Identification and Estimation of Interest Rate Rules in the New Keynesian Model

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Abstract

Conventional wisdom is that the Fed helped to stabilize the US economy in the early 1980s by switching to a policy in which it adjusted interest rates more strongly in response to variables such as inflation. This conclusion is supported by a wide range of empirical estimates. In a recent paper, Cochrane (2007b) argues against this interpretation of the historical record, asserting that the parameters of nominal interest rate rules are not identified in the baseline New Keynesian model that has become the workhouse for academic discussions of monetary policy. I show that Cochrane’s finding of non-identification is not a generic implication of the model, but is rather the result of a particular assumption on the policy rule. Under standard specifications of the nominal interest rate rule the policy parameters are identified and may be estimated consistently using conventional techniques. The assumption which leads to non-identification is that the central bank adjusts the interest rate one for one with fluctuations in the Wicksellian natural rate of interest. While an interesting normative prescription, it is difficult to envision how actual central banks would implement such a policy. I argue that such a rule is in fact likely to be welfare dominated by more conventional specifications of the interest rate rule. I also present indirect empirical evidence to suggest that the Fed does not, in fact, follow a rule in which it adjusts its interest rate one for one with the natural rate.

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1 Introduction

One of the most celebrated papers in economics is by Taylor (1993), in which he shows that a simple linear rule relating the Fed Funds Rate to the deviation of GDP from trend and the deviation of inflation from target characterizes US monetary policy well. Taylor argues that it is important that the central bank commit to raising nominal rates more than one for one with inflation. This so-called “Taylor Principle” essentially calls for the Fed to raise real rates in response to inflation, thereby eliminating the possibility of self-fulfilling inflations.

While the Taylor Principle was first espoused in the context of an “old” Keynesian model, researchers over the last fifteen years have adopted Taylor’s nominal interest rate rule (i.e. “Taylor Rules”) in the context of “new” Keynesian models. As Cochrane (2007a and 2007b) eloquently points out, in many respects the “new” Keynesian models are substantially different from their predecessors, and the “old” Keynesian intuition does not always carry over into the “new” models. Nevertheless, something very similar to the Taylor Principle obtains in the “new” model. Although the exact conditions are somewhat more complicated, a unique and non-explosive equilibrium in the New Keynesian model still requires that the central bank respond aggressively enough to movements in inflation and/or output.

Whether or not the Fed’s nominal interest rate target has always responded sufficiently enough to inflation is potentially a key issue in understanding the moderation of the US economy since the 1970s. In a provocative paper, Clarida, Gali, and Gertler (2000, hereafter CGG) develop a New Keynesian model with a policy rule based on Taylor (1993) and derive conditions for the determinacy of a rational expectations equilibrium. They then estimate regressions based on their model’s policy rule and argue that the Fed failed to satisfy the modified Taylor Principle in the 1970s but did so afterwards. If their empirical findings are robust, this suggests that policy itself might have been an important source of instability in the 1970s. Not only would the economy be subject to indeterminate “sunspot” equilibria were the Taylor Principle not satisfied, the economy’s responses to fundamental shocks might also have been affected in important and welfare-reducing ways.

In a recent paper, Cochrane (2007b) argues that CGG’s interpretation of macroeconomic history is flawed. In particular, he asserts that the parameters of the nominal interest rate rule are not identified in the New Keynesian model. His essential argument is that satisfaction of the Taylor Principle in the model is tantamount to a threat which is never actually carried out in equilibrium. As such, he argues, data generated from the model can never reveal the values of the policy parameters in the rule which lend credibility to the threat.

The present paper delves deeper into this issue of identification, and considers the conditions under which Cochrane’s claim of non-identification holds and the conditions under
which it does not. To foreshadow my conclusions, I show that Cochrane’s non-identification result is not a generic implication of the model, but rather obtains only under a particular (and likely unrealistic) specification of the central bank’s reaction function. For more standard specifications of policy, I show that the central bank’s key policy parameters are in fact identified and may be estimated by conventional means.

The key ingredient allowing identification of the central bank’s policy parameters is for non-policy shocks to lead to equilibrium movements in inflation and the output gap. In the standard New Keynesian model with a conventional Taylor Rule, non-policy shocks do affect inflation and the gap through the Phillips Curve. Thus, the New Keynesian model generates movements in inflation orthogonal to structural policy errors. Further, these movements in inflation are monotonically related to the parameters of the Taylor Rule. As such, those parameters are in fact identified. I demonstrate that a standard linear regression, with proper instrumental variables, will in fact consistently estimate the central bank’s policy parameters. Nevertheless, I document that the small sample properties of single equation estimates of the Taylor rule are likely rather poor. In particular, the further the central bank is from the Taylor principle cutoff, the more imprecise are the estimates.

Cochrane obtains his non-identification result in a framework in which the policy rule contains a “stochastic intercept” term which shifts endogenously in response to non-policy shocks in such a way as to completely stabilize the output gap and inflation. In particular, he assumes that the “stochastic intercept” moves one for one with fluctuations in the Wicksellian natural rate of interest, which Woodford (2003) defines as the real interest rate which would obtain in the absence of nominal frictions. In such a world, the only observable movements in inflation will are due to policy shocks, and hence the model yields no exogenous variation in inflation or the output gap off of which to identify the central bank’s policy parameters.

Such a specification of the policy rule is potentially appealing from a normative perspective. As shown in Blanchard and Gali (2007), stabilization of the output gap (up to first order approximation) maximizes welfare. Implementation of such a rule requires that the central bank know the natural rate of interest, which is not directly observable, but is rather a theoretical construct that is a potentially highly complex function of several structural shocks and parameters. As such, it is unlikely that any central bank would be able to implement such a rule with much precision. This fact can lead to a reinterpretation of policy shocks not as “helicopter drops of money”, but rather as errors in the observation of the natural rate of interest.

Cochrane argues that it is restrictive to make the common assumption of a constant intercept in the nominal interest rate rule. While it is true that the stochastic intercept rule welfare dominates the more standard specification of a Taylor type rule, the relative
welfare losses from a constant intercept rule are small. I show that the standard deviation of the welfare relevant output gap in a constant intercept specification of the rule is only slightly higher than it is under the stochastic intercept rule for plausible calibrations of the parameters of the model. As such, it takes only a very slight reduction in the variance of policy shocks for the constant intercept rule to welfare dominate the stochastic intercept rule. Since the constant intercept rule does not require the central bank to react to anything which is not directly observable, it seems likely that such a rule would be associated with a significantly lower variance of policy shocks, and thus with higher welfare.

