Monetary Policy in a Macro Model
ECON 40364: Monetary Theory & Policy

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Readings

- Mishkin Ch. 20
- Mishkin Ch. 21
- Mishkin Ch. 22
- Mishkin Ch. 23, pg. 553-569
Want to study role of monetary policy in the aggregate economy.

For that reason, we will not bother to “micro-found” the macro model of the economy (unlike, for example, in intermediate macro).

We will just posit a bunch of demand and supply relationships so as to get a useable demand-supply macro model.

These demand and supply relationships (or something that looks like them) can be motivated from micro principles.
Aggregate Demand

- Aggregate demand is the sum of planned expenditure by the four primary actors in an economy:
  1. Households (consumption)
  2. Firms (investment)
  3. Government (government purchases)
  4. Rest of the world (net exports)

- Total (real) aggregate demand is the sum of planned expenditure by each of these actors:

\[ Y^{ad} = C + I + G + NX \]

- In equilibrium, \( Y^{ad} = Y \) (expenditure equals income)

- This is somewhat complicated because expenditure categories on right hand side may depend on \( Y \)

- These are flow concepts but will follow book with no explicit time subscripts
Consumption

- It is assumed that aggregate consumption is governed by a consumption function:

\[ C = \bar{C} + mpc \times (Y - T) \]

- \( Y \) is income and \( T \) is taxes, so \( Y - T \) is disposable income
- \( \bar{C} \) is *autonomous* consumption. “Autonomous” means the exogenous component of an endogenous variable. \( \bar{C} \) is consumption independent of disposable income. Potential factors impacting it (which are not modeled explicitly):
  - Future income
  - Interest rates
  - Uncertainty
  - Government spending (via Ricardian Equivalence)
- \( mpc \) is the marginal propensity to consume and is between 0 and 1
- Something like this can be derived from micro principles
Investment

- Investment refers to expenses by business on new physical capital (as well as new residential construction and inventory accumulation).
- Investment function:
  \[ I = \bar{I} - dr \]

- \( r \) is “the” real interest rate, \( d \) is a parameter governing sensitivity to the real interest rate, and \( \bar{I} \) is autonomous investment (investment independent of the real interest rate).
- Possible factors:
  - Expectations about the future
  - Efficiency of producing new capital
  - Government regulations and taxes
- Book includes a “credit spread” variable here. We will return to that later.
We assume that government spending and taxes are completely exogenous. Hence:

\[ G = \bar{G} \]  

\[ T = \bar{T} \]  

We do not think explicitly about government debt, future taxes, or future spending. Also, taxes are “lump sum”

Not modeling Ricardian Equivalence
Net Exports

- Net exports depend on the exchange rate (how much one currency buys of another), as well as on other non-modeled things.

- A high US interest rate results in an appreciation of the dollar other things being equal (foreigners want to buy US assets, pushing up the value of the dollar relative to other currencies).

- An appreciation of the dollar leads to more imports (cheaper for US citizens to buy foreign goods) and fewer exports (more expensive for foreigners to buy US goods) and hence reduces net exports.

- Net export function, where $\bar{NX}$ is autonomous net exports and $x$ is a parameter governing sensitivity to real interest rate:

$$NX = \bar{NX} - xr$$
Putting it all together

- Total planned (or desired) expenditure:

\[ Y^{ad} = \bar{C} - mpc \bar{T} + \bar{I} + \bar{G} + \bar{NX} + mpcY - (d + x)r \]

- For simplicity, let \( \bar{A} = \bar{C} - mpc \bar{T} + \bar{I} + \bar{G} + \bar{NX} \)
- Then:

\[ Y^{ad} = \bar{A} + mpcY - (d + x)r \]

- One equation in three “unknowns” (\( Y^{ad}, Y, \) and \( r \)); \( d, x, \) and \( mpc \) are parameters, and \( \bar{A} \) is exogenous
- In equilibrium, total output must equal total planned expenditure, \( Y = Y^{ad} \), so:

\[ Y = \frac{1}{1 - mpc} \bar{A} - \frac{d + x}{1 - mpc} r \]
Keynesian Cross

\[ Y^{ad} = \bar{A} + mpc \cdot Y - (d + x) \cdot r \]

\[ Y^{ad} = Y \]

\[ Y^{ad} = Y^* \]

\[ \bar{A} - (d + x) \cdot r \]

\[ Y^* \]
The IS Curve

- The condition:

\[ Y = \frac{1}{1 - mpc} \bar{A} - \frac{d + x}{1 - mpc} r \]

