason to move away from Friedman rule