I present empirical evidence that the Fed does not, in fact, follow a stochastic intercept rule. There are testable implications of the stochastic intercept rule which do not depend on the exact parameter values. In particular, such a rule has the stark prediction that the only source of variation in inflation are policy shocks/errors. This prediction can be tested by examining the conditional relationships between inflation and non-policy shocks. I identify a “supply” shock from a bivariate VAR using a long run restriction. The shock explaining the unit root in output accounts for well more than half of the innovation variance in inflation. This empirical finding is incompatible with a nominal interest rate rule featuring a stochastic intercept which adjusts one for one with fluctuations in the Wicksellian natural rate of interest.

While not consistent with the predictions of a stochastic intercept rule, the pattern of responses from the bivariate VAR to “supply” shocks are also not consistent with more standard specifications of the interest rate rule. The impulse responses from the bivariate VAR suggest that there is both a large disinflation and a significant predictable increase in output following a positive “supply” shock. The predictable increase in output would have to be associated with an increase in real interest rates to be consistent with the Euler equation. The standard New Keynesian model with the common Taylor type nominal interest rate rule is simply not capable of delivering a substantial increase in real rates and disinflation at the same time. This result suggests that modifications to the New Keynesian/Taylor rule framework are necessary in order to better fit the data.

2 The New Keynesian Model with a Taylor Rule

The canonical New Keynesian model is firmly rooted in the pillars of modern macroeconomics: explicit micro foundations, agent optimization, rational expectations, and market clearing. It differs from its neoclassical counterparts in that it introduces nominal frictions into the model, thereby inducing non-trivial distortions from the first best. These frictions serve two purposes. First, they allow for real effects of monetary disturbances. Second,
these frictions fundamentally alter the economy’s response to real shocks. Modern research in monetary economics focuses on the second point, and in particular the question of how to better design the systematic component of policy so as to minimize the distortions associated with the economy’s response to non-policy shocks in a world with nominal frictions.

I present an extremely stylized version of the basic New Keynesian model. Other than the monetary policy rule, the two primary equations of the model are the Euler/IS equation and the Phillips Curve. The log-linearized “IS” equation is given by:

\[ E_t y_{t+1} - y_t = \sigma(i_t - E_t \pi_{t+1}) \]  

(1)

Reflecting the accounting identity that all output must be consumed, the equation relates expected consumption growth to the \textit{ex ante} real interest rate, with \( \sigma \) denoting the intertemporal elasticity of substitution.\(^1\) For simplicity, I assume a constant discount factor, no preference shocks, and no government consumption.

The New Keynesian Phillips Curve is derived under the assumption of Calvo (1983) style price-setting and relates inflation to the output gap and expected future inflation:

\[ \pi_t = \kappa(y_t - y^f_t) + \beta E_t \pi_{t+1} \]  

(2)

\( \kappa \) is a reduced form parameter reflecting the degree of exogenous price stickiness, the rate of time preference, and the output elasticity of real marginal cost.\(^2\) \( \beta \) is a subjective discount factor, and \( y^f_t \) refers to the flexible price equilibrium level of output (i.e. that level of output that would obtain in the absence of price stickiness).

One can think of the flexible price equilibrium level of output as either being exogenous or as endogenously determined given exogenous shock processes. For the sake of simplicity, I choose the former, and model the flexible price level of output as obeying an autoregressive process:

\[ y^f_t = \gamma y^f_{t-1} + \varepsilon_t \]  

(3)

In this simple environment, one can think of the flexible price equilibrium level of output as being driven by fluctuations in technology, though in a more complicated setting it would

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\(^1\)The New Keynesian model usually abstracts from capital and investment. This abstraction simplifies matters a great deal, and turns out to be largely irrelevant to the issues studied in the current paper.

\(^2\)Formally, \( \kappa = \frac{(1-\theta)(1-\theta\beta)}{\eta} (1/\sigma + 1/\eta) \), where \( \theta \) is the exogenous probability of a firm being able to change its price in any period and \( \eta \) is the Frisch labor supply elasticity. The term \( (1/\sigma + 1/\eta) \) is the elasticity of the output gap with respect to real marginal cost. I abstract from features which would introduce real rigidities (also known as strategic complementarities). Such features would affect the slope of the Phillips Curve, but would not affect the analysis concerning the identification of parameters from the monetary policy rule.
reflect any of several forces that would affect output in a standard RBC model – government purchase, preference, or tax shocks, to name but a few. It is also helpful to define another theoretical construct which I will refer to as the Wicksellian natural rate of interest (following Woodford (2003)):

\[ r_t^f = \frac{1}{\sigma}(E_t y_{t+1}^f - y_t^f) \]  

This is the real interest rate which would obtain in the absence of nominal rigidities, and is found by solving the Euler equation for the real interest rate consistent with output being at the flexible price equilibrium level.

The full solution to the model requires the introduction of an additional equation describing monetary policy. Early variants of the model included a money demand function and specified an exogenous time path for the money supply. Most recent research closes the model with a nominal interest rate rule. In the spirit of Taylor (1993), one might suppose that the central bank sets nominal interest rates according to a rule similar to:

\[ i_t = i^* + \phi_y(y_t - y_t^f) + \phi_\pi(\pi_t - \pi^*) + v_t \]  

Taylor argues that such a rule is both a good description of historical Fed policy as well as a good normative prescription for how the Fed ought to conduct policy. \( i^* \) is the Fed’s target nominal rate when both output and inflation are at their targets. Both response coefficients on the gap and inflation are assumed to be non-negative. \( v_t \) is some exogenous disturbance. One may think of this disturbance either as an exogenous shift in the stance of policy or, perhaps more realistically, as reflecting misperceptions of current inflation and/or the gap or the influence of some omitted variable(s).

The positive fact that nominal interest rates are highly persistent and the normative observation that banks may find it desirable to smooth rates over time has led researchers to consider modifications of (5) in which there is an explicit smoothing parameter. One simple way to do this is to assume that \( v \) is autocorrelated:

\[ v_t = \alpha v_{t-1} + \zeta_t \]  

Another is to include a lagged interest rate term on the right hand side of (5), leading to a reinterpretation of the policy rule as one of partial adjustment:

\[ i_t = \rho i_{t-1} + (1 - \rho)i^* + (1 - \rho)\left(\phi_y(y_t - y_t^f) + \phi_\pi(\pi_t - \pi^*)\right) + v_t \]  

While both (5) with an autocorrelated error term and (7) with a white noise error term will
produce persistent effects of policy shocks, they are not the same. In particular, (7) assumes that the bank desires to smooth rates in response to all shocks, whereas in (5) rates are only explicitly smoothed in response to policy shocks. The partial adjustment specification is the more popular of the two in the literature, though Rudebusch (2002) argues that the standard Taylor rule with an autocorrelated disturbance is more consistent with the data.