Is one equation in two endogenous variables, \( r \) and \( Y \)

- The IS curve is the set of \((r, Y)\) pairs where this condition holds

- It can be derived graphically:
  - It is downward sloping
  - It shifts right whenever \( \bar{A} \) increases
\[ Y^{ad} = Y \]
\[ Y^{ad} = \bar{A} + mpc \cdot Y - (d + x) \cdot r \]
\[ \bar{A} - (d + x) \cdot r_2 \]
\[ \bar{A} - (d + x) \cdot r_0 \]
\[ \bar{A} - (d + x) \cdot r_1 \]

- IS

- \( Y_1 \)
- \( Y_0 \)
- \( Y_2 \)

- \( r \)
- \( r_1 \)
- \( r_0 \)
- \( r_2 \)
\[ Y^{ad} = \bar{A} + mpc \cdot Y - (d + x) \cdot r \]

\[ A_0 - (d + x) \cdot r_0 \]

\[ A_1 - (d + x) \cdot r_0 \]
The MP Curve

- To get an aggregate demand curve, which shows a relationship between aggregate output and the inflation rate or the aggregate price level (where \( \pi = \frac{\Delta P}{P} \)), we need to combine the IS curve with some description of monetary policy.
- Traditionally, this is done through something called the LM curve, which combines the liquidity preference of money demand with a money supply rule.
- Given central bank’s modern focus on interest rates rather than money supply, we instead do so with the monetary policy curve (MP).
- Difference: with the LM curve, AD curve is downward-sloping in a graph with \( (P, Y) \). With MP curve, AD curve is downward-sloping with \( (\pi, Y) \).
- We need not model money at all, though we could figure out how Fed must adjust \( M \) to meet money demand if we wanted – the economy is “cashless”.
The MP curve and the Taylor Principle

- Recall the Fisher relationship, \( r = i - \pi^e \). Fed can control \( i \), but not necessarily \( r \)
- Assume adaptive expectations, so that \( \pi^e = \pi \).
- Then assume Fed sets nominal rate according to:

\[
i = \bar{r} + a\pi
\]

- Similar to the Taylor rule. \( \bar{r} \) is “autonomous monetary policy” (movements in rates unrelated to inflation). \( a > 1 \) – the “Taylor Principle”
- Can write the real interest rate as:

\[
r = \bar{r} + \lambda \pi
\]

- Where \( \lambda = (a - 1) \). Taylor principle requires \( \lambda > 0 \). Idea: when \( \pi \) increases, Fed responds by raising \( i \) by sufficiently much so as to raise \( r \)
The AD Curve

- Simply combine the IS and MP curves. You get:

\[ Y = \frac{1}{1 - mpc} \bar{A} - \frac{d + x}{1 - mpc} (\bar{r} + \lambda \pi) \]

- Can also re-arrange with \( \pi \) on LHS:

\[ \pi = -\frac{1 - mpc}{(d + x)\lambda} Y + \frac{1}{(d + x)\lambda} \bar{A} - \frac{1}{\lambda} \bar{r} \]

- Provided \( \lambda > 0 \) (Taylor principle satisfied):
  1. AD curve is downward-sloping
  2. AD curve is flatter the bigger is \( \lambda \)
  3. AD curve shifts out/up when \( \bar{A} \) increases
  4. AD curve shifts down/in when \( \bar{r} \) increases
Shifts of the AD Curve

- The AD curve is drawn holding $\bar{r}$ (which governs the position of the MP curve) and $\bar{A}$ (which governs the position of the IS curve) fixed.

- An increase in $\bar{r}$ will result in an increase in the real interest rate for a given inflation rate; from the IS curve, this results in a lower level of output. So the AD curve shifts left.

