Consumption, Dividends, and the Cross-Section of Equity Returns

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Abstract

A central economic idea is that an asset’s risk premium is determined by its ability to insure against fluctuations in consumption (i.e., by the consumption beta). Cross-sectional differences in consumption betas mirror differences in the exposure of the asset’s dividends to aggregate consumption, an implication of many general equilibrium models. Hence, cross-sectional differences in the exposure of dividends to consumption may provide valuable information regarding the cross-sectional dispersion in risk premia. We measure the exposure of dividends to consumption (labeled as consumption leverage) by the covariance of ex-post dividend growth rates with the expected consumption growth rate, and alternatively by relying on stochastic cointegration between dividends and consumption. Cross-sectional differences in this consumption leverage parameter can explain about 50% of the variation in risk premia across 30 portfolios— which include 10 momentum, 10 size, and 10 book-to-market sorted portfolios. The consumption leverage model can justify much of the observed value, momentum, and size risk premium spreads. For this asset menu, alternative models proposed in the literature (including time varying beta models) have considerable difficulty in justifying the cross-sectional dispersion in the risk premia. Our measures of consumption leverage are driven by the exposure of dividend growth rates to low frequency movements in consumption growth. We document that it is this exposure that contains valuable information regarding the cross-sectional differences in risk premia across assets.
1 Introduction

The idea that differences in exposure to systematic risk should justify differences in risk premia across assets is central to asset pricing. The static CAPM (see Sharpe (1964), Lintner (1965)) implies that assets’ exposures to aggregate wealth should determine cross-sectional differences in risk premia. The work of Lucas (1978) and Breeden (1979) argues that the risk premium on an asset is determined by its ability to insure against consumption fluctuations. Hence, the exposure of asset returns to movements in aggregate consumption (i.e., the consumption betas) should determine cross-sectional differences in risk premia. Evidence presented in Hansen and Singleton (1982, 1983) for the consumption based models, and in Fama and French (1992) for the CAPM, shows that these models have considerable difficulty in justifying the differences in rates of return across assets. Consequently, identifying economic sources of risks that justify differences in the measured risk premia across assets continues to be an important economic issue.

An implication of general equilibrium models is that consumption betas are determined by preference parameters and the exposure of dividends to consumption. In particular, Abel (1999), Campbell (2000), and Bansal and Yaron (2000) present economic models where differences in consumption betas in the cross-section of assets mirrors differences in the exposure of the asset’s dividends to consumption. Dividend flows which have larger consumption exposure (we label this consumption leverage) have a larger consumption beta, and consequently also carry a higher risk premium. Using data on consumption and dividends, we directly measure the consumption leverage of 30 asset portfolios: 10 size, 10 book-to-market, and 10 momentum sorted portfolios. We show that the cross-sectional dispersion in measured consumption leverage explains approximately 50% of the cross-sectional variation in observed risk premia. Further, the estimated market price of consumption risk is sizable and positive in all cases. Our estimated model can duplicate much of the spread in the mean returns of the extreme momentum portfolios (winner minus loser), the size spread (small capitalization minus large), and the value spread (high book-to-market minus low). For the same collection of assets, the benchmark Fama and French (1993) three factor model explains about 25% of the cross-sectional differences in the risk premia.

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1 A voluminous literature over the past 20 years documents the difficulties of asset pricing models in explaining cross-sectional differences in measured risk premia. For an extensive recent survey see Campbell (2000).

2 This is not surprising; Fama and French (1993) document that the dispersion in mean returns of mo-
We measure the exposure of a portfolio’s dividend stream to consumption (consumption leverage) in the time-series in two different ways. First, we measure consumption leverage by the regression coefficient (OLS slope coefficient) of the future ex-post dividend growth rate on a moving average of past consumption growth rates. In the second approach, we measure consumption leverage by relying on stochastic cointegration (for this notion of cointegration see Campbell and Perron (1991) and Ogaki and Park (1998)). The stochastic cointegration regression entails a time series regression of the log level of the dividends on a constant, a deterministic time trend, and the log level of consumption; the parameter estimate on the log level of aggregate consumption is taken as the measure of consumption leverage. As further discussed below, our two approaches of measuring consumption leverage focus on measuring the exposure of dividends to a small, persistent (low frequency) component in the consumption process.

We focus on size, book-market, and momentum sorted portfolios as the test assets. These assets form the basis of common risk factors used to explain differences in risk premia of other assets (see Fama and French (1993) and Carhart (1997)). Further, the dispersion in cross-sectional mean returns of these 30 assets are particularly challenging for many benchmark asset pricing models. In our empirical work we also compare our model to alternative models proposed in the literature. In particular, we report results for the three factor Fama-French model, the static CAPM, the C-CAPM, the time-varying beta version of the CAPM (Jagannathan and Wang (1996)) and the consumption based C-CAPM considered in Lettau and Ludvigson (2001b). As stated above, our consumption leverage developed in the paper can capture approximately 50% of the cross-sectional variation in risk premia. In general, betas associated with unconditional factor models cannot explain the cross-sectional variation in observed risk premia. For example, the constant beta versions of the static CAPM and the C-CAPM can capture no more than 5% of the cross-sectional dispersion. In many cases, the premium associated with the risk factor is negative. The time varying beta versions of the CAPM and C-CAPM can account for no more than 10% of the cross-sectional variation in measured risk premia. In contrast to the multifactor and time-varying beta models, our consumption leverage model \textit{a priori} restricts the price of risk to be positive. In addition, our specification provides a mapping between the cash-flow dynamics, the risk sources, and the betas.

Our approach to measuring consumption leverage is motivated by arguments presented

\textit{momentum assets is particularly challenging for the their three factor specification.
in Hall (2001), Barsky and DeLong (1993), and Bansal and Lundblad (2002). These authors argue that small persistent predictable variation in aggregate economic growth rates are important to understand asset price fluctuations. In all, our empirical evidence suggests that exposure to “low-frequency” components in consumption growth rates may indeed be an important source of systematic risk. Dividends of different assets have different exposures to this non-diversifiable source of risk; quantifiable differences in this exposure determine the consumption leverage and consequently the cross-section of risk premia. It is important to note (we provide this evidence) that measures of consumption leverage based on regressing dividend growth rates on contemporaneous consumption growth rates capture little to none of the cross-sectional differences in risk premia. Hence, it seems that valuable information regarding mean rates of return on different assets is encoded in the exposure of dividends to low frequency movements in aggregate consumption.

Section 2 provides the solution for the consumption leverage model, as well as an analytical expression for the fundamental consumption beta (risk exposure). Section 3 provides data description and empirical evidence for the degree of consumption leverage between portfolio dividends and aggregate consumption. Section 4 details the ability of the consumption leverage model to explain cross-sectional variation in risk premia in comparison to standard factor and consumption based models. Finally, Section 5 concludes.

2 Modeling Asset Returns

In this section, we briefly present a general equilibrium model which relates an asset’s consumption beta, and the hence the asset’s risk premium, to the exposure of dividends to aggregate consumption. In this model, dividend yields are not constant and fluctuate due to predictable variation in aggregate consumption growth along with fluctuations in asset specific predictable variation in dividends. The model is adapted from Bansal and Yaron (2000). It is important to note that the broad economic implication that consumption betas are endogenous and mirror the differences in the consumption exposure of dividends is not unique to the model presented below—Abel (1999) presents a habit based model where the same holds true.

For the preferences developed in Epstein and Zin (1989), the Intertemporal Marginal
Rate of Substitution (IMRS) as derived in their paper is,

\[ M_{t+1} = \delta^\theta G_{t+1}^{\frac{\theta}{\psi}} (R_{c,t+1})^{-(1-\theta)}. \]  

(1)

\( G_{t+1} \) is the aggregate gross growth rate of consumption and \( R_{c,t+1} \) is the total return on an asset that pays off aggregate consumption each period. Further, \( \delta \) is a time preference parameter, \( \psi \) denotes the intertemporal elasticity of substitution, and \( \theta \equiv \frac{1-\gamma}{1-\frac{\psi}{\theta}} \), where \( \gamma \) represents the coefficient of relative risk aversion.

All asset returns in this endowment economy must satisfy the standard asset pricing condition that

\[ E_t[M_{t+1} R_{i,t+1}] = 1 \]  

(2)

The one step ahead innovation in the log of the IMRS (that is, \( \ln(M_{t+1}) - E_t[\ln(M_{t+1})] \)), is given by

\[ \eta_{M,t+1} = -\frac{\theta}{\psi} \eta_{t+1} - (1-\theta)\eta_{c,t+1} \]  

(3)

where \( \eta_{t+1} \) is the innovation in log aggregate consumption growth and \( \eta_{c,t+1} \) is the innovation in the log return \( r_{c,t+1} \) (note, \( r_{c,t+1} \equiv \ln(R_{c,t+1}) \)). Risk premia are determined by computing an asset’s return covariance with the innovation in equation (3). It is well recognized that \( r_{c,t+1} \) is endogenous to the model (see Cochrane and Hansen (1992), Campbell (2000)), and the innovation \( \eta_{c,t+1} \), as shown below, depends only on the consumption growth innovation, \( \eta_{t+1} \). Hence, all risk premia are determined by the assets’ exposures to the uncertainty in aggregate consumption.

### 2.1 Aggregate Consumption

We refer the log of aggregate consumption as \( c_t \) and assume that the log level of consumption follows an ARIMA(1,1,1) process (such a process for consumption is also considered in Campbell (2000)). We refer to \( \eta_t \) as the consumption news at time \( t \); as the aggregate consumption is modeled as an univariate process, this also represents economic uncertainty.

The growth rate of consumption, \( g_{t+1} = c_{t+1} - c_t \), can then be represented as an ARMA(1,1),

\[ g_{t+1} = \mu_c (1-\rho) + \rho g_t + \eta_{t+1} - \omega \eta_t \]  

(4)
where \( g_t \) is stationary, and consequently \( \rho \) and \( \omega \) are less than one in absolute value. The above growth rate process can be stated in terms of \( x_t \), a variable that determines the conditional mean of the consumption growth rate:

\[
x_t = (\rho - \omega) \frac{g_t}{(1 - \omega L)}
\]

(5)

\[
g_{t+1} = \mu_x + (x_t - \mu_x) + \eta_{t+1},
\]

(6)

where \( \mu_x = \mu_x \frac{(\rho - \omega)}{(1 - \omega)} \) is the unconditional mean of \( x \). Substituting equation (6) into (5), it follows that \( x_t \) evolves as an AR(1) process,

\[
x_{t+1} = (1 - \rho)\mu_x + \rho x_t + (\rho - \omega)\eta_t
\]

(7)

From equation (5), it is evident that \( x_t \) is proportional to an exponential weighted average of the past growth rates.\(^3\) When \( \rho \) and \( \omega \) are large (with \( \rho > \omega \)), then \( x_t \) is small in size and quite persistent. Barsky and DeLong (1993) and Hall (2001) argue that such small, persistent fluctuations in growth rates (the low frequency movements) are important for understanding asset price fluctuations.