The full linear system of equations satisfies the Markov property and features two jump variables and two state variables. The determinacy and boundedness of the equilibrium of the model depend on the eigenvalues of the transition matrix. A unique and non-explosive rational expectations equilibrium requires one unstable eigenvalue for each jump variable. As shown by Woodford (2003) and others, a necessary condition for the uniqueness of the equilibrium is that the parameters of the nominal interest rate rule satisfy:

$$\phi_\pi + \frac{1-\beta}{\kappa} \phi_y > 1$$

This is slightly more complicated than the Taylor principle as originally espoused, which simply calls for the coefficient on inflation in the policy rule to exceed unity. That being said, with the discount factor close to one, the condition requisite for the uniqueness of equilibrium is still approximately that the coefficient on inflation in the rule be greater than one.

While cosmetically similar, the underlying economics behind this condition are quite different than what Taylor (1993) had in mind. The Taylor principle was originally cast in the context of an “old” Keynesian model, which differs from the “new” model in that inflation is a state, rather than a jump, variable. In such a world, a coefficient on inflation in the policy rule in excess of unity is necessary to head off nominal explosions. In particular, the central bank must raise real interest rates whenever inflation increases to prevent inflation from accelerating. In the “new” Keynesian model, however, we typically rule out nominal explosions by assumption. There, the satisfaction of the modified Taylor principle is necessary not to rule out explosions, but rather to ensure the uniqueness of the equilibrium.

As Cochrane (2007a and 2007b) stresses quite eloquently, though apparently similar, the

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Footnotes:
3. The jump variables are output and inflation. If one takes the rule to be given by (5), then the nominal interest rate can be eliminated, with $v$ being a state variable. Under a partial adjustment rule like (7) the nominal interest rate is a state variable. The other state variable is the flexible price equilibrium level of output.
4. Note that this condition holds for both (5) and the partial adjustment rule given by (7), since I wrote the $(1-\rho)$ term as multiplying the response coefficients to inflation and the output gap in (7).
5. In his companion paper, Cochrane (2007a) criticizes the assumption of ruling out nominal explosions, arguing that there is nothing in economic theory which justifies this (i.e. there is no transversality condition for inflation). Allowing for non-bounded solutions would require analyzing the non-linear system of equations. The log-linearization as shown here is only valid in the neighborhood of the zero inflation steady state.
Taylor principle in the old and new variants of the Keynesian model is quite different, and applying the “old” logic to the new model can be misleading. He offers an intriguing interpretation of satisfaction of the Taylor principle in the New Keynesian model. In particular, he argues that parameterizations of the policy parameters in the region of determinacy are tantamount to an off-the-equilibrium path threat to “blow up the world”. Since we rule out nominal explosions by assumption, we never see this threat actually carried out. Thus, Cochrane argues, data generated by the model can never reveal the parameters of the policy rule lending credibility to this threat, and concludes that these parameters are thus not identified.

3 Identification and Estimation

Cochrane’s finding of non-identification of policy parameters is not a generic implication of the satisfaction of the Taylor principle in the New Keynesian model. While it is true that we never see the world “blow up” in the model, we do observe equilibrium fluctuations of the endogenous variables in response to exogenous shocks, and for policy rules like (5) or (7), these equilibrium fluctuations do reveal information about the underlying parameters of the rule.

In order to see this, it is helpful to first construct theoretical impulse responses to the two exogenous shocks of the model. Doing so requires picking values of the structural parameters. I set the discount factor, $\beta$, to 0.99, implying an annualized steady state discount rate of roughly four percent. I set the elasticity of intertemporal substitution, $\sigma$, to 1, corresponding to the common case of log utility over consumption. The slope of the Phillips Curve is set to 0.3. I assume that the autocorrelation coefficient of the flexible price equilibrium level of output, given by $\gamma$ in (3) above, is 0.95. So as to focus in on the primary coefficient governing determinacy in the Taylor rule, I set the coefficient on the gap, $\phi_y$, in the rule equal to zero. To highlight the role of different parameterizations in the equilibrium fluctuations of the model, I consider two values of the parameter on inflation in the region of determinacy, 1.25 and 1.5. I consider both Taylor’s original specification of the rule given by equation (5) and the partial adjustment specification, (7). In the specification given by (5), I assume that the error term is autocorrelated as given by (6), with $\alpha = 0.8$. Under the partial adjustment specification in (7), I assume that the smoothing parameter, $\rho$, is also 0.8.

Figure 1(a) shows responses to a one percent technology shock and twenty-five basis points this calibration corresponds with a Calvo parameter of 2/3, implying that firms get to change their prices on average once every three quarters, and a Frisch elasticity of labor supply of roughly 1. See Footnote 2. This is done for simplicity and does not affect any of my conclusions. Most empirical evidence suggests that the Fed does not respond strongly to gap measures anyway.
point policy shock under specification (5) for both parameterizations. The first panel shows that both inflation and the output gap fall in response to the technology shock. The fall in both variables is monotonically decreasing in the policy rule parameter – that is, for higher values of $\phi_\pi$, the downward jumps in both inflation and the gap in response to a favorable technology shock are smaller. The second panel shows the response to the monetary policy shock. Again, the impact effects on both inflation and the output gap from the shock are decreasing in the size of the policy rule coefficient. Figure 1(b) repeats the same exercise for the partial adjustment description of monetary policy as given by specification (7). Here again, we see that the impact jumps in both inflation and the gap are smaller for larger values of $\phi_\pi$, though the discrepancy in the jumps for the two calibrations is smaller than under specification (5).

We thus see that both policy and non-policy shocks affect inflation and the output gap, and further that the effects of shocks on these variables depend on the values of the parameters in the policy rule. As such, the policy rule parameters ought, in principle, to be identified in the model. As noted in the Introduction, the key for identification is that non-policy shocks affect both inflation and the gap. If the only observed variation in these two variables was due to policy shocks, we would not be able to recover the values of the policy parameters. While it is true that the impact responses of inflation and the gap following a policy shock are dependent on the policy parameters, the size of the responses also depends on the size of the policy shock. In the second panels of Figure 1, I could have simply altered the size of the policy shock to force the responses of both inflation and the gap to lie everywhere on top of one another for both $\phi_\pi = 1.25$ and $\phi_\pi = 1.50$. Without knowledge of the variance of policy shocks, we can thus not identify the policy parameters off of variation in the variables due to policy shocks. Identification must come from the interaction of non-policy shocks with inflation and the gap.

One can go about estimating the policy rule parameters either by single or multiple equation methods. Simple inspection of (5) and (7) reveals that OLS will not produce consistent estimates of the parameters of the rule. This is because both inflation and the gap are jump variables, and are thus contemporaneously correlated with the structural error term in both specifications. As such, consistent estimation of the policy rule parameters requires the use of valid instrumental variables.

