Taylor Rules and Technology Shocks

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Abstract

In a standard New Keynesian model, a Taylor-type interest rate rule moves the equilibrium real interest rate in the same direction as the "natural" rate of interest, but less than one-for-one. This means that monetary policy is relatively expansionary in response to shocks which raise the natural rate and contractionary following shocks which lower the natural rate. Permanent increases in productivity are likely to temporarily raise the natural rate of interest. This means that a Taylor rule following central bank exerts an expansionary effect in response to a permanent technological improvement – output and hours rise by more than they would if prices were flexible.

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1 Introduction

The New Keynesian model has become a workhorse for analyzing monetary policy and economic fluctuations. Most variants include a description of central bank behavior in which interest rates are set according to simple rules, referred to as "Taylor Rules" after Taylor (1993). The basic Taylor rule calls for central banks to adjust nominal rates more than one for one to deviations of inflation from target and positively to deviations of output from potential. It is thus widely believed and frequently asserted that Taylor rules embody a "lean against the wind" feature – the central bank figuratively puts its foot on the brake whenever the economy begins to soar but hits the gas pedal when things start to sour.

Interpreting how Taylor rules work in general equilibrium is more difficult than the simple "lean against the wind" intuition, however. This is because the Taylor rule calls for interest rates to respond to endogenous variables – inflation and the output gap. Because movements in these variables are all simultaneously determined in equilibrium, it is not a well-defined thought experiment to ask, for example: "what does the central bank do if the output gap rises, holding inflation fixed?"

In the context of the textbook version of the New Keynesian model augmented with a Taylor rule, I derive an analytical expression that relates the equilibrium real interest rate to the "natural rate of interest," an object which can be treated as effectively exogenous within the model. The natural rate is the real interest rate that would obtain in the absence of price rigidity. The linear mapping from the natural rate to the equilibrium real rate has a coefficient that is bound between zero and one. This means that, in equilibrium, a Taylor rule moves real rates in the same direction as the natural rate, but less than one-for-one. In response to shocks which raise the natural rate, monetary policy exerts an expansionary effect relative to the flexible price equilibrium – it raises interest rates but not by enough, so the real interest rate is too low. In contrast, following shocks which lower the natural rate, monetary policy is relatively contractionary – it lowers interest rates but not by enough, so the real rate is too high.

I apply the intuition of this result to study the effects of technology shocks in the model. Many VAR studies find that positive, permanent technology shocks lower hours worked, and take this finding as empirical support for New Keynesian models with price-stickiness (e.g., Gali, 1999). This conclusion is inconsistent with the implications of the basic model augmented with a Taylor rule. Permanent positive technology shocks likely raise the natural rate of interest; thus monetary policy guided by a Taylor rule is overly accommodative. Conditional on such a shock, hours and output rise by more than they would if prices were flexible, and the amount by which they do so is increasing in the amount of price rigidity.

2 The Basic NK Model

The NK model is a fully general equilibrium model derived from first principles. For a formal derivation see Woodford (2003) or Gali (2008). Under standard assumptions on preferences and production, the log-linearized equilibrium conditions are:[1]

$$-y_t = -E_t y_{t+1} + r_t \tag{1}$$

$$\pi_t = \gamma\left(y_t - y_t^f\right) + \beta E_t \pi_{t+1} \tag{2}$$

$$i_t = \phi_\pi \pi_t + \phi_y \left(y_t - y_t^f\right) \tag{3}$$

$$r_t = i_t - E_t \pi_{t+1} \tag{4}$$

$$y_t^f = a_t \tag{5}$$

(1) is the intertemporal Euler equation and has the flavor of an IS relation; (2) is the Phillips Curve; (3) is the Taylor rule; (4) is the Fisher relationship; and (5) says that flexible price output is equal to the exogenous level of productivity, a_t.[2] β is a discount factor, γ reflects the amount of price stickiness (the bigger is γ, the more flexible are prices), $\phi_\pi > 1$, and $\phi_y > 0$. The natural rate is the real rate that would obtain with flexible prices. Combine (1) with (5) to get: $r_t^f = E_t a_{t+1} - a_t$. In other words, the natural rate is the expected growth rate of productivity.