### 2.2 Restrictions on the Risk Premia

Under this consumption growth specification, the innovation to the IMRS is as follows (see Appendix):

\[
-\eta_{M,t+1} = \left[-\frac{\theta}{\psi} - (1 - \theta)B_c\right] \eta_{t+1} \equiv B_M \eta_{t+1}
\]

(8)

where \( B_M = \left\{ \frac{\theta}{\psi} + (1 - \theta)[1 + \kappa_{c,1} \frac{1 - \frac{1}{\psi}}{(1 - \kappa_{c,1} \rho)}] \right\}^4 \). The risk premium on any asset, where the return and the IMRS are log-normally distributed, satisfy \( E_t[r_{i,t+1} - r_{f,t}] = -\text{var}_t(r_{i,t+1})/2 + \text{cov}_t(-\eta_{M,t+1}, r_{i,t+1}) \). The term \( \text{var}_t(r_{i,t+1})/2 \) is the Jensen’s adjustment, and shifting this term to the left hand side leads to the usual arithmetic risk premium restriction:

\[
E_t[R_{i,t+1} - R_{f,t}] = \beta_i [B_M \sigma_n^2]
\]

(9)

\(^3\)Using equation (5), it follows that \( x_t = (\rho - \omega) \frac{g_t}{(1 - \omega L)} = \frac{(\rho - \omega)}{(1 - \omega L)} g_t \), the second equality shows that \( x \) is proportional to an exponentially weighted average of the growth rate.

\(^4\)The parameter \( \kappa_{c,1} \) is an approximation constant that comes out of the Campbell-Shiller linearization of the log return and is typically very close to 1 (see Appendix).
where,

\[ \beta_i = \frac{\text{cov}(r_{i,t+1}, \eta_{t+1})}{\sigma^2_{\eta_{t+1}}} \]  

(10)

The risk premium on an asset that has an exposure of \( \eta_{t+1} \) to consumption innovation risk, equals \( B_M \sigma^2_{\eta} \); the market price of consumption risk. The consumption beta of the asset determines the risk premium on the asset.

It is important to note that the beta of an asset is not an exogenous variable, rather it is determined in equilibrium by the exposure of the underlying dividends to aggregate consumption risk. To show this, we discuss alternative specifications that relate dividends to consumption.

### 2.3 Consumption Leverage Model: Growth Rates

In the context of a model of habits, Abel (1999) argues that the risk premium on different assets can be viewed as a result of differences in their consumption leverage. He considers risk premia on assets where dividends (in logs) are expressed as \( d_{i,t} = \varphi_i c_t \); \( \varphi_i \) is the leverage of the asset. He shows that assets with larger \( \varphi \) also have larger risk premia.

The relationship between dividends and consumption can be modeled using a growth projection:

\[
d_{i,t+1} - d_{i,t} = g_{i,t+1} = \delta_i + \varphi_i x_t + \eta_{i,t+1}
\]

(11)

\[
\varphi_i = \frac{\text{cov}(g_{i,t+1}, x_t)}{\text{var}(x_t)}
\]

(12)

where \( x_t \) is the expected consumption growth rate and \( \varphi_i \) is a standard regression coefficient that measures the covariance between dividend growth and expected consumption growth.\(^5\) We show below that an asset’s beta critically depends on \( \varphi_i \), its exposure to the predictable variation in the expected consumption growth rate \( x_t \).

We first solve for the log price-dividend ratio of an asset with a dividend stream as in equation (12). Given the dividend growth rate process, it is shown in the appendix that an

\(^5\)We can assume that the asset specific shock, \( \eta_{i,t} \), is related to the aggregate consumption shock as follows, \( \eta_{i,t} = \tau_t \eta_t + u_{i,t} \).
respectively, where \( \bar{z}_i \) is the unconditional mean of \( z_{i,t} \), and \( A_{i,1} = \frac{\varphi_i - 1}{1 - \kappa_i \rho} \). The exposure of the ex-post return to the consumption shock is magnified by the term \( A_{i,1} \)—an asset with a large \( \varphi_i \) will have a larger degree of magnification, and consequently, the ex-post return will carry a larger compensation for consumption risk. The arithmetic risk premium on the asset will be determined by the risk premium expression (9), where the consumption beta of the asset is

\[
\beta_i = [\tau_i + \kappa_i A_{i,1}(\rho - \omega)]
\]

As can be seen, the beta of an asset is not exogenous. Rather, the asset’s risk is explicitly determined by the preference parameters and, critically, its dividend growth exposure to expected consumption growth. For simplification, we further assume that \( \tau_i = \tau \). Note that the dividend yield in this model is not constant, and reacts to changing expected consumption growth rates. It is also straightforward to show that the ex-post excess return on the asset depends on the news components of consumption and dividends and does not explicitly depend on \( x_t \).

### 2.4 Consumption Leverage: Cointegration

In this section, we model consumption and dividends as cointegrated processes. Log dividends, \( d_{i,t} \), and log consumption, \( c_t \) are related in the following manner:

\[
d_{i,t+1} = \mu_i + \delta_i \cdot (t + 1) + \phi_i c_{t+1} + \epsilon_{i,t+1}
\]

In equation (16), \( \phi_i \) describes the long-run stochastic relationship between consumption and dividends and is our alternative measure of consumption leverage. It is assumed that \( d_{i,t} \) and \( c_t \) are I(1), but stationary departures from this relationship, \( \epsilon_{i,t} \), are I(0). This specification, as in Campbell and Perron (1991) and Ogaki and Park (1998), implies that asset-specific dividends and aggregate consumption are stochastically cointegrated. That is, \( \phi_i \) measures the exposure of the stochastic trend in dividends to the stochastic trend in consumption (see
Campbell and Perron (1991)). Note that with this notion of cointegration $d_{i,t} - \phi_i c_t$ can only contain a deterministic trend (i.e., no stochastic trend) if $\delta_i$ is different from zero. The stochastic cointegration parameter $\phi_i$ can also be measured by first removing a deterministic time trend from the level of both $c_t$ and $d_{i,t}$ and then utilizing the resulting detrended series. Indeed, in practice we measure the $\phi_i$’s by deterministically detrending all the dividend and consumption series, and regressing the detrended dividends on a constant and the detrended consumption. This procedure ensures that our estimated measure of consumption leverage $\phi_i$, is not driven by deterministic trends in consumption and dividends.  

Using stochastic cointegration to measure consumption leverage is particularly valuable when one considers the possibility that both consumption and dividends are measured with error. Asymptotically, the presence of such measurement errors will not affect the estimates of the stochastic cointegration parameter, $\phi_i$. Taking the first difference of equation (16), and substituting the assumed consumption growth rate process (6), it follows that

$$g_{i,t+1} = \delta_i + \phi_i x_t + \phi_i \eta_{i,t+1} + \epsilon_{i,t+1} - \epsilon_i$$  (17)

Using equation (17), it is shown in the appendix that the consumption beta of the asset is

$$\beta_i = [\phi_i + \kappa_{i,1} A_{i,1} (\rho - \omega)]$$  (18)

Hence, as in the growth rate specification, the asset’s beta reflect its consumption leverage.

### 2.4.1 Cross-Sectional Restrictions

The asset’s consumption beta, in conjunction with the restriction on the asset risk premium (see equation (9)), leads to cross-sectional implications for risk premia. Typically, these cross-sectional restrictions are tested by regressing the average return on a constant and the beta on the asset. Given the link between the consumption leverage and the beta of the asset, the same theoretical restrictions can be tested by a cross-sectional regression on

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6Note that in equation (16), if $\delta_i$ is set to zero, then it follows that $d_{i,t}$ and $c_t$ are cointegrated. In this case the estimated $\phi_i$ (with $\delta_i$ set equal to 0) would equal the ratio of the mean dividend growth rate relative to mean consumption growth rate (see Hamilton (1994)). That is, the $\phi_i$ would equal the ratio of mean growth rate of dividends to the mean growth rate of consumption. By deterministically detrending $d_t$ and $c_t$ and then using the resulting detrended series to run a cointegration regression to estimate $\phi_i$, we ensure that this parameter is estimated by the exposure of the stochastic trend in dividends to the stochastic trend in consumption. In other words deterministic trends do not explicitly play a role in the estimation of $\phi_i$. 

consumption leverage. Assuming that $\kappa_{i,1}$ is identical across all assets (in the data, these differences are very small), it follows that the cross-sectional correlation between $\beta_i$ and $\varphi_i$ is one.

The perfect correlation between $\varphi_i$ and $\beta_i$ implies that the cross-sectional regression of the average return on a constant and $\varphi_i$, provides the same predicted (i.e, theoretical) mean return as a cross-sectional regression of the average return on $\beta_i$. Further, the $R^2$ in the cross-sectional regression based on $\varphi_i$ is equal to that from using $\beta_i$ directly. Consequently, substituting the consumption beta, equation (15), into the expression for the risk premium, equation (9), leads to the following cross-sectional regression,

$$E[R_{i,t}] = \lambda_0 + \varphi_i \lambda_c.$$  \hfill (19)

If the above cross-sectional regression used $\beta_i$ instead of $\varphi_i$, then $\lambda_0$ would equal the mean risk-free rate and $\lambda_c$ the risk-premium on the asset with unit consumption beta, that is $B_M \sigma_\eta^2$. When $\varphi_i$ is used, then the estimated $\lambda_0 = E[R_f] - \frac{1}{\varphi} q$, and $\lambda_c = (1 + q) B_M \sigma_\eta^2$, with $q = \frac{\rho - \omega}{1 - \kappa_1 \rho}$. Our estimates of $\lambda_0$ and $\lambda_c$ correspond to these quantities. Using the consumption leverage directly obviates the need to estimate additional preference and consumption growth rate parameters that go into the construction of $\beta_i$; these parameters, as stated above, do not alter the predicted (theoretical) mean return for various assets. Equation (19) will be used extensively to evaluate the empirical plausibility of the consumption leverage model. The above logic directly applies to the cointegration based measures of consumption leverage as well, the only difference being that we replace $\varphi_i$ in equation (19) with $\phi_i$.

3 Cash Flow Dynamics

3.1 Data

3.1.1 Aggregate Cash Flows and Factors

In our empirical tests, we consider the consumption leverage model stated in equation (19) as well as alternative pricing models in capturing cross-sectional variation in average returns. Our empirical exercise is conducted on data sampled on a quarterly frequency. Following many past studies [e.g. Hansen and Singleton (1983)], we define aggregate consumption as
seasonally adjusted real per capita consumption of nondurables plus services. The quarterly real per capita consumption data are taken from the NIPA tables available from the Bureau of Economic Analysis. To convert returns and other nominal quantities, we also take the associated personal consumption expenditures (PCE) deflator from the NIPA tables. The mean of the inflation series is 0.0113 per quarter with a standard deviation of 0.0065. The mean of the quarterly real consumption growth rate series over the period spanning the 3rd quarter of 1967 through the 4th quarter of 1999 is 0.0052 with standard deviation of 0.0045.