The set of permissible instruments depends on the specification of the policy rule. The most commonly used instruments in empirical papers include lagged values of inflation, the gap, and nominal interest rates. While potentially valid under the partial adjustment specification, lagged endogenous variables are not acceptable instruments under (5). To see
this, we can lag the specification one period and write it with a white noise error term:

$$i_t = (1 - \alpha) i^* + \alpha i_{t-1} - \alpha \phi_y(y_{t-1} - y^f_{t-1}) - \alpha \phi_\pi(\pi_{t-1} - \pi^*) + \phi_y(y_t - y^f_t) + \phi_\pi(\pi_t - \pi^*) + u_t$$

Because $u_t$ above is white noise, the lagged values of inflation, the interest rate, and output are econometrically exogenous. The current values of inflation and output still respond to the white noise policy innovation, and thus we still need instruments. Once lagged values of the endogenous variables will not work, as they already (implicitly, at least) appear in the rule. Because of the Markovian structure of the model, twice or more lagged endogenous variables are not permissible instruments either. This is because, conditional on the first lag of the endogenous variables, twice or more lagged variables convey no information about the current state of the system.

The only permissible instruments under Taylor’s original specification of the policy rule with an autocorrelated error term are thus current and lagged values of the flexible price equilibrium level of output itself (or shocks to it). If the policy error is not autocorrelated, then lagged values of the endogenous variables are valid instruments, provided there is some persistence to natural rate shocks.\(^8\) This presents a potential complication – both to central bankers and to economists trying to understand central bank behavior – as the flexible price equilibrium level of output is not directly observable. In reality, however, there are many factors affecting the economy’s natural rate – among them government spending, tax rates, natural disasters, etc. – which are observable and which may serve as valid instruments for both inflation and the output gap. For the purposes of this paper, I simply assume that I can observe the flexible price equilibrium level of output.

To assess the ability of a simple linear regression to estimate the policy rule parameters, I simulate a data set of 500 observations using the calibration described above under the rule given by (5) with $\phi_\pi = 1.5$. The standard deviation of the technology shock is normalized to be unity and I set the standard deviation of the policy shock to be 0.5 (both innovations are drawn from a standard normal distribution). I discard the first 100 observations so as to limit the influence of arbitrary initial conditions. I then estimate a regression of the nominal interest rate on current inflation and the lags of inflation and the nominal interest rate. I include the lags so as to force the error term to be white noise, as discussed above, and instrument for current inflation with the current level of the flexible price equilibrium level of output. I repeat this process 1000 times.

The mean estimate of $\phi_\pi$ from the 1000 Monte Carlo simulations is 1.56, which is very close to the true value of 1.5. The mean value of $\alpha$ comes out to be 0.81, almost exactly

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\(^8\)If neither policy nor natural rate shocks have any persistence, then the correlation between current and lagged endogenous variables will be zero.
equal to the true value. Figure 2 depicts the histogram of estimates of across the Monte Carlo simulations. While approximately unbiased, the implied standard errors of the estimate are rather large, with the 95 percent empirical confidence bands given by [1.19, 2.36]. Nevertheless, only one percent of the estimates lie below the critical value of unity. One can easily verify that, letting the size of the sample become arbitrarily large, the empirical distribution of point estimates collapses on the true value. The precision of the estimates (as measured by the confidence bands) is decreasing in the size of the true coefficients. As the policy rule coefficients become large, the amount of exogenous variation in inflation and the gap becomes smaller (see the theoretical impulse responses in Figure 1), and thus estimation of the policy parameters becomes more difficult.

I next consider a similar Monte Carlo exercise when policy is governed by the partial adjustment mechanism. Here the set of permissible instruments is different. Because we now interpret the error term to be white noise, lagged values of inflation and the output gap are valid instruments, because lags of these variables do not otherwise appear on the right hand side of the rule. Lagged values of the interest rate are not permissible, nor are twice or more lagged values of inflation or the gap once the first lags are included in the instrument set. Current and lagged values of the flexible price equilibrium level of output remain valid. There is a potential incongruity between valid instruments in the model and the instruments used in actual estimation in a number of empirical papers. CGG (2000), for example, assume a policy rule nearly identical to (7). Their instrument set includes multiple lags of the nominal rate, inflation, and an empirical measure of the gap, as well as lags of other variables which do not appear in their model. As noted above, however, multiple lags of the endogenous variables are not permissible instruments due to the Markovian structure of the model.

I examine the properties of the IV estimator using both lagged inflation and the current flexible price equilibrium level of output as instruments in isolation and together. The mean estimates and empirical distributions of estimates are virtually identical using the different instrument sets. For all three instrument sets, the mean estimate of φπ across the Monte Carlo simulations is about 1.65, while the average point estimate of the smoothing parameter comes out to be 0.79. Figure 3 depicts the empirical distribution of estimates of φπ using either once lagged inflation or the current flexible price equilibrium level of output as instruments in isolation. Here the small sample bias appears to be more significant than under Taylor’s original specification of the rule. The dispersion of the estimates also appears

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9CGG assume that the Fed reacts to expected inflation, not current inflation as presented here. There is no explicit policy shock in their rule, but there is a structural error term in their regression due to forecast errors. The assumption that the Fed reacts to expected inflation as opposed to current has little bearing on any of my results.
to be greater under rule (7) than rule (5). Indeed, the empirical 95 percent confidence bands come out to be roughly [0.74, 3.60] for either instrument, which is significantly wider than before.

The intuition for the less precise estimates under the partial adjustment rule is made obvious by close inspection of the theoretical impulse responses to the shocks shown in Figures 1 (a) and (b). The difference in jumps of inflation and the gap in response to the technology shock for the different calibrations of the policy parameter is significantly smaller under the partial adjustment rule relative to the original Taylor rule, thus likely accounting for the poor estimates under rule (7). Even with a sample size of 400 (which is much larger than the samples to which researchers typically have access), while we can reject that it is greater than zero, we cannot reject that \( \phi_\pi \) is greater than unity at any sensible level of statistical significance. As with the original Taylor Rule, as I let the sample size become arbitrarily large, the distribution of estimates collapses on the true value.

As noted by Lubik and Schorfheide (2004), full system based estimates of the policy parameters are more efficient than single equation IV estimates. The drawback of multiple equation based estimates is that they are more susceptible to misspecifications elsewhere in the model. Systems based estimation is also complicated by the fact that, as written, the model suffers from stochastic singularity. Because there are more variables than shocks, and more parameters than variables, identification of the complete parameter vector of the model is extremely difficult, if not impossible. Though I do not fully pursue this type of estimation in this paper, I have verified that system based GMM estimates of the policy parameters are consistent once I calibrate some of the other parameters of the model.