Suppose that a follows in AR(1) in first differences, abstracting from trend growth:

$$\Delta a_t = \rho \Delta a_{t-1} + \varepsilon_t, \ \ 0 \leq \rho < 1 \tag{6}$$

ε_t is a shock. Given the non-stationarity in a_t, output inherits a stochastic trend and must be re-scaled in order to solve the model. Define $\widehat{y}_t \equiv y_t - a_t$. Recalling that $r_t^f = E_t \Delta a_{t+1}$, the model can be re-written:

$$-\widehat{y}_t = -E_t \widehat{y}_{t+1} + \left(r_t - r_t^f\right) \tag{7}$$

$$\pi_t = \gamma \widehat{y}_t + \beta E_t \pi_{t+1} \tag{8}$$

$$r_t = i_t - E_t \pi_{t+1} \tag{9}$$

$$i_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t \tag{10}$$

$$r_t^f = \rho r_{t-1}^f + \rho \varepsilon_t \tag{11}$$

(11) follows from combining the definition of r_t^f with the process for Δa_t, and means that the natural rate of interest can effectively be taken to be exogenous.

Since the effects of nominal rigidities eventually vanish, $\lim_{T\to\infty} \ E_t \widehat{y}_{t+T} = 0$, and (7) can be solved forward for the output gap as a function of current and future gaps between the actual and natural rates of interest:

[1] Flow utility is assumed to be: $u(C_t, N_t) = \ln C_t - \psi \frac{N_t^{1-1/\eta}}{1-1/\eta}$ and the production technology is $y_t = A_t N_t$. A unitary elasticity of intertemporal substitution is necessary for balanced growth with separability between consumption and labor. These assumptions are inessential for the analysis to follow.

[2] Given that preferences are assumed consistent with balanced growth and the model has no capital, hours are constant in the flexible price equilibrium.

$$\widehat{y}_t = -\sum_{j=0}^{\infty} E_t \left(r_{t+j} - r_{t+j}^f \right) \tag{12}$$

Since the central bank indirectly controls r_t through its rule for i_t, monetary policy can be understood to be "expansionary" relative to the flexible price case if interest rate gaps are negative and relatively "contractionary" if interest rate gaps are positive.

The model features one state variable, r_t^f, two forward-looking variables, π_t and $\widehat{y}_t$, and two static variables, r_t and i_t. The static variables can be eliminated, leaving a three equation system. The solution is a pair of policy functions mapping the state variable into the jump variables. Using the method of undetermined coefficients, these policy functions can be solved analytically:

$$\pi_t = \left(\frac{\gamma}{\gamma \left(\phi_\pi - \frac{1}{\beta} \right) + (1 - \beta\rho) \left(1 - \rho + \phi_y + \frac{\gamma}{\beta} \right)} \right) r_t^f \tag{13}$$

$$\widehat{y}_t = \left(\frac{1 - \beta\rho}{\gamma \left(\phi_\pi - \frac{1}{\beta} \right) + (1 - \beta\rho) \left(1 - \rho + \phi_y + \frac{\gamma}{\beta} \right)} \right) r_t^f \tag{14}$$

Given these policy functions, the equilibrium real interest rate is:

$$r_t = \left(\frac{\gamma \left(\phi_\pi - \rho \right) + (1 - \beta\rho)\phi_y}{\gamma \left(\phi_\pi - \rho \right) + (1 - \beta\rho) \left(\phi_y + 1 - \rho \right)} \right) r_t^f \tag{15}$$

Since $0 \leq \rho < 1$, the coefficient on the natural rate of interest is positive but less than one. Hence the Taylor rule moves the real interest rate in the same direction as the natural rate, but less than one-for-one. Monetary policy is thus relatively expansionary in response to increases in the natural rate (the interest gap is negative), and relatively contractionary when the natural rate falls. As $\gamma \to \infty$, which occurs as prices become flexible, the coefficient on the natural rate goes to one, so the actual and natural rates of interest are always the same. Similarly, as either of the Taylor rule coefficients get arbitrarily large the coefficient on the natural rate also goes to one. When productivity follows an exact random walk, with $\rho = 0$, the natural rate of interest is constant, and hence the equilibrium real rate is always equal to the natural rate.

3 Technology Shocks in the NK Model

In the model above, if $\rho > 0$ permanent positive technology shocks raise the natural rate of interest. The natural rate is equal to the expected growth rate of technology; given autocorrelation in the growth rate, a positive shock today portends higher growth into the future, and hence pushes up the natural rate.