The second set of models that we investigate are referred to as unconditional factor models. The particular models that we consider are the Consumption Capital Asset Pricing Model (C-CAPM), the Capital Asset Pricing Model (CAPM), and a Three-Factor Model. The factor in the C-CAPM is the growth rate of consumption, defined as the first difference in log real per capita consumption. The priced source of risk in the CAPM is the return on a value-weighted index of stocks, obtained from CRSP. The three-factor Fama and French (1993) model posits that the priced risk factors are market, size, and value factors. The market risk premium is the excess return (over the return on a Treasury Bill with one month to maturity) on the value-weighted market return. The size factor is the difference in the return on a portfolio of small capitalization stocks and the return on a portfolio of large capitalization stocks. The value factor is the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks.7 Market capitalization and return data are taken from CRSP, and book values are formed from Compustat data.

In addition to the unconditional C-CAPM and CAPM, we also investigate conditional versions of these models. Lettau and Ludvigson (2001b) suggest a measure of the consumption-wealth ratio, \( k_t \), as a conditioning variable.8 The conditioning variable, \( k_t \), allows for time-variation in the betas. As in Jagannathan and Wang (1996), we estimate the conditional models by augmenting the factors in the C-CAPM and CAPM with the cross-product of the factors and the lagged \( k_t \) variable.9

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7For more detail on the formation of these factors, please see Fama and French (1993). Thanks to Ken French for making these data available.
8We thank Martin Lettau for providing these data. Details on the construction of \( k_t \) are provided in Lettau and Ludvigson (2001a).
9Menzly (2001) explores the ability of Lettau and Ludvigson (2001b)’s “scaled” models to explain the cross-sectional variation in average returns. Menzly both challenges the informational content of the \( k \) variable, and argues that influential data points are driving the model’s apparent cross-sectional power.
Throughout the paper, all of the coefficients and standard errors of both the time series and cross-sectional parameters are calculated via GMM; all of the risk exposures ($\varphi_i$ or $\beta_i$) and cross-sectional risk prices are jointly estimated in one step (see Appendix for details). The GMM procedure that we follow is similar to that proposed in Cochrane (2001).

3.1.2 Benchmark Portfolios

The portfolios employed in our empirical tests sort firms on dimensions that lead to dispersion in measured risk premia in the cross-section. The particular characteristics that we consider are firms’ market value, book-to-market ratio, and past returns (momentum). Our rationale for examining portfolios sorted on these characteristics is that size, book-to-market, and momentum based sorts are the basis for factor models examined in Fama and French (1993) and Carhart (1997) to explain the risk premia on other assets. Consequently, understanding the risk premia on these assets is an economically important step toward understanding the risk compensation of a wider array of assets. We focus on one-dimensional sorts on these characteristics as this procedure typically results in over 150 firms in each decile portfolio and over 300 firms in each quintile portfolio. To better measure the consumption exposure of dividends, it is important to try and to limit the portfolio specific variation in dividend growth rates — a larger number of firms in a given portfolio helps achieve this.

**Market Capitalization Portfolios**

We form a set of portfolios on the basis of market capitalization. The set of all firms covered by CRSP are ranked on the basis of their market capitalization at the end of June of each year using NYSE capitalization breakpoints. In Table 1, we present means and standard deviations of market value-weighted returns for size quintile portfolios. The data evidences a small size premium over the sample period; the mean real return on the lowest quintile firms is 247 basis points per quarter, contrasted with a return of 221 basis points per quarter for the highest quintile. The means and standard deviations of these portfolios are similar to those reported in previous work.\(^{10}\)

**Book-to-Market Portfolios**

Book values are constructed from Compustat data. The book-to-market ratio at year $t$ is

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\(^{10}\)We thank Ken French for providing these data.
computed as the ratio of book value at fiscal year end \( t - 1 \) to CRSP market value of equity at calendar year \( t - 1 \).\(^{11}\) All firms with Compustat book values covered in CRSP are ranked on the basis of their book-to-market ratios at the end of June of each year using NYSE book-to-market breakpoints. Sample statistics for these data are also presented in Table 1. The data evidence a higher book-to-market than size spread; the highest book-to-market firms earn average real quarterly returns of 306 basis points, whereas the lowest book-to-market firms average 215 basis points per quarter.

**Momentum Portfolios**

The third set of portfolios investigated are portfolios sorted on the basis of past returns. Jegadeesh and Titman (1993) use NYSE and AMEX listed firms to document that a “momentum” strategy that purchases the best-performing firms and shorts the worst over a past horizon earns a substantial profit. To construct our momentum-based portfolio returns, we follow a procedure analogous to Fama and French (1996) and sort CRSP-covered NYSE and AMEX firms on the basis of their cumulative return over months \( t - 12 \) through \( t - 1 \). Summary statistics for value-weighted portfolios formed at time \( t \) on the basis of these past returns are presented in Table 1. As shown, this sort provides the highest dispersion in mean returns among the firm characteristics. The highest quintile firms earn an average real return of 342 basis points per quarter, whereas the lowest quintile firms earn an average real return of 48 basis points per quarter. The spread of 294 basis points and the reported volatility of returns is comparable to the data in Fama and French (1996).

### 3.2 Portfolio Dividends

To explore the long-run relationships between portfolio cash flows and consumption, we also need to extract dividend payments associated with these value-weighted portfolios. Our construction of the dividend series is the same as that in Campbell (2000). Let the total return per dollar invested be

\[
R_{t+1} = h_{t+1} + y_{t+1}
\]

\(^{11}\)We thank Ken French for providing us the book to market-sorted portfolio data. For a detailed discussion of the formation of the book to market variable, refer to Fama and French (1992).
where $h_{t+1}$ is the price appreciation and $y_{t+1}$ the dividend yield (i.e., dividends at date $t+1$ per dollar invested at date $t$). We observe $R_{t+1}$ and the price gain series $h_{t+1}$ for each portfolio; hence, $y_{t+1} = R_{t+1} - h_{t+1}$. The level of the dividends we use in the paper is computed as

$$D_{t+1} = y_{t+1}V_t$$

where

$$V_{t+1} = h_{t+1}V_t$$

with $V_0 = 100$. Hence the dividend series that we use, $D_t$, corresponds to the total dividends given out by a mutual fund at $t$ that extracts the dividends and reinvest the capital gains. The ex-dividend value of the mutual fund is $V_t$ and the per dollar return for the investors in the mutual fund is

$$R_{t+1} = \frac{V_{t+1} + D_{t+1}}{V_t} = h_{t+1} + y_{t+1}$$

From this equation, it is evident that $V_t$ is the discounted value of the dividends that we use.

We construct the level of the dividends $D_t$ for all the portfolios on a monthly basis. From this we construct quarterly levels of dividends by summing the level of dividends within a quarter. As the dividend yields have strong seasonalities, we employ a trailing four quarter average of the quarterly dividends to construct the deseasonalized quarterly dividend series. This procedure is consistent with the approach in Hodrick (1992), Heaton (1993), and Bollerslev and Hodrick (1995). These series are converted to real by the personal consumption deflator. Log growth rates are constructed by taking the log first difference of the quarterly deseasonalized series. Summary statistics for the dividend growth rates of the portfolios under consideration are presented in Table 1. An analogous construction is applied for decile portfolios; summary statistics for these portfolios are presented in Table 2.

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12 To be precise, $h_{t+1}$ represents the ratio of the market value of the firm at time $t+1$ to time $t$, $\frac{V_{t+1}}{V_t}$, and $y_{t+1}$ represents the total dividends paid by the firm at time $t+1$ divided by firm value at time $t$, $\frac{D_{t+1}}{V_t}$.

13 We thank Ken French for providing the total return and price appreciation series for both the size and book-to-market quintile and decile portfolios.
4 Empirical Evidence

4.1 Measuring Consumption Exposure of Dividends

To measure the time-series relationship between dividends and consumption, we explore the consumption leverage specification:

\[ g_{i,t+1} = \delta_i + \varphi_i x_t + \eta_{i,t+1} \]  \hspace{1cm} (20)

where \( g_{i,t+1} = d_{i,t+1} - d_{i,t} \) and \( \varphi_i \) measures the covariance between portfolio dividend growth and the expected consumption growth rate, \( x_t \). Given \( x_t \), this relationship is estimated by standard OLS. We construct \( x_t \) as a simple trailing eight-quarter moving average of past real consumption growth.\(^{14}\)

To measure the consumption leverage parameter \( \phi_i \) via stochastic cointegration we first detrend the log consumption and dividend series by regressing the log level of consumption and the log level of dividends on a constant and a time trend. The resulting de-trended time series, \( c^*_{t} \) and \( d^*_{t} \), are then used to measure \( \phi_i \). More specifically, we follow Stock and Watson (1993) and use DOLS to estimate \( \phi_i \), that is we estimate \( \phi_i \) via

\[ d^*_{i,t} = \mu_i + \phi_i c^*_{t} + \sum_{k=1}^{K} \left( \alpha_{-k} \Delta c^*_{t-k} + \alpha_k \Delta c^*_{t+k} \right) + \epsilon_{i,t} \]  \hspace{1cm} (21)

with \( K = 4 \). By first deterministically detrending consumption and dividends, and then using the de-trended series to measure \( \phi_i \) we ensure that deterministic trends in dividends and consumption do not drive our measures of \( \phi_i \).

Estimates of \( \varphi_i \) for the characteristic-sorted portfolios are presented in Table 1. As shown in the table, a clear pattern emerges in the covariance of portfolio dividend growth rates with the smoothed consumption growth rate. Sorting on past returns produces extremely large dispersion in \( \varphi_i \); the sensitivity of winner portfolio dividend growth to consumption growth is 9.33 (S.E. 5.71) compared to -2.65 (S.E. 5.50) for the loser portfolio. Book-to-market sorts also produce a large spread; the high book-to-market firms’ sensitivity to consumption

\(^{14}\)We also estimate an ARMA (1,1) for the consumption growth series. The AR(1) parameter estimate of this process is 0.730 (S.E. = 0.116) and the MA(1) estimate is 0.404 (S.E. = 0.178). The smoothed consumption growth series and the resulting conditional mean implied by the estimated ARMA process are very similar.
growth is 5.58 (S.E. 2.88) compared to -0.83 (S.E. 1.82) for the low book-to-market firms. Interestingly, the table suggests that loser and low book-to-market portfolio growth rates vary negatively with smoothed consumption growth, as reflected in the negative consumption leverage coefficient estimate. The evidence based on cointegration also mirrors exactly this pattern. In particular, the loser portfolio and the low book-to-market portfolios have small \( \phi \)'s and low mean returns, while the opposite is true for the high mean return portfolios. As a measure of the empirical content of the economic argument, note that in Table 1 panel B we report results from regressing the mean returns on to consumption leverage measures. Throughout the paper, all risk prices are expressed in quarterly percentage terms. For the growth rate based measure on consumption leverage the cross-sectional \( R^2 \) is 58\%, and for the cointegration based measure it is 68\%. The slope coefficients in both cases are positive and highly significant. The quarterly compensation for consumption risk is about 0.15\% (annual is 0.6\%) in both cases. For a benchmark comparison, the three factor model of FF for these 15 assets has a cross-sectional \( R^2 \) of 11\%.