In spite of the fact that the small sample properties of the single equation IV estimators are not particularly good, it is clear that one can, in principle, consistently estimate the policy parameters from data generated from the baseline New Keynesian model with a standard description of the central bank’s nominal interest rate rule. How, then, does Cochrane (2007b) obtain his non-identification result? He assumes a policy rule that looks nearly identical to (5), but with one small (and important) difference – there is a time subscript on the target nominal rate:

\[
i_t = i_t^* + \phi_y(y_t - y_t^*) + \phi_\pi(\pi_t - \pi^*) + \nu_t
\]

Cochrane refers to the time-varying target rate as a “stochastic intercept”. This is an unfortunate and potentially misleading use of terminology. After all, one could interpret

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10 One can see that the impact jumps in both inflation and the gap with \( \phi_\pi = 1.25 \) are roughly double those with \( \phi_\pi = 1.50 \) under Taylor’s original specification (Figure 1(a)). Under the partial adjustment rule (Figure 1(b)), the difference in jumps is closer to a factor of 1.5.
innovations to $\nu$ as a time-varying intercept as well, so it is not immediately clear that (8) is conceptually any different than (5). Rather than allowing $i^*$ to vary randomly, however, Cochrane assumes that it tracks fluctuations in the Wicksellian natural rate of interest, as defined in (4) above:

$$i^*_t = r^f_t$$

It is easy to see that, if the target rate varies one for one with the Wicksellian natural rate of interest, then a no gap/no inflation outcome is a potential equilibrium of the model absent policy shocks. The modified Taylor principle necessary for the uniqueness of the equilibrium is unaffected by the presence of the stochastic intercept. Hence, so long as the policy rule parameters are chosen so as to satisfy the modified Taylor principle, the equilibrium of the model will be unique, and both inflation and the output gap will be completely stabilized in response to any non-policy shock. Note that it is not, in general, possible for a partial adjustment rule such as (7) to implement the no gap/no inflation equilibrium, even with the stochastic intercept term tracking fluctuations in the Wicksellian natural rate.

Figure (4) shows theoretical impulse responses to the two exogenous shocks of the model under the same calibration as above but with the stochastic intercept rule. As before, I show responses under two values of the policy parameter on inflation, 1.25 and 1.5. As noted, the non-policy shock induces no equilibrium movement in either inflation or the gap for either value of the policy rule parameter. Because the stochastic intercept is unaffected by a policy shock, the responses to a policy shock are identical to what is shown in Figure 1(a). It is thus obvious that the policy rule parameters do appear in the equilibrium dynamics of data generated from the model, even under the stochastic intercept rule.

Nevertheless, the policy parameters are not identified. While the model does produce variation in the variables of the model which is influenced by those parameters, this variation is not exogenous. In particular, the lack of identification results because the assumptions on the policy rule ensure that non-policy shocks have no effect on either inflation or the output gap. Thus, the model yields no permissible instruments and hence no way of consistently estimating the policy parameters in a linear regression.

There is also no hope of estimating these parameters through systems based estimation.

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11 Following King (2000), Cochrane usually write the rule in terms of deviations from the Fed’s chosen equilibrium, e.g. the term on the RHS of the rule would be $\pi_t - \pi^*_t$, where $\pi^*_t$ denotes the central bank’s desired level of inflation at that particular moment in time. Under a stochastic intercept rule, the central bank solely determines the equilibrium level of inflation. Thus, the rule allows the Fed to insure that the actual level of inflation is equal to its desired level, so that $\pi_t = \pi^*_t$ at all times, and the policy parameter is not identified. Under the more standard rules such as (5) or (7), the central bank is not able to always implement its desired level of inflation in the face of external shocks.
In particular, the likelihood function is not single-peaked for this specification of the model. Intuitively, the volatilities of inflation and the gap are both influenced by the variance of policy shocks and the magnitude of the policy rule parameters. The model is capable of matching given volatilities either with a relatively high value of the variance of policy shocks and a low value of the policy rule parameters, or with a low variance of the shock and high coefficients in the rule. Without knowledge of the variance of policy shocks, it is simply not possible to recover the true value of the policy rule parameters under this specification of the rule.

Because the modified Taylor principle itself is unaffected by the presence of the stochastic intercept, it is clear that non-identification is not a generic implication of the satisfaction of the Taylor principle in the New Keynesian model, but is rather the result of this particular specification of the policy rule. Cochrane’s assertion of non-identification does not apply to existing empirical work, since most or all existing empirical papers assume interest rate rules like (5) or (7).

All of the above results assume that the policy rule parameters are such that the economy is in the region of determinacy – in other words, I have assumed throughout that the modified Taylor principle is always satisfied and therefore that the equilibrium of the economy is unique. I close this section with a brief discussion about estimation and identification when the Taylor principle is not satisfied.

Although it may at first seem non-intuitive, policy coefficients in the region of indeterminacy ought to be better identified (and more precisely estimated) than those yielding uniqueness. In the simple setup above, with and \( \phi_y = 0 \) and \( \phi_x < 1 \), the policy function mapping states into output and inflation would not be unique. In other words, for a given state of the world, there are an infinite number of combinations of output and inflation consistent with the equations of the model holding and with non-explosion. Pinning down the actual equilibrium from this set of possible equilibria requires the introduction of an extrinsic coordinating variable – a “sunspot” – which is otherwise completely independent of the rest of the model. For a clear description of how to incorporate sunspots into the model, the interested reader is referred to the working paper version of CGG (available as NBER WP # 6442, 1998).

The mapping between the sunspot realizations and the jump variables (output and inflation) of the model is completely arbitrary, so it is difficult to say much about the actual behavior of the economy in the region of indeterminacy. We can, however, make a few general observations. First, the sunspot will, in general, lead to fluctuations in both inflation and output which are unrelated to the two fundamental shocks. Second, the presence of sunspots will alter the response of output and inflation to the fundamental shocks. As it
pertains to identification and estimation, the first observation is important. In particular, the sunspot introduces an additional source of exogenous variation to both inflation and the output gap. Under policy rules like (5) or (7), this source of additional variation has no bearing on whether or not the policy rule parameters are identified. Because it is an additional source of exogenous variation in both inflation and the gap, however, the sunspot itself is now also in the set of permissible instruments (the other valid instruments remain the same as in the discussion above). Even if the sunspot is unobservable, the additional source of exogenous variation in both inflation and the gap is likely to improve the precision of IV estimates of the policy rule coefficients.

In the region of determinacy, Cochrane’s specification of the rule given by (8) eliminates all exogenous variation in both inflation and the gap, thus leading to his non-identification result. Once the coefficients place the economy in the region of indeterminacy, however, the policy shock is no longer the only source of variation in these variables. In particular, the non-fundamental sunspot shock will, in general, affect both inflation and the gap on impact. Furthermore, fundamental non-policy shocks (which manifest themselves as fluctuations in the Wicksellian natural rate of interest), will also, in general, affect both inflation and the output gap. Thus, Cochrane’s non-identification result only applies under rule (8) when the Taylor principle is satisfied. If the Taylor principle is not satisfied, there will exist exogenous variation in inflation and the gap off which to identify the policy parameters. Thus, these parameters may, as a matter of principle, be consistently estimated.