- An increase in $\bar{A}$ will shift the IS curve right. For a given inflation rate, the real interest rate is given, which means that output increases. So the AD curve shifts right.
\[ \pi = \pi \]

\[ r = \bar{r} + \lambda \pi \]

\[ Y \]

\[ Y_0 \]

\[ Y_1 \]
The Taylor Principle and the Slope of the AD Curve

- The Taylor principle \((a > 1\) or \(\lambda > 0\)) calls for the Fed to raise the real interest rate when inflation increases.
- The higher real interest rate causes output to decline from the IS curve, which makes the AD curve downward-sloping.
- What if the Taylor principle isn’t satisfied? i.e. \(\lambda < 0\)?
- Higher inflation results in lower real interest rates, which stimulates output from the IS curve.
- The AD curve is upward-sloping. For reasons we will talk about later, this is undesirable.
\[
\pi = \pi \\
Y = Y \\
\pi_0 = \pi_0 \\
\pi_1 = \pi_1 \\
\pi_2 = \pi_2 \\
\bar{r} = \bar{r} + \lambda \pi, \lambda < 0 \\
r = r_0 + \pi_0, r < 0 \\
Y_1 = Y_1 \\
Y_0 = Y_0 \\
Y_2 = Y_2 \\
\lambda < 0
\]
Aggregate Supply

- Relationship between $\pi$ and $Y$ (or $P$ and $Y$)
- Accepted paradigm:
  - Aggregate supply (AS) curve is vertical in the “long run” or “medium run.” The level of output where it’s vertical, or “potential output,” is determined by labor supply, available capital, and technology. **Classical dichotomy**: real variables independent of nominal
  - Prices and/or wages are “sticky” in the short run, so “short run” AS curve is upward-sloping instead of vertical. Classical dichotomy breaks down
- Many policy debates among macroeconomists are over the “shape” of the AS curve (i.e. how far from vertical is the AS curve in the short run) and how long it takes to transition from short run to medium/long run
- Let $Y^P$ denote potential output; take this to be exogenous (equilibrium concept in model without nominal rigidity)
- LRAS (long run aggregate supply) is vertical at this point
The assumed short run AS curve is:

$$\pi = \pi^e + \gamma(Y - Y^P) + \rho$$

Where:

1. $\pi^e$: how much inflation firms/households expected from “yesterday” to “today” (note slightly different than $\pi^e$ in the Fisher relationship, which is expected inflation from “today” to “tomorrow”. . . . this is where time subscripts would be handy)
2. $\gamma$: parameter governing how steep AS is (related to underlying price and/or wage stickiness)
3. $\rho$: “inflation shock” (e.g. increase in oil prices), also called “cost-push shock.” Zero on average; if it changes, only changes for one period (simplifying assumption)

If $Y > Y^P$, firms/households have pressure to increase prices/wages, which puts upward pressure on $\pi$. $\gamma$ big: it’s easy to do this. $\gamma$ small: prices/wages are very sticky
\[
\pi = \pi^e + \gamma(Y - Y^p) + \rho
\]
The following are relevant things to remember about the AS curve:

1. If $\gamma \to \infty$, no distinction between SRAS and LRAS
2. AS curve crosses point $Y = Y^p$ at $\pi = \pi^e + \rho$. The inflation shock $\rho$ is zero on average, so we’ll often think of this point as being where $\pi = \pi^e$. In other words, if households and firms are not surprised by inflation, then $Y = Y^p$ regardless of what $\gamma$ is.
   - In a sense, nominal rigidities matter only if agents are “fooled” (Lucas 1972)
3. The AS curve shifts up if $\pi^e$ or $\rho$ increase
4. The AS curve shifts right if $Y^p$ increases
\[
\pi = \pi^e + \gamma(Y - Y^P) + \rho
\]
\[ \pi = \pi^e + \gamma(Y - Y^P) + \rho \]
\[ \pi = \pi^e + \gamma(Y - Y^P) + \rho \]

Where \( \pi^e \) denotes the expected inflation rate, \( \gamma \) is the policy parameter, \( Y \) is output, \( Y^P \) is potential output, and \( \rho \) is a constant.
General Equilibrium