From the analysis in the previous section, equilibrium real rates move in the same direction as the natural rate, but less than one for one. Hence, monetary policy is relatively expansionary following a positive technology shock, as it raises the actual real rate by less than the natural rate of interest. This means that output and hours will both expand by more than they otherwise would if prices were fully flexible. In the basic model hours do not respond to technology shocks if prices are flexible.[3] Hence, with price stickiness, hours should rise following a positive, permanent technology shock. Furthermore, this positive effect should be larger the more rigid are prices. As γ decreases (price rigidity increases), the coefficient on the natural rate in (15) becomes smaller, exacerbating the extent to which monetary policy is expansionary or contractionary.

To make this point quantitatively, I consider a fairly standard parameterization. $\beta = 0.99$. The parameters of the Taylor are in-line with many empirical estimates: $\phi_\pi = 1.5$ and $\phi_y = 0.5$. I set the persistence of the growth rate of productivity to $\rho = 0.5$; this is more for the point of illustration than it is meant to match the data. Figure 1 shows impulse responses to a one percent shock to the productivity growth rate for different levels of price rigidity. The coefficient in the Phillips Curve is related to deeper parameters of the model by: $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}(1+1/\eta)$, where $\phi \in (0,1)$ is the probability a firm is stuck with a pre-existing price in any period and η is the Frisch labor supply elasticity. Low values of ϕ correspond to more flexible prices and hence higher values of γ. I set $\eta = 1$ and consider three different values of ϕ: 0.2, 0.5, and 0.8.

The hours response to the technology shock is always positive and is increasing in the degree of nominal rigidity.[4] This is driven by the fact that the interest rate gap falls when productivity improves – while both the natural and actual real rates rise, the actual real rate rises by less than the natural rate. This gap is wider the larger is ϕ. Figure 2 shows the impact responses of hours and the interest rate gap to the same shock for values of ϕ ranging from 0.01 (flexible prices) to 0.99 (very rigid prices). The impact increase in hours is increasing in the degree of nominal rigidity, with the impact decline in the interest rate gap roughly the mirror image.

Many textbook treatments present the model where technology follows a stationary AR(1), e.g. $a_t = \rho a_{t-1} + \varepsilon_t,\ \rho < 1$. In such a case, it is straightforward to show that the equilibrium conditions in (7)-(11) only need to be modified such that the process for the natural rate is given by: $r_t^f = \rho r_{t-1}^f + (\rho - 1)\varepsilon_t$. With $\rho < 1$, positive technology shocks lead to declines in the natural rate of interest – the natural rate in the model equals the expected growth rate of technology, and positive stationary technology shocks portend persistent negative growth rates. With a decline in the natural rate the equilibrium real rate falls but not by enough, so that monetary policy is relatively contractionary. Hence, output and hours should rise by less than they otherwise would if prices were flexible.

Figure 3 shows impulse responses to a stationary technology shock for different values of ϕ, with $\rho = 0.9$. As suggested by the analytical solution, the interest rate gap rises after a temporary technology shocks and hours decline, with both magnitudes increasing in the amount of price rigidity. These effects would likely be reversed if the model featured capital, because the real rate would be tied to the marginal product, which rises with productivity increases.

[3] This is a consequence of preferences consistent with balanced growth and no capital. With capital hours would rise temporarily.

[4] Given a linear production technology (see Footnote 1), the hours response is just the output gap response: $n_t = y_t - a_t \equiv \widehat{y}_t$.

4 Conclusion

In the NK model with a Taylor rule the equilibrium real interest rate moves in the same direction as the natural rate, but less than one-for-one. This means that monetary policy is relatively expansionary in response to shocks which raise the natural rate and contractionary following shocks which lower the natural rate. Although derived in a simple framework, this result obtains quantitatively in extensions of the model with interest smoothing, capital, and other real rigidities.

Since permanent technology shocks likely raise the natural rate, empirical findings of positive technology shocks causing hours to fall cannot be used as evidence in support of NK models per se. Rather, such findings likely point to important real rigidities (such as habit formation or investment adjustment costs; see Francis and Ramey, 2008), which may cause hours to contract following technological improvement even when prices are flexible.

References

[1] Francis, N., Ramey, V., 2005. Is the techology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. Journal of Monetary Economics 52(8), 1379-1399.