I repeat the Monte Carlo exercise from above for the system in the region of indeterminacy assuming that I can observe the sunspot realizations. I assume that the policy rule is given by (8) and that \( \phi_y = 0 \) and \( \phi_\pi = 0.9 \). The rest of the parameters are calibrated as above, and I draw sunspot realizations from a standard normal distribution. For simplicity, I assume that the mapping between sunspot realizations and the jump variables is such that the innovation in current inflation is equal to the sunspot realization.\(^\text{12}\) I run a regression of the nominal interest rate on its own lag, the lag of inflation, and current inflation, which forces the structural error term to be white noise. I do not include the Wicksellian natural rate of interest in the regression. While this leads to a composite error term which is not white noise, it does not affect the consistency of the estimates of the policy rule coefficients, since the sunspot realization which serves as my instrument is uncorrelated with the natural rate of interest.

Figure (5) depicts the histogram of estimates of \( \phi_\pi \) under this policy rule in the region of indeterminacy. The estimates are approximately unbiased, with the mean estimate of

\(^{12}\)This assumption is made only for simplicity. My results are unaffected for more complicated mappings between sunspot realizations and the jump variables.
\( \alpha \) equal to 0.79 and the mean estimate of \( \phi_\pi \) equal to the truth at 0.90. The empirical confidence bands are much tighter than those in the region of determinacy, with 95 percent of the estimates of lying in the region \([0.80, 1.00]\). For the kind of question posed by CGG (2000) – did the Fed move from a "passive" policy in the 1970s to a more "active" one in the 1980s – the appropriate null hypothesis is that the policy parameters are in the region of indeterminacy. The Taylor rule parameters are always identified under this null hypothesis, even under the stochastic intercept rule.

4 Welfare Evaluation and Empirical Evidence

The previous section established that it is not a generic implication of the New Keynesian model that the policy rule parameters are not identified in the region of determinacy. Rather, non-identification only obtains in the case in which the central bank’s rule features a stochastic intercept term which adjusts one for one with movements in the Wicksellian natural rate of interest. In this section, I address both a normative and a positive question related to this kind of policy rule. First, should the central bank attempt to adhere to a stochastic intercept rule? Second, do central banks try to follow such a rule? My answer to the former is probably not, and almost certainly not to the latter.

There are two distortions reflected in the equilibrium level of output in the New Keynesian model as described above. One comes from the assumed price stickiness, which potentially leads to discrepancies between the actual and flexible price equilibrium levels of output. The second distortion is the monopolistic competition which gives rise to price-setting in the first place. Optimal policy should target the Pareto optimal level of output that would obtain in the absence of both of these distortions. As shown in Blanchard and Gali (2007), in the standard New Keynesian model, the first best and flexible price equilibrium levels of output differ by a constant, and thus stabilization of the output gap (the gap between actual and flexible price output) is equivalent to maximization of the welfare relevant gap (the gap between actual and first best output). As shown above, the stochastic intercept rule stabilizes both inflation and the output gap, and thus is the welfare maximizing policy rule in the basic model.

While the stochastic intercept rule maximizes welfare in the model, it may or may not be optimal once real world considerations are taken into account. In particular, such a rule requires the central bank to observe the Wicksellian natural rate of interest in real time. This natural rate is in fact not observable and is a potentially complex function of several underlying shock processes. If the central bank can only observe the natural rate with noise, then it is possible that the resulting increase in the variance of policy shocks could result in
lower welfare relative to the more standard constant intercept specification.

I compare welfare under both the standard Taylor rule (5) and the stochastic intercept rule (8). Because it is generally not possible to implement the no gap, no inflation equilibrium under a partial adjustment rule, a welfare comparison between the stochastic intercept rule and a rule similar to (7) is unnatural. I use the standard deviation of the output gap as a welfare metric. The first two columns of Table 1 show the model standard deviations of the gap under both the stochastic intercept and standard Taylor rules for different values of $\gamma$, which is the persistence of the flexible price equilibrium. The rest of the model parameters are calibrated as above. In particular, the policy rule parameters are set at $\phi_y = 0$ and $\phi_\pi = 1.5$.

Because the stochastic intercept rule stabilizes the gap in response to non-policy shocks, the standard deviation of the gap under such a rule is invariant to different values of $\gamma$. For high values of $\gamma$, the standard deviation of the gap under the standard Taylor rule is almost identical to the stochastic intercept rule (standard deviation of 0.6352 vs. 0.6331 when $\gamma = 0.95$). As $\gamma$ decreases the welfare differences between the two rules increase. The intuition for this is straightforward upon inspection of the definition of the Wicksellian natural rate (equation (4)). For high values of $\gamma$, fluctuations in the Wicksellian natural rate are small, and thus a constant intercept rule comes close to replicating the stochastic intercept rule.\footnote{In the model without capital, if real shocks obey an exact random walk ($\gamma = 1$), then the Wicksellian natural rate of interest is constant and the constant intercept rule is equivalent to a stochastic intercept rule.} As real shocks become less persistent, the natural rate of interest fluctuates more, and the constant intercept rule is further from the stochastic intercept rule. That being said, even when real shocks are almost white noise ($\gamma \to 0$), the standard deviation of the gap under the standard Taylor rule is only about 50 percent larger than under the stochastic intercept rule.

I next make the realistic assumption that the central bank can only observe a noisy signal of the Wicksellian natural rate. In particular:

$$ r^*_t = r^f_t + n_t \tag{9} $$

$r^*_t$ denotes the central bank’s observed natural rate and $n_t$ is the noise in their observation. I assume that $n_t$ is uncorrelated with all other shocks in the economy. I allow misperceptions of the natural rate to be persistent by modeling the noise term as a stationary AR(1):

$$ n_t = \delta n_{t-1} + \eta_t \tag{10} $$

With this specification of the observed natural rate, the stochastic intercept rule can now be
written as:

\[ i_t = \phi_n (\pi_t - \pi^*) + v_t + n_t \] (11)

This is identical to (8), but now with a composite error term reflecting exogenous shifts in policy \((v_t)\) and misperceptions of the natural rate of interest \((n_t)\).

The last three columns of Table 1 show the value of the standard deviations of noise shocks \(\text{std}(\eta)\) for different persistence parameters of noise for which the standard Taylor rule with a constant intercept welfare dominates the stochastic intercept rule. For highly persistent real shocks, one can see that it only takes a very small amount of noise in the central bank’s observation of the natural rate for the constant intercept rule to be optimal. To get an idea for how large the standard deviation of noise shocks must be, Table 2 shows the standard deviation of the Wicksellian natural rate under different calibrations of \(\gamma\). For highly persistent real shocks, one can see that the required standard deviation of noise disturbances can be quite a bit smaller than the standard deviation of the natural rate itself. As the persistence of real shocks rises, the variance of noise shocks required for the constant intercept rule to be welfare dominant also rises. If one believes that real economic shocks are highly persistent (which they appear to be in the US), then it is clear that it takes only a small amount of imprecision in the central bank’s observation of the Wicksellian natural rate for a standard Taylor rule to result in higher welfare than a stochastic intercept rule.