The summary statistics illustrate an important point; portfolios with high (low) \( \varphi_i \) are portfolios with high (low) average returns. That is, portfolios with high sensitivity to aggregate consumption growth are firms that have high average risk premia. This pattern is consistent with our model, which suggests that small shocks to the permanent component in consumption growth have large implications for risk premia that differ across assets in concert with assets’ consumption leverage. To analyze this relationship further, we display the extreme portfolio dividend growth rates and the smoothed consumption growth rate in Figures 1-3. In accordance with the large estimated \( \varphi \)'s, the winner and high book-to-market portfolio dividend growth rates demonstrate a close relationship to the smoothed consumption growth rate, generally falling during documented recessions and rising during consumption booms. However, the loser portfolio dividend growth rates demonstrates strong countercyclical movements. These plots suggest that the momentum and book-to-market portfolios are sorting along macroeconomic exposures across firms. Capitalization-sorted portfolios also demonstrate this pattern with respect to consumption, with the estimated leverage coefficient on small firms exceeding the large firms, but the difference is less pronounced in accordance with the reduced size premium observed in more recent years. Further note, given the estimated \( \lambda_c \), we can explain much of the momentum, size and book-to-market return spreads. For example, using the growth rate based estimate, \( \varphi_i \), the model-implied expected spreads on the extreme portfolios for momentum, size, and book-to-market are
207, 9, and 110 basis points per quarter, respectively. These compare very well with their counterparts observed in the data.

One noteworthy result from the regressions is the negative risk measure for low book-to-market portfolios. As shown in Figure 3, while the high book-to-market portfolio dividend growth displays a strong positive correlation with consumption growth, the low book-to-market portfolio cash flow does not. This result suggests that, although low book-to-market firms may have growth prospects, these opportunities bear little relation to systematic risk. That is, the risk inherent in these growth opportunities does not covary strongly with the permanent component in consumption. As a result, this risk is not priced, which is reflected in the relatively low average returns.

An additional issue that pertains to measures of $\varphi_i$ and $\phi_i$ is the relatively large time series standard errors reported of these parameters in Table 1. This is not surprising as most of the variation in dividend growth rates are not related to changes in aggregate consumption. That is, the $R^2$’s of the time-series regressions are small; in our data they range from less than $1\%$ to $9\%$. This suggests that $91$-$99\%$ of the variation in dividend growth rates is attributable to portfolio specific (i.e., not related to consumption growth) movements in dividend growth rates. Further, the variation in $x_t$ in the growth rate projection (equation (20)), the eight quarter moving average of consumption growth, is only $0.25\%$ per quarter. Given this, in sample sizes of 130 quarterly observations, the standard error is bound to be large.

To carefully evaluate this issue we conduct a Monte Carlo experiment of 10,000 replications of 140 quarters. We set $\varphi = 1$, $\sigma_x$ in equation (20) to 0.0025 and it is assumed that $x_t$ follows an AR(1) process with autoregressive coefficient of 0.97. The independent noise term in equation (20) is chosen to match a dividend growth rate volatility of 0.03, which represents the $low$ end of the range of the measured dividend growth rate volatility. Two results are relevant from this Monte Carlo. First, there is no small sample bias in estimating the $\varphi$ – the mean Monte Carlo estimate of $\hat{\varphi} = 1.00$. Second, as the percentage of the dividend growth rate volatility attributed to $x$ is small, the standard errors, as in the data, are large – the mean monte carlo $t$-statistics are well below 1.0. Only when we consider sample sizes of approximately 600 quarterly observations in our monte carlo do the $t$-ratios approach 2.0. In all, it seems that estimates of the dividend exposure to consumption movements in realistic samples do quite well in measuring their population values. The large standard errors in realistic samples are a reflection of the fact that the dividend growth rates contain very large portfolio specific noise. The economic value of the measured $\varphi$’s should be
determined by their ability to explain cross-sectional differences in measured risk premia. To underscore this point, note that market betas are estimated with small time series standard errors; however, these betas provide little economic information regarding the dispersion in the mean returns across assets. It is also worth noting that finer sorts of the assets on any given sorting dimension typically increase the standard errors with which the consumption exposures of dividends is measured. To measure these exposures with greater precision is the reason that we focus first on quintile sorts which yields more firms in each portfolio, and subsequently the decile based sorts (that consist of fewer firms per portfolio) which are discussed below.

4.2 Equity Risk Premia in the Cross-Section

In this section, we formally examine the relative performance of our consumption leverage model, standard unconditional factor models, and scaled conditional factor models in explaining the cross-section of equity risk premia. We perform standard cross-sectional regressions utilizing a finer sort of the portfolios detailed above. We form 30 portfolios (10 size, 10 momentum, and 10 book-to-market); summary statistics for these portfolios are presented in Table 2. As exhibited in the table, this finer sort exhibits similar cross-sectional patterns in risk exposures and mean returns as the quintile sort analyzed above.

4.2.1 Performance of Consumption Leverage Model

We begin our exploration by examining the ability of our consumption leverage model presented above to explain the cross-section of equity returns. The cross-sectional risk premia restriction is stated in equation (19), with $\lambda_0$ and $\lambda_c$ as the cross-sectional parameters of interest, given the consumption leverage.$^{15}$ Table 3 (Panel A) documents the cross-sectional performance of the consumption leverage model. The results show that the risk price of consumption leverage is positive and significant. Further, the adjusted $R^2$ is 51% suggesting, that the fundamental model can explain a considerable portion of the equity risk premia associated with this set of portfolios. This evidence is depicted graphically in Figure 4, which plots the predicted expected returns against the realized mean returns. A particular success

$^{15}$Note that in the case where the consumption leverage is estimated via the co-integration approach, based on Engle and Granger (1987), we ignore the estimation error in estimating the superconsistent leverage parameters $\phi_i$'s.
of the model is that it is capable of explaining much of the variation across momentum returns. This dimension is particularly challenging for the alternative models considered. These results are particularly intriguing since the model’s estimates of risk sensitivity are based solely upon the cash flows associated with a particular portfolio. That is, the high adjusted $R^2$'s are associated only with measures of the relationship between portfolio cash flows and aggregate consumption.

### 4.2.2 Performance of Unconditional Models

We continue our exploration by examining the ability of several standard unconditional (constant) $\beta$ representations to explain the cross-section of equity returns. Table 3 (Panel A) documents the results of cross-sectional regressions in the context of standard unconditional models: the C-CAPM, the CAPM, and Fama and French (1993) three factor model. The tables report estimated risk prices, $\lambda_k$, associated with each risk source. Since the GMM estimation is performed in one step, standard errors (reported in the parentheses) reflect first stage time-series estimation of risk exposures. The tables also report cross-sectional $R^2$'s, adjusted for degrees of freedom. To explore the ability of standard unconditional models to explain the cross-section of equity returns, the factors explored are $g_t$, the consumption growth rate, $R_{vw,t}$, the excess return on the CRSP value-weighted index, $R_{SMB,t}$, the return on the size factor from Fama and French (1993), and $R_{HML,t}$, the return on the book-to-market factor from Fama and French (1993).

The first model we consider is the consumption based C-CAPM, for which the associated risk premium restriction is as follows:

$$E[R_{i,t+1}] = \lambda_0 + \beta_{g,i}\lambda_g$$  \quad (22)

where $\beta_{g,i}$ describes as asset’s exposure to aggregate consumption risk; for all models, the betas are estimated using a standard time series regression of the portfolio return on the fundamental risk factors. The estimated price of consumption risk, $\lambda_g$, is positive and statistically significant, but the adjusted $R^2$ of 4.6% suggests that this model explains some, but not a substantial portion of the cross-sectional variation in average returns. The inability of the unconditional C-CAPM to explain the portfolio returns is depicted graphically in Figure 4.
We next consider the static CAPM, where risk is embodied entirely in the portfolio return’s exposure to market risk. This model implies the following cross-sectional risk premium restriction

\[ E[R_{i,t+1}] = \lambda_0 + \beta_{vw,i} \lambda_v \]

where \( \beta_{vw,i} \) describes an asset’s exposure to market risk, and \( \lambda_v \) describes the price of market risk. As in previous studies, the estimate of \( \lambda_v \) is negative and not statistically significant. Further, the ability of the model to explain cross-sectional risk premia is limited, as demonstrated in the relatively low adjusted \( R^2 \) of 3.0%. Again, the difficulty of the static CAPM in explaining the cross-section of equity market returns is displayed graphically in Figure 4.

Finally, we present results for the Fama and French three-factor model. The cross-sectional risk premia restriction implied by this model is as follows:

\[ E[R_{i,t+1}] = \lambda_0 + \beta_{vw,i} \lambda_v + \beta_{SMB,i} \lambda_{SMB} + \beta_{HML,i} \lambda_{HML} \]

This model exhibits substantial improvement over the single-factor models in explaining cross-sectional variation in returns, as the adjusted \( R^2 \) rises to 24.3%. However, with the exception of the market risk premium, the model parameters are imprecisely estimated. Fama and French (1996) demonstrate that the model cannot explain momentum portfolio returns; the graphical depiction of model fit in Figure 4 reinforces this evidence. In Table 3, we show that when the momentum assets are dropped the three factor model indeed does quite well; the adjusted \( R^2 \) is 67%. The consumption leverage model continues to perform well with a positive and significant price of risk and an adjusted \( R^2 \) of 38%.

### 4.2.3 Performance of Conditional (Scaled) Models

Despite the difficulty the simple constant-\( \beta \) models have in explaining the cross-section of risk premia for our challenging assets menu, there is evidence that conditional (scaled) factor models, which essentially facilitate time-varying risk exposures, are capable of describing the cross-section of equity returns (see Ferson and Harvey (1991), Jagannathan and Wang (1996), and Lettau and Ludvigson (2001b), for example). We augment the C-CAPM, CAPM and labor income models with a single scaling variable, \( k_t \), and explore the effects of including scaled factors in explaining the cross-section of risk premia. As before, the factors are \( g_t \), the
growth rate of aggregate consumption and $R_{vw,t}$, the return on the CRSP value-weighted index. Additional risk premia are incorporated into the model by multiplying these primitive factors by $k_{t-1}$.

Table 3 documents parameter estimates and cross-sectional $R^2$'s associated with these specifications. We first consider a conditional (scaled) version of the C-CAPM:

$$E[R_{i,t+1}] = \lambda_0 + \beta_{g,i} \lambda_g + \beta_{gk,i} \lambda_{gk}$$ (25)

Although the scaled factor risk premium is precisely estimated, the $R^2$ suggests that the conditional C-CAPM has little additional explanatory power relative to the static C-CAPM. As shown graphically in Figure 5, the fit of the model is comparable to that of the standard C-CAPM. The evidence is similar for a scaled version of the CAPM:

$$E[R_{i,t+1}] = \lambda_0 + \beta_{k,i} \lambda_k + \beta_{vw,i} \lambda_{vw} + \beta_{vwk,i} \lambda_{vwk}$$ (26)

In this case, the scaling variables appear to add little to the ability of the model to explain cross-sectional variation in returns. The parameters are not precisely estimated and the explanatory power of the conditional model is similar to that of the unconditional model. However, it is important to note that when the momentum portfolios are removed from the test assets that the conditional C-CAPM performs quite well; the adjusted $R^2$ of the model rises to 28%. This evidence suggests that a role for time-variation in risk measures remains.