- Equilibrium occurs where AD and AS intersect
- For simplicity, assume these intersect initially at the LRAS
- Exogenous variables cause curves to shift and change the equilibrium
  - Demand shocks:
    - IS shocks: changes in \( \bar{\bar{A}} \) (\( \bar{\bar{C}}, \bar{\bar{I}}, \bar{\bar{T}}, \bar{\bar{G}}, \bar{\bar{N\bar{X}}} \))
    - Monetary shocks: changes in \( \bar{r} \)
  - Supply shocks:
    - Potential output shocks: changes in \( Y^P \)
    - Inflation shocks: changes in \( \rho \)
    - Expected inflation shocks: changes in \( \pi^e \)
Demand Shock

\[ \pi_0 \rightarrow \pi_1 \]

\[ Y_0 \rightarrow Y_1 \]
IS Shock: What Happens to $r$

\[ \pi = \pi \]

(a) to (b): direct effect
(b) to (c): indirect effect due to higher $\pi$
MP Shock: What Happens to $r$

(a) to (b): direct effect
(b) to (c): indirect effect due to lower $\pi$

$r = \bar{r} + \lambda \pi$

$\pi = \pi$

$\pi_0$

$\pi_1$

$\pi_0$

$\pi_1$

$r_0$

$r_0'$

$r_1$

$\bar{r}_1$

$\bar{r}_0$

$\pi_0$

$\pi_1$

$Y_0'$

$Y_1$

$Y_0$

$Y$
Monetary Neutrality: MP Shock When AS is Vertical

\[ \pi = \pi \]

\[ r = \bar{r} + \lambda \pi \]

(a) to (b): direct effect
(b) to (c): indirect effect
due to lower \( \pi \)
Supply Shock: Increase in $\pi^e$ or $\rho$
Supply Shock: Increase in $Y^P$
Decrease in $Y^P$: Effect on $r$

(a) to (b): direct effect
(b) to (c): indirect effect
due to higher $\pi$

\[ r = \bar{r} + \lambda \pi \]

\[ Y = \pi \]

\[ \pi = \pi \]

\[ r = \bar{r} + \lambda \pi \]

\[ Y = Y \]

\[ Y = Y \]

\[ Y = Y \]

\[ Y = Y \]

\[ Y = Y \]
Algebraically Solving for Equilibrium

- **AS with AD:**

  \[
  \pi = \pi^e + \gamma \left( \frac{1}{1 - mpc} \bar{A} - \frac{d + x}{1 - mpc} (\bar{r} + \lambda \pi) - Y^p \right) + \rho
  \]

- **Solve for \(\pi\):**

  \[
  \pi = \frac{1 - mpc}{1 - mpc + (d + x)\lambda \gamma} (\pi^e + \rho) + \frac{\gamma}{1 - mpc + (d + x)\lambda \gamma} \bar{A}
  \]

  \[
  - \frac{(d + x)\gamma}{1 - mpc + (d + x)\lambda \gamma} \bar{r} - \frac{\gamma(1 - mpc)}{1 - mpc + (d + x)\lambda \gamma} Y^P
  \]

- **Once you have this, you can get \(Y\) from the AS curve:**

  \[
  Y = Y^P + \frac{1}{\gamma} (\pi - \pi^e - \rho)
  \]

- **Then can get \(r = \bar{r} + \lambda \pi\) and components of output**
Assume $\rho = 0$ (its average value)

From AS, if $Y \neq Y^P$, then $\pi \neq \pi^e$

$Y > Y^p \Rightarrow \pi > \pi^e$: agents were surprised with more inflation than they expected

Stands to reason that, going forward in time, they will then revise up expectations

$\pi^e$ going up: shifts AS curve up, which makes $Y$ fall with no effect on $Y^P$

This process will continue until $Y = Y^P$ and inflation stops changing

And vice-versa in the other direction
Adjustment from Positive Gap: Qualitative

\[ Y_1 = Y^p \]

\[ \pi \]

\[ \pi_0 \]

\[ \pi_1 \]

\[ \pi^e \]

\[ Y \]

LRAS

AS'