There are additional reasons why a stochastic intercept rule may be either infeasible or undesirable. For one thing, central banks cannot implement such a rule in times in which the natural rate of interest is negative, given the zero lower bound on the nominal interest rate. Secondly, as stressed repeatedly throughout Woodford (2003), it is not so much the actual coefficients and structure of the policy rule which matter, but rather that the agents in the economy know the structure of the rule and believe that the central bank will remain committed to it. For the same reasons that the parameters cannot be identified by an econometrician, there is no mechanism by which agents in the economy could ever learn the values of the response coefficients under a perfect stochastic intercept rule. Even if the central bank were to publicly announce values of \(\phi_x\) and \(\phi_y\), there would be no observable action lending credibility to the announcement, and thus no reason for households to believe them. Lastly, there are many realistic modifications of the baseline model in which the stochastic intercept rule is not necessarily optimal. Such modifications frequently involve a “cost push” shock in the Phillips Curve, which makes it impossible for the central bank to simultaneously stabilize inflation and the gap. More generally, there are several realistic features in which the distance between the flexible price and first best equilibrium levels of output is not constant; such features include real wage stickiness or time-varying monopoly
power (see Blanchard and Gali (2007)). In such instances, optimal policy would not even attempt to stabilize the actual output gap or inflation in response to external shocks.

I now turn to a positive question: does the Fed attempt to follow a stochastic intercept rule? It is not possible to directly estimate the policy rule under the null hypothesis of the stochastic intercept term. There are, however, testable implications of the model with such a rule which do not directly depend on the values of the policy parameters (as long as the economy is in the region of determinacy). In particular, the model with the stochastic intercept rule has the sharp prediction that the only source of variation in inflation is the policy shock. Real shocks (by which I mean shocks which affect the flexible price equilibrium of output) have no impact on inflation in the model augmented with a stochastic intercept rule.

It is thus possible to test whether or not the Fed obeys a stochastic intercept rule by examining the conditional relationships between real shocks and inflation. To identify real shocks I estimate a bivariate VAR featuring real GDP and inflation and orthogonalize the innovations into “demand” and “supply” through a long run restriction that “demand” shocks have no long run effect on the level of output. I use the terms “demand” and “supply” only begrudgingly so as to facilitate comparison with the literature. In reality, the identified “supply” shock is a compilation of anything which can have a long run effect on output, while the “demand” shock reflects forces only having a temporary effect. This VAR is similar to the one in Gali, Lopez-Salido, and Valles (2003), with the exception that their VAR features output per hour. My identifying assumption is that real shocks may have a permanent effect on output, but monetary policy shocks may not. Thus, the structural policy shock from the model above is subsumed in the identified “demand” shock while other factors imparting a unit root on output are reflected in the “supply” shock.\footnote{In the baseline model of the previous section there is no unit root in output. In a version of the model in which there are shocks inducing a unit root in output it is still the case that monetary policy shocks have no long run effect on output.}

I estimate the VAR using quarterly data on real GDP growth and CPI inflation from 1960-2006 with four lags of each variable.\footnote{I arrive at a quarterly inflation measure by calculating the percentage change in the seasonally adjusted CPI from the last month of each quarter. Output enters the VAR in first differences and I show the cumulated level response in the figures.} The impulse responses of both the level of real GDP and inflation to the identified “demand” and “supply” shocks are show in Figure 6. The dashed lines are 90 percent confidence bands from the bias-corrected bootstrap procedure of Kilian (1998). The shock identified as having a permanent effect on output explains the bulk of the inflation innovation variance. In particular, a shock raising output in the long run is associated with a reduction in inflation (at an annualized rate) of more
than one and a half percent on impact. This effect is statistically significant for roughly six quarters. By comparison, the “demand” shock induces an increase in inflation, though the impact effect is not statistically significant. After a few quarters the demand shock explains inflation more significantly. These results are clearly at odds with the predictions of a stochastic intercept rule, under which there should be no relationship between the identified “supply” shock and inflation. The point estimates are somewhat smaller and the statistical significance is weaker in the post-Volcker part of the sample, though the same qualitative pattern emerges. That the response of inflation to “supply” shocks is smaller in the latter half of the sample is consistent with larger response coefficients in the policy rule.

While clearly not consistent with a stochastic intercept rule, the impulse responses from the empirical VAR are also not consistent with a more standard Taylor type rule. The impulse responses suggest that there is a significant predictable increase in output following a “supply” shock. To be consistent with the Euler equation, there must also be an increase in real interest rates following such a shock. However, it is extremely difficult for a standard Taylor rule with coefficients in the region of determinacy to produce a simultaneous decrease in inflation and increase in real rates in response to non-policy shocks. With \( \beta \approx 1 \) and \( \theta \) not too close to zero, the Phillips Curve (2) suggests \( \pi_t \approx E_t \pi_{t+1} \). Using this approximation, subtracting \( E_t \pi_{t+1} \) from both sides of (5), and ignoring the gap term, we see that:

\[
rt = it - E_t \pi_{t+1} \approx (\phi_{\pi} - 1) \pi_t
\]

With \( \phi_{\pi} > 1 \), an increase in real interest rates absent a policy disturbance requires an increase in inflation.\(^{18}\) I should stress that this is an approximate result — there are values of the parameters in which there is a simultaneous increase in real rates and decrease in inflation, but this only occurs when the increase in real interest rates is small. For the amount of predicted increase in output shown in Figure 6, however, the increase in real interest rates would have to be rather large. In that case, it is almost certainly true that the model with a Taylor rule would require an increase in inflation, not a decrease as we see in the data. These

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\(^{16}\)Gali, Lopez-Salido, and Valles (2003) find an insignificant response of inflation to identified technology shocks in the latter half of the sample, and interpret this as evidence in favor of the Fed following a stochastic intercept rule in that time period. The point estimates even in that sample are far too large for such an interpretation. It is extremely difficult to ascertain statistical significance on the basis of roughly 80 quarterly observations with relatively persistent data series. While not necessarily statistically significant, the identified response of inflation to “supply” shocks remains economically significant in the post-Volcker part of the sample.

\(^{17}\)Ignoring the influence of the gap is relatively innocuous here. With this same approximation, fluctuations in the gap are always very small and tend to co-move positively with inflation anyway.

\(^{18}\)The exact same condition also obtains under the partial adjustment specification. Assuming that both \( it \approx it_{t-1} \) and \( \pi_t \approx E_t \pi_{t+1} \) and simplifying yields the same approximate relationship between real interest rates and inflation under (7) as (5).
empirical results thus indicate not only that the Fed likely does not adhere to a stochastic intercept rule, but that the more commonplace specifications of interest rate rules may also be misspecified.