### 4.2.4 Diagnostics and Economic Interpretation

It is worth noting that standard consumption based models, unconditional or conditional, obtain the asset’s beta by projecting returns on the ex-post consumption growth in the time-series. As is well recognized and reinforced above, this approach has considerable difficulty in explaining risk premia in the cross-section. Hence, in the presence of measurement error, there is every reason to believe that the standard return-based consumption betas will fail to capture the important exposures facilitated by our regression methodology. For example, we find that the correlation between the estimated consumption beta (using returns) and the growth rate based exposures ($\varphi_i$) is 0.21. Within the sorting dimensions, this correlation is 0.21 for momentum, 0.37 for size, and 0.64 for book-to-market. If there are considerable errors or other difficulties in measuring the appropriate level of consumption (as argued in
Campbell (1996)), then indeed the usual consumption beta may be a poor estimate of the true consumption beta that is needed to explain risk premia. Our measures of consumption leverage model are fairly robust to stationary measurement errors in consumption and dividends as these should not alter long-run relationships. As discussed earlier, stochastic cointegration relations are unaffected by stationary measurement error; further, even our growth rate projections employing a moving average of consumption growth rates which will mitigate the contaminating effects of mis-measuring consumption.

To evaluate the importance of low frequency movements in consumption growth, we conduct an additional experiment. We estimate leverage parameters where consumption growth is now smoothed over various horizons. The details are presented in Table 4. For each portfolio, we regress portfolio dividend growth rates on different measures of expected consumption growth: contemporaneous consumption, consumption smoothed over the previous 1 year (4 quarters), and 2 years (8 quarters). In Panel A of Table 4, we present the estimated cash flow exposures, $\varphi_i$, for each case. The relationship between the exposures and average returns is visibly present only in the cases where consumption is smoothed over 4 or 8 quarters (note that 8 quarter results are already presented in Table 3). Indeed, for regressions on contemporaneous consumption, the relationship between cash flow exposures and average returns are inverted for the momentum sort. As shown in Panel B, if $\varphi_i$ is estimated via a regression of dividend growth rates on contemporaneous consumption growth, the cross-sectional $R^2$'s are negative. That is these contemporaneous projections provide little information regarding the dispersion in mean returns on assets. As we smooth the consumption growth rates and regress dividend growth rates on these smoothed growth rates, the estimated $\varphi_i$'s are better able to justify the cross-sectional differences in mean returns. This evidence, along with that for cointegration based measures, suggests that dividend exposures to low frequency movements of consumption are economically important to understand differences in risk premia across assets. The failure of contemporaneous comovements in dividends and consumption growth to matter more significantly is puzzling, but could very well be a measurement issue.\footnote{Daniel and Marshall (1997) argue that at longer horizons, a habits-based model can better justify the risk premium on the (value-weighted) market portfolio and the risk free rate. They argue that the measurement errors in consumption may be the reason for the poor performance of the model at shorter horizons.}

In Table 5, Panel A, we report results which give a sense of the relative merits of the different factors in explaining the cross-section of risk premia. In particular, we augment the consumption leverage model to include additional factors and inquire if the consumption...
leverage continues to be important in their presence. First, note that the risk premium on consumption in highly significant and positive in all cases. With the inclusion of the consumption leverage, we are unable to reject the hypothesis that the factors in addition to the consumption leverage parameter are jointly zero. In Panel B, we show that for the reduced menu of only size and book-to-market sorted portfolios that the additional factors contribute substantial explanatory power. This result, as argued in Campbell and Cochrane (1999) and Lettau and Ludvigson (2001b), suggests that the additional risk measures and time-varying betas contain potentially valuable information on risk-return relationships. Note that even in this reduced menu of assets the consumption exposure of dividends continues to have significant explanatory power.

4.3 Robustness Checks

We conduct several robustness checks for our empirical evidence. Our first check is to compute rolling estimates of the $\varphi$’s as in Fama and MacBeth (1973). We utilize a set of data that is analogous to that used in the previous section, but sample the data at a monthly frequency. The $\varphi$’s are estimated as in the previous section over a rolling window of 60 months prior to cross-sectional estimation of the $\lambda_c$, the market price of consumption risk. For brevity, the details of this estimation are not reported. The average of the time series of $\lambda_c$ is 0.0062 with a robust standard error of 0.0031. Analogous evidence for the static CAPM and the three-factor model suggest that these models’ associated factor risks are not priced.

An additional robustness check that we have investigated is an extension of our sample period to the first quarter of 1953. These results are presented in Table 6, and duplicate the results for the shorter sample in Table 1. The main difference in this sample is that the spread in the small and large size portfolio mean returns widens to 60 basis points per quarter. As can be seen in Table 6, both the growth rate and cointegrating estimates of the consumption risk measure captures the relationship between mean rates of return and consumption leverage. In particular, small firms have high mean returns and high consumption leverage measures relative to large firms. Given the growth rate based estimate of $\lambda_c$, the consumption exposures, $\varphi_i$, imply an average return spreads of 64 basis points per quarter for the extreme size portfolios. In addition, along the momentum and book-to-market dimensions, our evidence is broadly similar to that presented for the shorter sample.
We have also analyzed annual data from 1929 through 1999 and the results (not reported) are materially unchanged.\textsuperscript{17} Hence, it appears that these measured relationships are robust across different frequencies and samples.

5 Conclusion

The idea that differences in exposures to sources of systematic risk should justify differences in risk premia across assets is fundamental to financial economics. We present a simple, parsimonious general equilibrium model, in which consumption betas directly mirror the exposure of dividends to consumption (consumption leverage). We show that the measured consumption exposure of dividends do quite well in terms of explaining the cross-sectional differences in the risk premia on 30 portfolios comprised of 10 size, 10 momentum, and 10 book-to-market portfolios. We measure the consumption leverage of a given dividend stream by relying the projection coefficient of ex-post the dividend growth rate on a measure of the expected consumption growth rate. and alternatively on the measure of stochastic cointegration between consumption and dividends.

Our consumption leverage model can account for about 50\% of the cross-sectional differences in risk premia across 30 assets, and the risk premium associated with the consumption uncertainty is positive and highly significant. This performance compares very favorably against standard factor and time-varying beta models. Our evidence suggests that there is a small predictable, and very persistent, component in consumption growth rates. Small shocks to this component have very long lasting effects for future expected growth rates, and hence these shocks have a large impact on asset prices. Dividends of different assets have varying exposures to this aggregate source of non-diversifiable risk that requires different risk compensation. We find that the extreme loser and low book-to-market portfolio dividends have negative consumption leverage and low risk premia. In sharp contrast, the winner portfolio and the high book-to-market portfolio have large positive consumption leverage and large positive risk premia. We document that our specification can duplicate much of the value spread (high book-to-market less low book-to-market), the momentum spread (winner firm less looser firms), and the size spread (small firm less large firm return).

Our overall empirical evidence suggests that dividend exposures to low frequency move-

\textsuperscript{17}These data were provided by Ken French for size and book-to-market sorts.
ments in consumption provide very valuable information regarding the cross-sectional differences in risk premia across assets. Assets which have large exposures to this non-diversifiable source of risk also carry a large risk premia.
References


6 Appendix

We have defined preferences (Epstein-Zin) and consumption/aggregate cash flow dynamics (ARIMA) for the economy. An equilibrium will then be a price function that, given preferences and consumption dynamics, will clear the market. In order to move to this equilibrium, we begin by noting that, by definition, the return on any asset is given by

\[ R_{i,t+1} = 1 + \frac{P_{i,t+1}}{D_{i,t}} \frac{D_{i,t+1}}{D_{i,t}} \]  

(27)

Let \( G_{i,t+1} = \frac{D_{i,t+1}}{D_{i,t}} \) and \( Z_{i,t} = \frac{P_{i,t}}{D_{i,t}} \). Campbell and Shiller (1988a) derive a Taylor series approximation to (27), which is expressed in log form as

\[ r_{i,t+1} = \kappa_{i,0} + \kappa_{i,1} Z_{i,t+1} - z_{i,t} + g_{t+1} \]  

(28)

where lowercase letters represent logs of their uppercase counterparts.\(^{18}\) Thus, the return on any asset at time \( t + 1 \) is a function of its price-dividend ratio at times \( t \) and \( t + 1 \), and the growth rate in its cash flows.

6.1 Equilibrium

We make two assumptions in order to solve for the wealth consumption portfolio as functions of the expected consumption growth. First, we conjecture that the log wealth consumption ratio, \( z_{c,t} \), is linear in the state variable, \( x_t \):

\[ z_{c,t} = A_{c,0} + A_{c,1} x_t \]  

(30)

Our second assumption is that the return on the portfolio that pays consumption and the IMRS are joint lognormally distributed. We then observe that, as shown in Bansal and

\[^{18}\kappa_{i,0} \text{ and } \kappa_{i,1} \text{ are constants from the Taylor series approximation:} \]

\[ \kappa_{i,1} = \frac{\exp(\bar{z}_i)}{1 + \exp(\bar{z}_i)}, \quad \kappa_{i,0} = -\log(\kappa_{i,1}) - (1 - \kappa_{i,1})\bar{z}_i \]  

(29)
Yaron (2000), we can solve for the coefficient $A_{c,1}$ and $A_{i,2}$ using the relationship

$$E_t[\exp(m_{t+1} + r_{c,t+1})] = 1$$

and the fact that, for a normally distributed random variable $X$,

$$E[e^X] = e^{E[X] + \frac{1}{2} \text{Var}[X]}$$

Aggregate consumption growth follows an ARMA(1,1) process as follows:

$$c_{t+1} - c_t = g_t = (1 - \rho)\mu_c + \rho g_t + \eta_{t+1} - \omega\eta_t$$  \hspace{1cm} (31)$$

$$x_{t+1} = (1 - \rho)\mu_x + \rho x_t + (\rho - \omega)\eta_{t+1}$$  \hspace{1cm} (32)$$

where $x_t$ is the expected consumption growth rate. Using the Epstein-Zin equilibrium pricing restriction and the approximated definition of return:

$$\theta \ln \delta - \frac{\theta}{\psi}((1 - \rho)\mu_c + x_t + x_t)$$

$$+ (\theta)[\kappa_{c,0} + \kappa_{c,1}\{A_{c,0} + A_{c,1}((1 - \rho)\mu_x + \rho x_t)\} - A_{c,0} - A_{c,1}x_t + (1 - \rho)\mu_c + x_t]$$  \hspace{1cm} (33)$$

Isolate the terms related to the expected consumption growth, $x_t$,

$$-\frac{\theta}{\psi} + (\theta)[\kappa_{c,1}A_{c,1}\rho - A_{c,1} + 1] = 0$$  \hspace{1cm} (34)$$