[2] Gali, J., 1999. Technology, employment, and the business bycle: do technology shocks explain aggregate fluctuations? American Economic Review 89(1), 249-271.

[3] Gali, J., 2008. Monetary Policy, Inflation, and the Business Cycle. Princeton: Princeton University Press.

[4] Taylor, J., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[5] Woodford, M., 2003. Interest and Prices. Princeton: Princeton University Press.

Figure 1: IRFs to Permanent Technology Shock

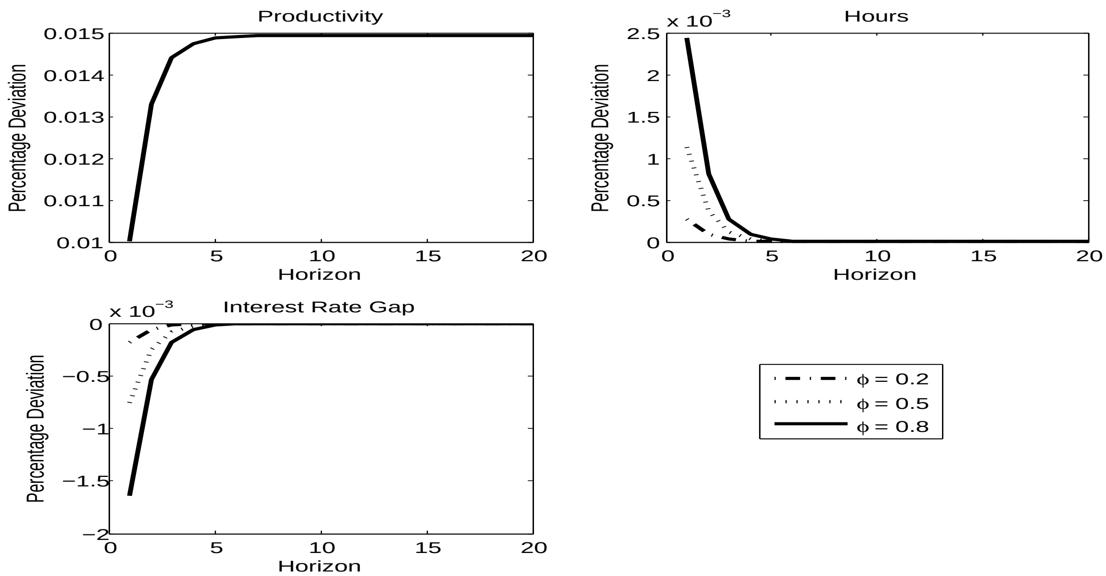

Figure 2: Impact Effects

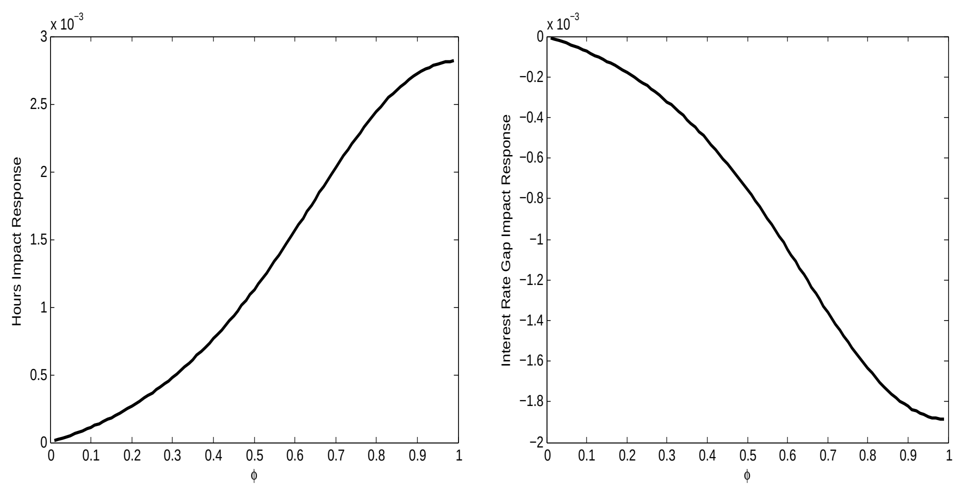

Figure 3: IRFs to Stationary Technology Shock