The Zero Lower Bound

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1 Introduction

In the standard New Keynesian model, monetary policy is often described by an interest rate rule (e.g. a Taylor rule) that moves the interest rate in response to deviations of inflation and some measure of economic activity from target. Nominal interest rates are bound from below by 0 – since money is storable, one would never accept a negative nominal return. How does the behavior of the NK model change when interest rates hit zero and cannot freely adjust in response to changing economic conditions?

To answer this question, we consider the implications of an interest rate peg in the model. This isn’t literally what happens at the zero lower bound, but what matters in the model is not that the interest rate is zero per se, but rather that it becomes unresponsive to economic conditions. In the experiments I consider, the nominal interest rate is pegged at a fixed value for a finite (and deterministic) period of time. After the peg, monetary policy obeys a simple Taylor rule. It turns out to be relatively straightforward to modify a Dynare code to take this into account. I include a government spending shock in the model so that we can analyze the effects of government spending shocks at the zero lower bound, which has been a topic of much recent interest.

The interest rate peg ends up exacerbating the effects of price stickiness. In particular, output responds even less to a positive supply shock (productivity) and more to “demand” shocks (government spending) than under a standard Taylor rule. This operates through an inflation channel and the Fisher relationship: positive supply shocks lower inflation, which raises real interest rates if nominal rates are unresponsive, with the reverse holding for a demand shock like government spending.

2 The Model

The model is a standard New Keynesian model with price stickiness. To make things a little more interesting, as well as to tie into some of the recent literature, I included government spending in the model.

I abstract from money altogether. Monetary policy is characterized by an interest rate rule. The money supply is determined passively in the background so as to equate demand and supply
of real balances at the central bank’s target interest rate. The full set of equilibrium conditions are given by:

\[ C_t^{-\sigma} = \beta E_t C_{t+1}^{-(1 + \sigma)}(1 + i_t)(1 + \pi_{t+1})^{-1} \]  \hfill (1)

\[ \psi N_t^n = C_t^{\sigma} w_t \]  \hfill (2)

\[ mc_t = \frac{w_t}{A_t} \]  \hfill (3)

\[ Y_t = C_t + G_t \]  \hfill (4)

\[ Y_t = \frac{A_t N_t}{v_t^p} \]  \hfill (5)

\[ v_t^p = (1 - \phi)(1 + \pi_t^#)^{-\epsilon}(1 + \pi_t) + (1 + \pi_t)^\epsilon \phi v_{t-1}^p \]  \hfill (6)

\[ (1 + \pi_t)^{1 - \epsilon} = (1 - \phi)(1 + \pi_t^#)^{1 - \epsilon} + \phi \]  \hfill (7)

\[ 1 + \pi_t^# = \frac{\epsilon}{\epsilon - 1}(1 + \pi_t^#) \frac{x_{1,t}}{x_{2,t}} \]  \hfill (8)

\[ x_{1,t} = C_t^{-\sigma} mcY_t + \phi \beta E_t (1 + \pi_{t+1}) \epsilon x_{1,t+1} \]  \hfill (9)

\[ x_{2,t} = C_t^{-\sigma} Y_t + \phi \beta E_t (1 + \pi_{t+1}) \epsilon x_{2,t+1} \]  \hfill (10)

\[ \ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \]  \hfill (11)

\[ \ln G_t = (1 - \rho_g) \ln G^* + \rho_g \ln G_{t-1} + \varepsilon_{g,t} \]  \hfill (12)

\[ i_t = i^* + \phi_{\pi}(\pi_t - \pi^*) \]  \hfill (13)

\( G^* \) is an exogenously chosen steady state value of government spending, and \( i^* = \frac{1}{\beta}(1 + \pi^*) - 1 \) is the steady state nominal interest rate. This is a standard interest rate rule and model. I use the following parameter values in quantitative solutions of the model: \( \sigma = 1, \ \psi = 1, \ \eta = 1, \ \epsilon = 10, \ \phi = 0.75, \ \beta = 0.99, \ \rho_a = 0.95, \ \rho_g = 0.95, \ \pi^* = 0, \ \phi_{\pi} = 1.5, \) and \( G^* = 0.2. \) This parameterization ends up implying that government spending is about 20 percent of steady state output. I set the standard deviations of both the government spending and productivity shocks to 0.01. I abstract from a smoothing term in the interest rate rule and do not model an explicit monetary policy shock.