By solving for the coefficients, the solution for the log wealth consumption ratio is given by

$$A_{c,1} = \frac{1 - \frac{1}{\psi}}{1 - \kappa_{c,1}\rho}$$  \hspace{1cm} (35)$$

This implies the following for the innovations to the wealth portfolio return:

$$r_{c,t+1} - E_t[r_{c,t+1}] = \eta_{c,t+1} = B_c\eta_{t+1}$$  \hspace{1cm} (36)$$

where $B_c = 1 + \kappa_{c,1}(\rho - \omega)A_{c,1}$. Given the solution for the log wealth consumption ratio, the pricing kernel innovation can be rewritten solely as a function of the consumption growth
rate innovation:

\[ \eta_{M,t+1} = -\frac{\theta}{\psi} \eta_{t+1} - (1 - \theta) \eta_{c,t+1} = -\left[ -\frac{\theta}{\psi} - (1 - \theta) B_c \right] \eta_{t+1} \equiv B_M \eta_{t+1} \]  

(37)

where \( B_M = -\left[ -\frac{\theta}{\psi} - (1 - \theta) B_c \right] \). The geometric risk premium on any asset is given by

\[ E_t \left[ r_{i,t+1} - r_{f,t} \right] = \text{cov}_t \left( -\eta_{M,t+1}, r_{i,t+1} \right) - \text{var}(r_{i,t+1})/2 \]  

(38)

Substituting the form of the solution coefficients, and converting to the arithmetic risk premium yields

\[ E_t \left[ R_{i,t+1} - R_{f,t} \right] = \beta_i \left[ B_M \sigma^2 \eta \right] \]  

(39)

where the fundamental \( \beta_i = \frac{\text{cov}(r_{i}, \eta)}{\sigma^2} \). Note that \( B_M \sigma^2 \eta = -\left[ -\frac{\theta}{\psi} - (1 - \theta) B_c \right] \sigma^2 \), is the equilibrium market price of consumption risk.

### 6.2 Consumption Leverage: Growth Rates

We specify the dynamics for asset-specific real log cash flow growth, \( g_{i,t} \), in relation to the expected growth rate of consumption, \( x_t \):

\[ d_{i,t+1} - d_{i,t} = g_{i,t+1} = \delta_i + \varphi_i x_t + \eta_{i,t+1} \]  

(40)

Given this specification, we conjecture that \( z_{i,t} \) is linear in the state variable, \( x_t \):

\[ z_{i,t} = A_{i,0} + A_{i,1} x_t \]  

(41)

Using the solution for the log wealth consumption ratio and the approximated definition of the return, we isolate the terms in the Epstein-Zin first order condition \( E_t \left[ \exp(m_{t+1} + r_{i,t+1}) \right] = 1 \) related to the expected consumption growth, \( x_t \):

\[ -\frac{1}{\psi} + \varphi_i - A_{i,1} [1 - \kappa_{i,1} \rho] = 0 \]  

(42)

The solution for the coefficient is given by

\[ A_{i,1} = \frac{\varphi_i - \frac{1}{\psi}}{1 - \kappa_{i,1} \rho} \]  

(43)
As above, the arithmetic risk premium on any asset is given by

$$E_t[R_{i,t+1} - R_{f,t}] = \beta_i[B_M\sigma^2_{\eta}]$$  \hspace{1cm} (44)$$

Since the return innovation is

$$r_{i,t+1} - E_t[r_{i,t+1}] = [\tau_i + \kappa_{i,1}A_{i,1}(\rho - \omega)]\eta_{t+1} + \eta_{i,t+1},$$  \hspace{1cm} (45)$$

it follows that the consumption beta of the asset

$$\beta_i = \tau_i + \kappa_{i,1}A_{i,1}(\rho - \omega)$$ \hspace{1cm} (46)$$

### 6.3 Consumption Leverage: Cointegration

To solve for individual equilibrium asset prices, we conjecture that $z_{i,t}$ is linear in the state variables, $x_t$ and $\epsilon_{i,t}$:

$$z_{i,t} = A_{i,0} + A_{i,1}x_t + A_{i,2}\epsilon_{i,t}$$  \hspace{1cm} (47)$$

As with the log wealth consumption ratio, we can solve for the coefficients $A_{i,1}$ and $A_{i,2}$. Asset dividend and aggregate consumption are stochastically cointegrated as follows:

$$d_{i,t+1} = \mu_i + \delta_i \cdot (t + 1) + \phi_i \epsilon_{t+1} + \epsilon_{i,t+1}$$ \hspace{1cm} (48)$$

Taking the first difference, and substituting for the assumed consumption growth rate process (6), it follows that

$$g_{i,t+1} = \delta_i + \phi_i x_t + \phi_i \eta_{t+1} + \epsilon_{i,t+1} - \epsilon_{i,t}$$ \hspace{1cm} (49)$$

where we assume

$$\epsilon_{i,t+1} = \xi_i \epsilon_{i,t} + u_{i,t+1}$$  \hspace{1cm} (50)$$

Using the solution for the log wealth consumption ratio and the approximated definition of the return, we isolate the terms in the Epstein-Zin first order condition $E_t[\exp(m_{t+1} + r_{i,t+1})] = 1$ related to the expected consumption growth, $x_t$,

$$-\frac{1}{\psi} + \phi_i - A_{i,1}[1 - \kappa_{i,1}\rho] = 0$$ \hspace{1cm} (51)$$
Second, isolate the terms related to the asset specific cyclical component, $\epsilon_{i,t}$,

$$\kappa_{i,1} A_{i,2} \xi_i - A_{i,2} + \xi_i - 1 = 0$$

(52)

By solving for the coefficients, the solution for the log price dividend ratio is given by

$$A_{i,1} = \frac{\phi_i - \frac{1}{\psi}}{1 - \kappa_{i,1} \rho} A_{i,2} = \frac{\xi_i - 1}{1 - \kappa_{i,1} \xi_i}$$

(53)

From the equilibrium solution, the geometric risk premium on any asset is given by

$$E_t [R_{i,t+1} - R_{f,t}] = \beta_i [B_M \sigma^2_i]$$

(54)

Substituting the solution for $z_i$ in to the expression for the ex-post return implies that the return innovation is

$$r_{i,t+1} - E_t [r_{i,t+1}] = [\phi_i + \kappa_{i,1} A_{i,1} (\rho - \omega)] \eta_{t+1} + (1 + \kappa_{i,1} A_{i,2}) u_{i,t+1}$$

(55)

The asset’s beta is, $\beta_i = \frac{\text{cov}(r_{i,t+1}, \eta_{t+1})}{\sigma^2_{\eta_{t+1}}}$. Given the return innovation, it follows that

$$\beta_i = \phi_i + \kappa_{i,1} A_{i,1} (\rho - \omega)$$

(56)

### 6.4 GMM Estimation

Let the true parameter vector be given by:

$$\Psi_0 = \begin{bmatrix} \alpha_1 & \cdots & \alpha_N & \beta'_1 & \cdots & \beta'_N & \lambda_0 & \lambda' \end{bmatrix}$$

(57)

where the $\beta_i$ and $\lambda$ vectors are determined by the model specification.

Let $R_{i,t}$ denote the return on the $i$th portfolio. There are $N$ portfolios in total. The basic regression that we run for each portfolio’s return is

$$R_{i,t+1} - R_{f,t} = \alpha_i + \beta'_i f_{t+1} + e_{i,t+1}$$

(58)

for a vector of $K$ risk factors, $f_{t+1}$, determined by each model. $f_{k,t}$ represents the realization of factor $k$ at time $t$, and $\beta_k$ indicates the sensitivity associated with risk factor $k$. $e_{i,t}$ is
assumed to be conditionally mean independent of the risk factors \( f_{k,t} \). We formulate the following moment conditions to estimate the risk sensitivities (\( \beta_i \)'s):

\[
E[e_{i,t+1}] = 0 \quad \forall i = 1, \ldots, N \\
E[e_{i,t+1} f_{t+1}] = 0 \quad \forall i = 1, \ldots, N
\]

(59)

The \( \beta_i \)'s are identified in the time series. We also identify the risk prices in the cross-section by exploiting the following set of moment conditions:

\[
E[R_{i,t+1} - \lambda_0 - \beta_i' \lambda] = 0 \quad \forall i = 1, \ldots, N
\]

(60)

We can stack the sample counterparts to the moment conditions (59) and (60) as follows (Hansen (1982)):

\[
g_T(\Psi) = \frac{1}{T} \sum_{t=1}^{T} f(X_t, \Psi)
\]

(61)

This yields \([N + N \cdot K + (K + 1)]\) parameters to be estimated with \([N + N \cdot K + N]\) moment conditions, where \( N > K + 1 \). We construct an exactly identified system by setting linear combinations of \( g_T \), an \([N(K + 1) + N] \times 1\) vector, equal to zero. Specifically, we write the moment conditions as

\[
A_T' g_T = 0
\]

(62)

Our choice of \( A_T \), an \([N(K + 1) + N] \times [N(K + 1) + (K + 1)]\) matrix, is designed to ensure that the estimates are consistent with OLS.

\[
A_T = \left[ \begin{array}{cccc}
I_{N(K+1)} & 0_{N(K+1)\times 1} & 0_{N(K+1)\times 1} & \cdots & 0_{N(K+1)\times 1} \\
0_{N,N} & 1_{N \times 1} & \hat{\beta}_1 & \cdots & \hat{\beta}_K
\end{array} \right]
\]

(63)

where \( I_{N(K+1)} \) is the identity matrix, \( 0_{N(K+1)\times 1} \) and \( 1_{N(K+1)\times 1} \) denote column vectors of zeros and ones, respectively, and \( \hat{\beta}_k \) is an \( N \times 1 \) vector of the estimated sensitivities to risk factor \( k \). We then estimate the parameters, \( \Psi_T \), of the exactly identified system to ensure that \( A_T' g_T(\Psi_T) = 0 \). Based on Hansen (1982), we know that

\[
\sqrt{T}(\Psi_T - \Psi_0) \sim N(0, (AD)^{-1}(ASA')(AD)^{-1'})
\]

(64)

where \( D \) is the gradient of the stacked moment conditions in equation (61), and \( S \) is the
variance-covariance matrix of the moment conditions, for which the sample counterpart is estimated using Newey and West (1987) with 4 lags.
Table 1: Cross-Sectional Evidence (1967.3-1999.4): 15 Portfolios