AS

AD
Being More Specific

- Put time subscripts on all variables: \( t \) is present, \( t - 1 \) is one period in past, \( t + 1 \) one period in future, and so on.
- Assume adaptive expectations, so that \( \pi^e \) in the AS curve is \( \pi_{t-1} \).
- Then AS curve is:

\[
\pi_t = \pi_{t-1} + \gamma \left( Y_t - Y_t^P \right) + \rho_t
\]

- AD curve is:

\[
Y_t = \frac{1}{1 - mpc} \bar{A}_t - \frac{d + x}{1 - mpc} \left( \bar{r}_t + \lambda \pi_t \right)
\]

- The interesting dynamics are on the supply side. As \( \pi_t \) changes, the position of the AS curve will change dynamically over time. Assume \( \rho_t = 0 \) (unconditional mean).
- Adjustment occurs slowly, only eventually do you get to \( Y^P \).
Adjustment from Positive Gap: More Specific
Create an Excel file

Assume the following parameters and exogenous variables:

\[
mpc = 0.7, \quad d = 0.3, \quad x = 0.1, \quad \bar{r} = 1, \quad \lambda = 1, \quad \bar{A} = 4.95,
\gamma = 0.5, \quad Y^P = 12.5
\]

Assume \(\pi_{t-1} = 0\)

Implies that \(\pi_t = 0.8\) and \(Y_t = 14.1\)

Use Excel to trace out dynamic paths
Adjustment from Positive Gap: Quantitative

Inflation

Output
Positive IS Shock: Dynamic Adjustment

\[ Y_{t+1} \]

\[ Y_{t+2} \]

\[ Y_P \]

\[ LRAS \]

\[ AS' \]

\[ AS'' \]

\[ AD' \]

\[ AD \]

\[ \pi \]

\[ \pi_{t-1} \]

\[ \pi_t \]

\[ \pi_{t+1} \]

\[ \pi_{t+2} \]
Quantitative Adjustment from Positive IS Shock

Inflation

Output

r
Monetary Shock

- An exogenous monetary tightening (increase in $\bar{r}$) has effects in terms of output and inflation similar to an IS shock (exogenous increase in $\bar{A}$).
- Difference: monetary shock does not affect $r_t$ in medium/long run.
- “Natural rate of interest”: $r$ consistent with IS curve holding at $Y^P$. Call it $r^P$ (potential or long run real interest rate):
  \[
  r^P = -\frac{1 - mpc}{d + x} Y^P + \frac{1}{d + x} \bar{A}
  \]
- Does not depend on $\bar{r}$
Adjustment from Positive $\bar{\sigma}$ Shock

![Graph of Inflation](image1)

![Graph of Output](image2)

![Graph of $\bar{\sigma}$](image3)
Application: Volcker Disinflation

![Graphs showing FFR, Inflation, and Output over time.](image)
Adjustment from Positive $Y^P$ Shock
Quantitative Adjustment from Positive $Y^P$ Shock

- **Inflation**
  - Graph showing inflation over time.
  - Initial inflation peaks at around 2.2, then decreases gradually over time.

- **Output**
  - Graph showing output over time.
  - Output increases significantly at around time 0 and remains relatively flat thereafter.

- **r**
  - Graph showing the effective interest rate $r$ over time.
  - The rate decreases sharply at time 0 and stabilizes at a lower level over time.
Adjustment from One Period $\rho$ Shock
Quantitative Adjustment from Positive $\rho$ Shock

![Graph of Inflation](image)

![Graph of Output](image)

![Graph of $r$](image)
Application: Oil Shocks and Stagflation
Monetary Policy Objectives

- When it comes to the business cycle, monetary policy has a **dual mandate**: it wants to stabilize inflation around some low and stable rate and achieve “maximum employment”
- In the context of our model, we can think about the first half of the mandate as keeping $\pi_t$ close to constant, and the second half as keeping $Y_t$ close to $Y^P_t$
- Define the output gap as $X_t = Y_t - Y^P_t$
- Hence, objectives are to keep $\pi_t$ close to constant and $X_t$ close to zero
Monetary Policy Tools

- The primary tool of monetary policy (in this simple model) is the parameter $\lambda$.
- Governs how strongly policy reacts to inflation.
- Recall algebraic expression for AD curve:

$$\pi = -\frac{1 - mpc}{(d + x)\lambda} Y + \frac{1}{(d + x)\lambda} \bar{A} - \frac{1}{\lambda} \bar{r}$$