### 2.1 An Interest Rate Peg

The zero lower bound refers to the fact that nominal interest rates cannot be negative (whereas real rates can). The argument for why this is the case is fairly straightforward, though without money explicitly in the model is not terribly transparent. The nominal interest rate tells you the dollar return on foregoing one dollar’s worth of current consumption, whereas the real interest rate tells you the consumption return on forgoing one unit of current consumption. If consumption goods are not storable, then you may be willing to accept a negative real return – giving up 1 unit of fruit today for 0.9 fruits tomorrow isn’t a great deal, but if your outside option is zero fruit
tomorrow, you may be willing to take this. But since money is a store of value, one would never take a negative return on money – you could simply hold your wealth in money between periods and have as many dollars tomorrow as you saved today. Hence, no one would ever take a negative nominal interest rate.

What is relevant in this model is not whether the nominal interest rate is zero or positive per se, but rather whether the nominal interest rate is responsive to changing economic conditions. Suppose that a central bank desired a negative nominal interest rate of 2 percent, given conditions. Then, given reasonably sized movements in inflation, the central bank would keep the nominal interest rate fixed at zero, until such a time as its desired interest rate (given its policy rule) is positive, after which time it would go back to its standard rule.

A simple way for us to model this behavior that requires only minor modification of our model and codes is to not worry about the zero lower bound per se, but rather to model an interest rate peg. A peg means that the interest rate is fixed for a known duration of time, after which time it follows the standard Taylor rule. This will simulate the situation described above of the Fed desiring a very negative nominal interest rate, and therefore keeping the actual nominal rate fixed for a period of time.

Formally, the policy rule under the peg is:

\[ i_{t+h} = i_{t-1}, \quad \text{for } h = 0, 1, \ldots, H - 1 \]  
\[ i_{t+s} = i^* + \phi \pi (\pi_{t+s} - \pi^*) \quad \text{for } j \geq H \]  

In other words, the nominal interest rate is fixed for the current and subsequent \( H - 1 \) periods (for a total of \( H \) periods, hence \( H \) is the length of the peg). Starting in period \( H \), the interest rate follows the usual Taylor rule and is expected to do so thereafter. The determinacy properties of the policy rule do not depend on the length of the peg, so long as it is finite, but rather only how interest rates are set after the peg ends (e.g. on \( \phi \pi \)).

It is reasonably straightforward to introduce this into Dynare. To do so, you simply need to create some auxiliary state variables. Suppose I want a four period peg, \( H = 4 \). I would introduce four new state variables, call them \( S1, S2, S3, \) and \( S4 \). I would write my code:

\[ i_t = s1(t - 1) \]  
\[ s1(t) = s2(t - 1) \]  
\[ s2(t) = s3(t - 1) \]  
\[ s3(t) = s4(t - 1) \]  
\[ s4(t) = i^* + \phi \pi (\pi_{t+4} - \pi^*) \]

\( s4(t) \) describes how the interest rate will be set in the fourth period from now (period \( t + 4 \), where we take period \( t \) to be the present. In period \( t + 1 \), \( S3 \) will equal this; in period \( t + 2 \), \( S2 \)
will equal this; in period $t + 3$, $S_1$ will equal this. Iterating forward, from the first line we then see that $i_{t+4} = s4(t) = i^* + \phi_\pi(\pi_{t+4} - \pi^*)$, which is just the standard Taylor rule. Furthermore, given this setup we will also have $i_{t+k} = i^* + \phi_\pi(\pi_{t+k} - \pi^*)$ for all $k \geq 4$.

3 Analysis

I solve the model in Dynare using a first order approximation under three different policy scenarios: the standard Taylor rule, a $H = 4$ period interest rate peg, and a $H = 8$ period interest rate peg. Below I plot impulse responses first to a productivity shock and then to the government spending shock under the different policy rules. The solid lines show the responses under a standard Taylor rule, the dashed lines under a four period peg, and the dotted lines under an eight period peg (the periods can be interpreted as quarters).
We see something pretty interesting here. In particular, the interest rate peg *exacerbates* the effects of price stickiness. For the case of the productivity shock, under a standard Taylor rule with sticky prices output rises by less than it would if prices were flexible. The longer the interest rate is pegged, the less output rises. For the case of the government spending shock, output would rise more with sticky prices under a standard Taylor rule than if prices were flexible. The longer the interest rate is pegged, the more output reacts. For both the productivity and government spending shocks, inflation also reacts more the longer is the peg – it falls by more in the case of the productivity shock, and rises by more in the case of the government spending shock. We clearly see how the nominal interest rate is fixed for the specified number of periods. After the peg is over, the IRFs lie on top of one another – this occurs because there is no endogenous state variable in the model, so once the peg is over it is irrelevant that the peg was ever in place in the first place.