Panel A: Summary Statistics

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Returns</th>
<th>Dividend Growth</th>
<th>Consumption Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>M1-2</td>
<td>0.48</td>
<td>13.01</td>
<td>-1.57</td>
</tr>
<tr>
<td>M3-4</td>
<td>1.65</td>
<td>9.81</td>
<td>-0.32</td>
</tr>
<tr>
<td>M5-6</td>
<td>1.71</td>
<td>8.45</td>
<td>0.15</td>
</tr>
<tr>
<td>M7-8</td>
<td>2.42</td>
<td>8.21</td>
<td>0.93</td>
</tr>
<tr>
<td>M9-10</td>
<td>3.42</td>
<td>9.98</td>
<td>1.84</td>
</tr>
<tr>
<td>S1-2</td>
<td>2.47</td>
<td>13.12</td>
<td>0.37</td>
</tr>
<tr>
<td>S3-4</td>
<td>2.59</td>
<td>11.87</td>
<td>0.30</td>
</tr>
<tr>
<td>S5-6</td>
<td>2.54</td>
<td>10.76</td>
<td>0.35</td>
</tr>
<tr>
<td>S7-8</td>
<td>2.41</td>
<td>10.01</td>
<td>0.32</td>
</tr>
<tr>
<td>S9-10</td>
<td>2.21</td>
<td>8.11</td>
<td>0.13</td>
</tr>
<tr>
<td>B1-2</td>
<td>2.15</td>
<td>9.93</td>
<td>0.19</td>
</tr>
<tr>
<td>B3-4</td>
<td>2.21</td>
<td>9.04</td>
<td>0.28</td>
</tr>
<tr>
<td>B5-6</td>
<td>2.18</td>
<td>7.85</td>
<td>0.16</td>
</tr>
<tr>
<td>B7-8</td>
<td>2.53</td>
<td>8.21</td>
<td>0.55</td>
</tr>
<tr>
<td>B9-10</td>
<td>3.06</td>
<td>9.25</td>
<td>1.05</td>
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</tbody>
</table>

Panel B: Unconditional Models

<table>
<thead>
<tr>
<th>Model</th>
<th>( \lambda_0 )</th>
<th>( \lambda_c )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage</td>
<td>1.947</td>
<td>0.172</td>
<td>0.584</td>
</tr>
<tr>
<td>(0.210)</td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coint</td>
<td>2.232</td>
<td>0.152</td>
<td>0.684</td>
</tr>
<tr>
<td>(0.097)</td>
<td>(0.027)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A of Table 1 presents descriptive statistics (in percentages) for the 15 characteristic sorted quintile portfolios. Value-weighted returns and log dividend growth rates are presented for portfolios formed on momentum (M), market capitalization (S), and book-to-market ratio (B). M1-2 represents the lowest momentum (loser) quintile, S1-2 the lowest size (small firms) quintile, and B1-2 the lowest book-to-market quintile. In the column labeled “Consumption Leverage,” the relevant measure, \( \phi_i \), is retrieved by performing the following regression:

\[
g_{i,t+1} = \delta_i + \phi_i x_t + \epsilon_{i,t+1}
\]

where \( g_{i,t} \) denotes the cash flow growth rate for portfolio \( i \) at time \( t \), and \( x_t \) denotes a two-year smoothed growth rate of real per capita consumption of nondurables and services at time \( t \). To measure the consumption leverage parameter \( \phi_i \) via stochastic cointegration we first detrend the log consumption and dividend series by regressing the log level of consumption and dividends on a constant and a time trend. The resulting de-trended time series, \( c_{i,t}^c \) and \( d_{i,t}^c \), are then used to measure \( \phi_i \). More specifically, we follow Stock and Watson (1993) and use DOLS to estimate \( \phi_i \), that is we estimate \( \phi_i \) via

\[
d_{i,t}^c = \mu_i + \phi_i c_{i,t}^c + \sum_{k=1}^{K} (a_k \Delta c_{i-k}^c + \alpha_k \Delta c_{i+k}^c) + \epsilon_{i,t}
\]

with \( K = 4 \). Robust standard errors are provided in the columns next to the parameter estimates. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the 3rd quarter 1967 through 4th quarter 1999. Panel B presents estimated risk premia and standard errors obtained from one-step GMM estimation of the risk measures and the consumption risk premium as discussed in the appendix. Risk prices are expressed in quarterly percentage terms.
Table 2: Summary Statistics

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Returns</th>
<th>Dividend Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>M1</td>
<td>-0.63</td>
<td>15.48</td>
</tr>
<tr>
<td>M2</td>
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<td>12.09</td>
</tr>
<tr>
<td>M3</td>
<td>1.29</td>
<td>10.57</td>
</tr>
<tr>
<td>M4</td>
<td>1.91</td>
<td>9.50</td>
</tr>
<tr>
<td>M5</td>
<td>1.59</td>
<td>8.62</td>
</tr>
<tr>
<td>M6</td>
<td>1.81</td>
<td>8.61</td>
</tr>
<tr>
<td>M7</td>
<td>2.27</td>
<td>8.63</td>
</tr>
<tr>
<td>M8</td>
<td>2.58</td>
<td>8.22</td>
</tr>
<tr>
<td>M9</td>
<td>3.18</td>
<td>9.38</td>
</tr>
<tr>
<td>M10</td>
<td>3.83</td>
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<td>13.53</td>
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<td>S8</td>
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<td>9.81</td>
</tr>
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<td>S9</td>
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<td>8.01</td>
</tr>
<tr>
<td>B1</td>
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<td>B2</td>
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</tr>
<tr>
<td>B3</td>
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<td>9.31</td>
</tr>
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<td>B5</td>
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<td>B10</td>
<td>3.19</td>
<td>10.49</td>
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</table>

Table 2 presents descriptive statistics (in percentages) for the 30 characteristic sorted decile portfolios. Value-weighted returns and log dividend growth rates are presented for portfolios formed on momentum (M), market capitalization (S), and book-to-market ratio (B). M1 represents the lowest momentum (loser) decile, S1 the lowest size (small firms) decile, and B1 the lowest book-to-market decile. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the 3rd quarter 1967 through 4th quarter 1999.
Table 3: Cross-Sectional Evidence (1967.3-1999.4): 30 Portfolios

Panel A: Unconditional Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda_0$</th>
<th>$\lambda_c$</th>
<th>$\lambda_g$</th>
<th>$\lambda_{vw}$</th>
<th>$\lambda_{SMB}$</th>
<th>$\lambda_{HML}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>(0.053)</td>
<td></td>
<td></td>
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</tr>
<tr>
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<td></td>
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<tr>
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<td>(0.022)</td>
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</tr>
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</tr>
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<td>(0.074)</td>
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<tr>
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</tr>
<tr>
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<td>(1.673)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>3-Factor</td>
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<tr>
<td></td>
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Panel B: Conditional Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda_0$</th>
<th>$\lambda_c$</th>
<th>$\lambda_g$</th>
<th>$\lambda_{vw}$</th>
<th>$\lambda_{g,k}$</th>
<th>$\lambda_{k,vw}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cond. CCAPM</td>
<td>1.870</td>
<td>0.260</td>
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<td>0.155</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.336)</td>
<td>(0.101)</td>
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<td>(0.059)</td>
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<td>Cond. CAPM</td>
<td>3.579</td>
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<tr>
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Panel C: 20 Portfolios (Excluding Momentum)

<table>
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<tr>
<th>Model</th>
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<th>$\lambda_c$</th>
<th>$\lambda_g$</th>
<th>$\lambda_{vw}$</th>
<th>$\lambda_{g,k}$</th>
<th>$\lambda_{SMB}$</th>
<th>$\lambda_{HML}$</th>
<th>$R^2$</th>
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<tr>
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<td>3-Factor</td>
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<td>0.673</td>
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<tr>
<td></td>
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<td></td>
<td>(0.803)</td>
<td>(0.082)</td>
<td>(0.148)</td>
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</tr>
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<td>0.034</td>
<td>0.023</td>
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<td>(0.060)</td>
<td>(0.036)</td>
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Table 3 presents results for cross-sectional regressions, utilizing a set of 30 portfolios (10 size, 10 momentum, and 10 book-to-market). Parameter estimates and robust standard errors are obtained in a single step via GMM as discussed in the appendix. The factors utilized in the analysis are: 1) The 2-year smoothed growth rate of log real per capita consumption of nondurables and services, $c$, 2) Log rate of change in real per capita consumption, $g$, 3) The value-weighted CRSP index return, $vw$, 4) The excess return on a portfolio of low market capitalization stocks over high market capitalization stocks, $SMB$, 5) The excess return on a portfolio of high book-to-market ratio stocks over a portfolio of low book-to-market ratio stocks, $HML$, and 6)-7) Cross-products of the conditioning variable, $k$, with the growth rate of consumption and the value-weighted index return. In Panel C, we repeat this exercise using a set of 20 portfolios (10 size and 10 book-to-market). $R^2$ represents the regression $R^2$ adjusted for degrees of freedom. Risk prices are expressed in quarterly percentage terms. The data cover the period 1967.3-1999.4, and are converted to real using the PCE deflator.
Table 4: Smoothed Growth Rate Risk Measures (1967.3-1999.4)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Quarter:1</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>-2.83</td>
<td>-3.97</td>
<td>-4.04</td>
</tr>
<tr>
<td>M2</td>
<td>2.91</td>
<td>1.59</td>
<td>-1.68</td>
</tr>
<tr>
<td>M3</td>
<td>1.58</td>
<td>0.34</td>
<td>-0.85</td>
</tr>
<tr>
<td>M4</td>
<td>1.58</td>
<td>-0.66</td>
<td>-0.25</td>
</tr>
<tr>
<td>M5</td>
<td>3.23</td>
<td>3.99</td>
<td>0.41</td>
</tr>
<tr>
<td>M6</td>
<td>0.62</td>
<td>0.72</td>
<td>3.32</td>
</tr>
<tr>
<td>M7</td>
<td>1.25</td>
<td>4.51</td>
<td>5.14</td>
</tr>
<tr>
<td>M8</td>
<td>-3.84</td>
<td>0.88</td>
<td>6.57</td>
</tr>
<tr>
<td>M9</td>
<td>-2.33</td>
<td>6.17</td>
<td>8.86</td>
</tr>
<tr>
<td>M10</td>
<td>-1.63</td>
<td>4.66</td>
<td>10.80</td>
</tr>
<tr>
<td>S1</td>
<td>2.18</td>
<td>3.25</td>
<td>1.57</td>
</tr>
<tr>
<td>S2</td>
<td>1.58</td>
<td>3.14</td>
<td>2.98</td>
</tr>
<tr>
<td>S3</td>
<td>0.93</td>
<td>2.57</td>
<td>0.89</td>
</tr>
<tr>
<td>S4</td>
<td>0.74</td>
<td>1.43</td>
<td>1.44</td>
</tr>
<tr>
<td>S5</td>
<td>-0.01</td>
<td>1.99</td>
<td>1.54</td>
</tr>
<tr>
<td>S6</td>
<td>1.30</td>
<td>2.53</td>
<td>2.33</td>
</tr>
<tr>
<td>S7</td>
<td>0.63</td>
<td>1.74</td>
<td>1.72</td>
</tr>
<tr>
<td>S8</td>
<td>0.06</td>
<td>-0.90</td>
<td>0.27</td>
</tr>
<tr>
<td>S9</td>
<td>0.34</td>
<td>1.97</td>
<td>1.03</td>
</tr>
<tr>
<td>S10</td>
<td>0.29</td>
<td>1.08</td>
<td>1.40</td>
</tr>
<tr>
<td>B1</td>
<td>1.30</td>
<td>2.40</td>
<td>4.00</td>
</tr>
<tr>
<td>B2</td>
<td>-1.34</td>
<td>-3.50</td>
<td>-3.54</td>
</tr>
<tr>
<td>B3</td>
<td>0.43</td>
<td>2.65</td>
<td>0.38</td>
</tr>
<tr>
<td>B4</td>
<td>-0.38</td>
<td>-1.40</td>
<td>-1.16</td>
</tr>
<tr>
<td>B5</td>
<td>-1.45</td>
<td>-1.23</td>
<td>-0.31</td>
</tr>
<tr>
<td>B6</td>
<td>-0.30</td>
<td>0.29</td>
<td>1.78</td>
</tr>
<tr>
<td>B7</td>
<td>-0.89</td>
<td>-0.23</td>
<td>0.96</td>
</tr>
<tr>
<td>B8</td>
<td>1.78</td>
<td>4.13</td>
<td>3.69</td>
</tr>
<tr>
<td>B9</td>
<td>0.79</td>
<td>3.95</td>
<td>5.15</td>
</tr>
<tr>
<td>B10</td>
<td>1.87</td>
<td>6.45</td>
<td>8.04</td>
</tr>
</tbody>
</table>