- $\lambda$ influences two things:
  - Slope of AD curve (bigger $\lambda$ and AD curve is flatter).
  - Shift of AD curve conditional on IS shock, $\bar{A}$ (bigger $\lambda$ and AD curve shifts up less; horizontal shift of AD curve unaffected by $\lambda$).
\( \lambda \) and the Effects of IS Shocks

\[ Y_0 \quad \pi_0 \quad Y_1 \quad \pi_1 \quad \pi_1' \quad Y_{1'} \]

\( LRAS \)

\( AS \)

\( AD (\lambda \ \text{big}) \)

\( AD (\lambda \ \text{small}) \)

\( Y \)
After an increase in $\bar{A}$, a bigger $\lambda$ results in:

1. Smaller increase in $\pi$
2. Smaller increase in $Y$. Since $Y^P$ doesn’t change, this means that the gap, $X$, goes up by less

Bigger $\lambda$ is a “win-win” from perspective of dual mandate conditional on IS shocks
IS Shock: $\lambda = 1$ vs. $\lambda = 10$
\( \lambda \) and the Effects of \( Y^P \) Shocks

\[
\begin{align*}
\lambda &\quad \text{LRAS} \\
\pi &\quad \text{LRAS'} \\
\pi_0 &\quad \text{AS} \\
\pi_1' &\quad \text{AS'} \\
\pi_1 &\quad \text{AD (} \lambda \ \text{small)} \\
\end{align*}
\]
After an increase in $Y^P$, a bigger $\lambda$ results in:

1. Smaller increase in $\pi$
2. Bigger increase in $Y$. Since $Y^P$ increases, this means that the gap, $X$, goes down by less the bigger is $\lambda$

Bigger $\lambda$ is a “win-win” from perspective of dual mandate conditional on potential output shocks shocks
$Y^P$ Shock: $\lambda = 1$ vs. $\lambda = 10$
Divine Coincidence

- Divine Coincidence: there is no tradeoff between objectives of monetary policy
- Responding strongly to inflation both stabilizes inflation and the output gap
- So named by Blanchard and Gali (2007)
- Forms the basis of the call for “inflation targeting,” either implicitly or explicitly done by several leading central banks
Average Inflation Targeting

- As of two weeks ago, in conclusion of its comprehensive review of its tools and strategies, the Fed has adopted average inflation targeting.

- Basic idea of average inflation targeting: targets average inflation rate of, say, 2 percent, over a several-year window.
  - e.g. if inflation runs at 1 percent one year, the Fed targets 3 percent the next year.

- Basically a way for the Fed to promise future monetary stimulus (i.e. future low value of $\bar{r}$).

- In current AD-AS model, which isn’t forward-looking, not obvious why this would matter.

- But in forward-looking model, credible promise of future stimulus could lower long-term rates today, providing stimulus in present and helping to achieve inflation target in present (particularly relevant when ZLB binds).
Divine coincidence does not hold conditional on inflation shocks ($\rho$).

- Sometimes called “inefficient supply shocks” (do not affect $Y^p$).
- Here, there is a tradeoff between the two aspects of the dual mandate.
λ and the Effects of ρ Shocks

The diagram illustrates the effects of shocks on the economy. The horizontal axis represents the level of output (Y), and the vertical axis represents the inflation rate (π). The LRAS curve represents the long-run aggregate supply, and the AS curve represents the aggregate supply in the short run. The diagram shows two scenarios:

1. AD (λ big): This curve represents a large aggregate demand shock, shifting the AS curve to the right, indicated by the upward arrow. This results in an increase in output from Y₀ to Y₁ and an increase in inflation from π₀ to π₁.

2. AD (λ small): This curve represents a small aggregate demand shock, shifting the AS curve upwards but not as much as in the previous case. The increase in output is from Y₀ to Y₁', and the increase in inflation is from π₀ to π₁'.

The diagram highlights the impact of different magnitudes of aggregate demand shocks on the economy's output and inflation rates.
ρ Shock: \( λ = 1 \) vs. \( λ = 10 \)