5 Conclusion

A Taylor type nominal interest rate rule has become ubiquitous in the monetary economics literature and is almost universally accepted by macroeconomists as both a good description and prescription for the conduct of monetary policy. Likewise, the New Keynesian model with explicit micro foundations and optimizing behavior has become one of the workhorse models for analyzing short run fluctuations. As such, fully understanding the interplay between interest rate rules and the New Keynesian model—and whether or not data generated from the model are informative about the structure of the policy rule—is an important task.

The received wisdom in macroeconomics is that the Fed helped to stabilize inflation and the US economy by switching to a more active monetary policy under Paul Volcker in the early 1980s. This conclusion is largely based on regression estimates of interest rate rules, which generally find response coefficients on inflation and other variables that were too low in the 1970s and much higher thereafter. In a recent paper, Cochrane (2007b) has challenged this conclusion, arguing that the policy rule coefficients are not identified in the New Keynesian model. Whereas in the “old” Keynesian model sufficiently large policy rule coefficients are necessary to prevent explosive dynamics, in the New Keynesian model the policy rule coefficients must be sufficiently large to render the equilibrium unique. Satisfaction of the Taylor principle in the New Keynesian model imparts an unstable root into the dynamic system of equations. Cochrane argues that, since New Keynesian modelers rule out explosive behavior \textit{a priori}, the equilibrium dynamics of the model cannot reveal any information about the unstable root. In other words, he argues, policy rule parameters in the region of determinacy are not identified.

I demonstrated that non-identification is not a generic implication of the model, but rather results from a particular (and unrealistic) assumption on the policy rule itself. For a standard specification of the interest rate rule—similar either to Taylor’s original specification or the more common partial adjustment specification—the policy rule parameters are in fact identified and may be estimated using standard techniques. The key for identification is for real shocks, by which I mean shocks which would affect output in a flexible price model, to influence inflation and the output gap. This condition is satisfied in the standard New Keynesian model, and thus the model produces exogenous variation in inflation and the output gap off which one may consistently estimate the policy rule parameters.
Cochrane’s non-identification result requires that the interest rate rule feature a “stochastic intercept” which tracks fluctuations in the Wicksellian natural rate of interest. Provided that the response coefficients on inflation and the gap satisfy the modified Taylor principle, such a rule serves to completely stabilize inflation and the output gap in response to any non-policy shock. By eliminating any exogenous variation in inflation and the gap, such a rule renders the policy rule parameters unidentified. Since the gap between the actual and flexible price levels of output is the appropriate welfare metric, such a rule is optimal provided that the central bank can in fact observe the Wicksellian natural rate in real time. Nevertheless, the welfare losses from a more standard constant intercept specification of the policy rule are small, and I showed that it requires only a small amount of noise in the bank’s observation of the Wicksellian natural rate for the stochastic intercept rule to actually result in lower welfare.

While it is not possible to directly estimate the policy rule under the null of a stochastic intercept, there are testable implications of such a rule which do not rely on the particular values of the rule’s response coefficients. In particular, the stochastic intercept rule leads to the stark prediction that the only source of variation in inflation are monetary policy shocks. Since policy shocks can only have temporary effects on output in the model, a direct test of the stochastic intercept rule is to see whether or not things permanently influencing output and spending affect inflation. The results from a structural bivariate VAR suggest that they do. In particular, I find that shocks permanently affecting output account for the bulk of the innovation variance in inflation. I interpret this finding as a rejection of the stochastic intercept rule. Without the stochastic intercept term, there is no inherent identification issue with respect to Taylor type rules in the New Keynesian model.

Nevertheless, the results from the bivariate VAR also pose challenges to a standard Taylor rule with a constant intercept, and suggest that either the policy rule or the model itself may be inconsistent with the data. The “supply” shock identified from the bivariate VAR is associated both with a large reduction in inflation and large predictable increases in output. For this to be consistent with the Euler equation, such permanent supply shocks must lead to higher real interest rates. It is extremely difficult for a standard Taylor rule satisfying the Taylor principle to simultaneously generate large declines in inflation and large increases in real interest rates. As this is exactly the pattern implied by the data, this suggests that there may be a more general misspecification issue regarding the interest rate rule or the New Keynesian model itself.
References


Figure 1(a)
Theoretical Responses to Shocks under Rule (5)
Figure 1(b)
Theoretical Responses to Shocks under Rule (7)
In both figures the true value of $\phi_\pi$ is 1.5.
Figure 4
Theoretical Impulse Responses to Shocks under Stochastic Intercept Rule (8)
Figure 5
Histogram of Estimates of $\phi_\pi$ under Stochastic Intercept Rule (8)
In Region of Indeterminacy

The true value of $\phi_\pi$ above is 0.9.
### Table 1
Standard Deviations of Output Gaps Under Different Policy Rules

<table>
<thead>
<tr>
<th>Stochastic Intercept</th>
<th>Standard Taylor</th>
<th>Noise Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>std($y - y^f$)</td>
<td>std($y - y^f$)</td>
<td>δ = 0.0</td>
</tr>
<tr>
<td>std(η)</td>
<td>std(η)</td>
<td>δ = 0.4</td>
</tr>
<tr>
<td>std(η)</td>
<td>std(η)</td>
<td>δ = 0.8</td>
</tr>
</tbody>
</table>

- \( \gamma = 0.95 \): 0.6331 0.6352 0.0770 0.0567 0.0407
- \( \gamma = 0.75 \): 0.6331 0.7053 0.4654 0.3430 0.2455
- \( \gamma = 0.50 \): 0.6331 0.8062 0.7474 0.5508 0.3942
- \( \gamma = 0.10 \): 0.6331 0.8996 0.9570 0.7052 0.9136

The table shows the analytical standard deviation of the output gap under the stochastic intercept and standard Taylor rules for different values of the persistence of the flexible price equilibrium level of output (\( \gamma \)). The last three columns show the required standard deviation of noise in the observation of the natural rate of interest for different persistence terms (\( \delta \)) for the standard Taylor rule to welfare dominate the stochastic intercept rule.

### Table 2
Standard Deviation of Wicksellian Natural Rate

<table>
<thead>
<tr>
<th>std($r^f_t$)</th>
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- \( \gamma = 0.95 \): 0.1601
- \( \gamma = 0.75 \): 0.3780
- \( \gamma = 0.50 \): 0.5774
- \( \gamma = 0.10 \): 0.9041
Figure 6
Impulse Responses of Output and Inflation from Bivariate VAR

The above are impulse responses from a bivariate VAR featuring real GDP growth and inflation. The shocks are identified by a long run restriction that only supply shocks may lead to a long run response of output. The dashed lines are 90 percent bias-corrected bootstrap confidence bands.