Panel B1: One Quarter  | Panel B2: Four Quarters
\[
\begin{array}{ccc}
\lambda_0 & \lambda_e & R^2 \\
2.264 & -0.022 & -0.033 \\
(0.238) & (0.127) & \\
\end{array}
\begin{array}{ccc}
\lambda_0 & \lambda_e & R^2 \\
1.940 & 0.188 & 0.341 \\
(0.259) & (0.078) & \\
\end{array}
\]

Table 4 presents risk measures and results for cross-sectional regressions, utilizing a set of 30 decile portfolios (10 momentum (M), 10 size (S), and 10 book-to-market (B)). M1 represents the lowest momentum (loser) decile, S1 the lowest size (small firms) decile, and B1 the lowest book-to-market decile. Parameter estimates and robust standard errors are obtained from a one-step GMM regression as discussed in the appendix. The factor utilized in the analysis is the log rate of change in real per capita consumption, g. Cash flow growth rates are regressed on one-quarter consumption growth (Panel B1), smoothed 4 quarters consumption growth (Panel B2), and smoothed 8 quarters consumption growth (as in Table 3). R^2 represents the regression R^2 adjusted for degrees of freedom. Risk prices are expressed in quarterly percentage terms. The data cover the period 1967.3-1999.4, and are converted to real using the PCE deflator.
Table 5: Relative Merits of Consumption Leverage and Factor Models (1967.3-1999.4)

Panel A: 30 Portfolios

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda_0$</th>
<th>$\lambda_c$</th>
<th>$\lambda_g$</th>
<th>$\lambda_{vw}$</th>
<th>$\lambda_{g-k}$</th>
<th>$\lambda_{k-vw}$</th>
<th>$\lambda_{SMB}$</th>
<th>$\lambda_{HML}$</th>
<th>$R^2$</th>
<th>Wald</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>6.478</td>
<td>0.141</td>
<td>-4.473</td>
<td>0.128</td>
<td>-0.117</td>
<td>0.599</td>
<td>4.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.510)</td>
<td>(0.035)</td>
<td>(2.484)</td>
<td>(0.222)</td>
<td>(0.345)</td>
<td>(0.23)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cond. CCAPM</td>
<td>1.690</td>
<td>0.165</td>
<td>0.061</td>
<td>0.037</td>
<td>0.482</td>
<td>1.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.292)</td>
<td>(0.058)</td>
<td>(0.090)</td>
<td>(0.054)</td>
<td>(0.51)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cond. CAPM</td>
<td>2.441</td>
<td>0.160</td>
<td>-0.376</td>
<td>-0.262</td>
<td>0.509</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.092)</td>
<td>(0.043)</td>
<td>(1.021)</td>
<td>(0.644)</td>
<td>(0.73)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: 20 Portfolios (Excluding Momentum)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda_0$</th>
<th>$\lambda_c$</th>
<th>$\lambda_g$</th>
<th>$\lambda_{vw}$</th>
<th>$\lambda_{g-k}$</th>
<th>$\lambda_{k-vw}$</th>
<th>$\lambda_{SMB}$</th>
<th>$\lambda_{HML}$</th>
<th>$R^2$</th>
<th>Wald</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>0.270</td>
<td>0.031</td>
<td>1.931</td>
<td>0.268</td>
<td>0.545</td>
<td>0.706</td>
<td>191.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.801)</td>
<td>(0.022)</td>
<td>(0.793)</td>
<td>(0.069)</td>
<td>(0.087)</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cond. CCAPM</td>
<td>2.073</td>
<td>0.068</td>
<td>-0.011</td>
<td>-0.004</td>
<td>0.545</td>
<td>7.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.036)</td>
<td>(0.032)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cond. CAPM</td>
<td>1.793</td>
<td>0.035</td>
<td>0.382</td>
<td>0.281</td>
<td>0.526</td>
<td>7.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(0.042)</td>
<td>(0.261)</td>
<td>(0.159)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5 presents results for cross-sectional regressions, utilizing a set of 30 portfolios (10 size, 10 momentum, and 10 book-to-market). We augment the consumption leverage model (using the growth rate based estimates) to include additional factors: 1) the FF factors, $vw$, $SMB$, and $HML$; 2) the conditional CCAPM, with both the log rate of change in real per capita consumption, $g$, and its cross product with the conditioning variable, $k$; and 3) the conditional CAPM, with the $vw$ market factor and its cross product with $k$. Risk prices are expressed in quarterly percentage terms. In Panel B, we repeat this exercise with a set of 20 portfolios (10 size and 10 book-to-market). All data are converted to real using the PCE deflator and are measured at the quarterly frequency. The column labeled “Wald” presents Wald statistics for the joint significance of the coefficients with respect to the additional factors. $p$-values for these statistics are displayed in parentheses.
Table 6: **Summary Statistics (1953.1-1999.4)**

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Returns</th>
<th>Dividend Growth</th>
<th>Consumption Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>M1-2</td>
<td>0.75</td>
<td>11.75</td>
<td>-1.36</td>
</tr>
<tr>
<td>M3-4</td>
<td>1.74</td>
<td>8.97</td>
<td>-0.21</td>
</tr>
<tr>
<td>M5-6</td>
<td>2.14</td>
<td>7.91</td>
<td>0.39</td>
</tr>
<tr>
<td>M7-8</td>
<td>2.63</td>
<td>7.90</td>
<td>0.93</td>
</tr>
<tr>
<td>M9-10</td>
<td>3.83</td>
<td>9.46</td>
<td>1.73</td>
</tr>
<tr>
<td>S1-2</td>
<td>3.00</td>
<td>12.13</td>
<td>0.58</td>
</tr>
<tr>
<td>S3-4</td>
<td>2.97</td>
<td>11.00</td>
<td>0.48</td>
</tr>
<tr>
<td>S5-6</td>
<td>2.87</td>
<td>9.97</td>
<td>0.53</td>
</tr>
<tr>
<td>S7-8</td>
<td>2.70</td>
<td>9.28</td>
<td>0.50</td>
</tr>
<tr>
<td>S9-10</td>
<td>2.39</td>
<td>7.69</td>
<td>0.35</td>
</tr>
<tr>
<td>B1-2</td>
<td>2.38</td>
<td>9.23</td>
<td>0.32</td>
</tr>
<tr>
<td>B3-4</td>
<td>2.28</td>
<td>8.31</td>
<td>0.33</td>
</tr>
<tr>
<td>B5-6</td>
<td>2.64</td>
<td>7.45</td>
<td>0.62</td>
</tr>
<tr>
<td>B7-8</td>
<td>2.79</td>
<td>8.05</td>
<td>0.65</td>
</tr>
<tr>
<td>B9-10</td>
<td>3.22</td>
<td>9.19</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Table 6 presents descriptive statistics (in percentages) for the 15 characteristic sorted quintile portfolios. Value-weighted returns and log dividend growth rates are presented for portfolios formed on momentum (M), market capitalization (S), and book-to-market ratio (B). M1-2 represents the lowest momentum (loser) quintile, S1-2 the lowest size (small firms) quintile, and B1-2 the lowest book-to-market quintile. In the column labeled “Consumption Leverage,” the relevant measure, \( \varphi_i \), is retrieved by performing the following regression:

\[
g_{i,t+1} = \delta_i + \varphi_i x_t + \epsilon_{i,t+1}
\]

where \( g_{i,t} \) denotes the cash flow growth rate for portfolio \( i \) at time \( t \), and \( x_t \) denotes a two-year smoothed growth rate of real per capita consumption of nondurables and services at time \( t \). To measure the consumption leverage parameter \( \phi_i \) via stochastic cointegration we first detrend the log consumption and dividend series by regressing the log level of consumption and dividends on a constant and a time trend. The resulting detrended time series, \( c_{t}^* \) and \( d_{t}^* \), are then used to measure \( \phi_i \). More specifically, we follow Stock and Watson (1993) and use DOLS to estimate \( \phi_i \), that is we estimate \( \phi_i \) via

\[
d_{t,i}^* = \mu_i + \phi_i c_{t}^* + \sum_{k=1}^{K} (\alpha_{-k} \Delta c_{t-k}^* + \alpha_k \Delta c_{t+k}^*) + \epsilon_{i,t}
\]

with \( K = 4 \). Robust standard errors are provided in the columns next to the parameter estimates. Data are converted to real using the PCE deflator. The data are sampled at the quarterly frequency, and cover the 1st quarter 1953 through 4th quarter 1999.
The figure presents the dividend growth series for the top and bottom momentum quintile portfolios, as well as the trailing eight quarter moving average of consumption growth. The correlation refers to the correlation between the dividend growth series and the trailing eight quarter moving average of consumption growth. \( \varphi \) refers to the regression slope coefficient from regressing the dividend growth rate series on the trailing eight quarter moving average of consumption growth.
The figure presents the dividend growth series for the top and bottom capitalization quintile portfolios, as well as the trailing eight quarter moving average of consumption growth. The correlation refers to the correlation between the dividend growth series and the trailing eight quarter moving average of consumption growth are presented. \( \varphi \) refers to the regression slope coefficient from regressing the dividend growth rate series on the trailing eight quarter moving average of consumption growth.
The figure presents the dividend growth series for the top and bottom book-to-market quintile portfolios, as well as the trailing eight quarter moving average of consumption growth. The correlation refers to the correlation between the dividend growth series and the trailing eight quarter moving average of consumption growth are presented. \( \varphi \) refers to the regression slope coefficient from regressing the dividend growth rate series on the trailing eight quarter moving average of consumption growth.
Figure 4: Scatterplots: Unconditional Models

The figure presents scatterplots for the unconditional models estimated in the paper. The fitted expected returns are plotted against mean realized returns.
Figure 5: Scatterplots: Conditional Models

The figure presents scatterplots for the conditional models estimated in the paper. The fitted expected returns are plotted against mean realized returns.