What’s going on here is fairly simple and operates through the real interest rate in conjunction with the Fisher relationship. The positive productivity shock is a “supply shock” that results in lower inflation. When the central bank follows a Taylor rule, it reacts to lower inflation by lowering the nominal interest rate, which prevents the real interest rate from rising by much (indeed, in the baseline scenario here under a Taylor rule it falls). But when the nominal interest rate is fixed, falling inflation means a higher real interest rate. This higher real interest rate works to “choke off” demand. The longer the interest rate is pegged, the more inflation initially falls. And with a fixed nominal rate, this translates into a bigger increase in the real interest rate and an even more contractionary effect on output.

We see the reverse pattern in the case of a government spending shock. This is a kind of “demand” shock that leads to an increase in inflation. Under a standard Taylor rule, higher inflation means a higher nominal interest rate, which translates into a higher real interest rate. But if the interest rate is fixed, higher inflation has the opposite effect on the real interest rate:
higher inflation means a lower real interest rate, which is stimulative. The longer the peg, the more inflation increases, which means the real interest rate falls by more, translating into an even bigger effect on output.

We can quantify the effects of the government spending shock via the “multiplier”: \( \frac{dY_t}{dG_t} \). To do this, simply calculate the ratio of the impact impulse responses of output and government spending, and multiply by the steady state ratio of output to government spending to put it in “dollar form” (this is because the impulse responses are in percentage terms, and the multiplier is usually expressed in levels, not log deviations). Under a standard Taylor rule, the multiplier under my parameterization is 0.60. In other words, output rises by less than government spending, meaning that consumption gets “crowded out.” The crowding out of consumption results because of the increase in the real interest rate. Under the interest rate peg, in contrast, the multiplier under a four period peg is 1.08, and under an 8 period peg is 2.31. The reason why the multiplier is bigger than one under the peg is because the government spending shock results in a decrease in the real interest rate under the peg, rather than an increase, which “crowds in” consumption. It is based on simple logic like this that many commentators have argued for more fiscal stimulus in our current low interest rate environment.

4 Policy Implications

To the extent to which a central bank doesn’t like output gaps (output being different from its natural, or flexible price, level) or inflation, it’s pretty clear that not being able to adjust the interest rate to shocks is a bad thing. The interest rate being pegged exacerbates the role of price stickiness: the economy responds even less to a supply shock, and even more to a demand shock, than if prices were flexible, the more so the longer is the peg. Inflation also reacts by more. Clearly, a central bank operating with an interest rate rule would like to avoid the situation in which it is unable to adjust the nominal interest rate.

Hence, central banks would clearly like to avoid the incidence of nominal interest rates hitting the zero lower bound. The most straightforward way to do this is to have a higher inflation target. The steady state nominal interest rate is \( 1 + i^* = \frac{1}{\beta} (1 + \pi^*) \). The central bank can’t control \( \beta \), but it can raise \( \pi^* \). Raising \( \pi^* \) raises \( i^* \), and if nominal interest rates fluctuate about a higher steady state level, it is natural that the incidence of hitting zero will be lower. So should central banks raise inflation targets? It’s not so obvious, and there are tradeoffs. First, raising inflation targets implies larger and larger deviations from the Friedman Rule, which would have deflation along trend and zero interest rates. To the extent to which the Friedman rule is optimal in a flexible price model, then a positive inflation target implies a larger steady state distortion. Secondly, in the sticky price model, positive trend inflation induces positive steady state price dispersion, which is strictly welfare reducing. Positive trend inflation also makes price dispersion a first order (as opposed to second order) phenomenon, so not only will positive trend inflation lead to a steady state distortion, it will result in an added distortion in terms of the dynamics of price dispersion about the steady state.
Coibion, Gorodnichenko, and Wieland (2012, *Review of Economic Studies*) study the question of the optimal inflation target in light of the zero lower bound in a standard New Keynesian model. In spite of the fact that hitting the zero lower bound is very costly, it occurs rarely enough in these models that it is not worth the cost of high positive trend inflation. They argue that the optimal level of trend inflation is typically 2 percent per year or less, which is remarkably close to recent US experience.