AIDS Crisis and Growth

Paul Corrigan  
Michigan State University

Gerhard Glomm  
Indiana University

Fabio Mendez  
University of Arkansas

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Abstract: We investigate an OLG economy where an AIDS epidemic influences human capital accumulation and growth through the creation of large numbers of orphans. We study how intra-family allocations regarding school and work time of children are adjusted in the face of AIDS within a family, and how, in turn, these adjustments influence accumulation of physical and human capital. We compute the aggregate effects of an AIDS epidemic on human and physical capital accumulation and growth. We find that growth effects of an AIDS epidemic are large. Some policies such as subsidization of AIDS medication have relatively small effects.

Corresponding Author: Gerhard Glomm, Department of Economics, Wylie Hall, Indiana University, Bloomington, IN 47405, phone: 812-855-7256, e-mail: gglomm@indiana.edu.

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I. Introduction

This paper addresses the question: What is the effect of the massive AIDS crisis in sub-Saharan Africa on economic growth? Answers to this question matter for two reasons: First, even moderate growth effects can have a sizeable impact on the welfare of future generations. Second, neither is there any cure, nor is there any chance of controlling the epidemic in the near future. Predicting the effects of the AIDS crisis, and evaluating the usefulness of proposed policies to mitigate its economic effects (such as subsidizing AIDS medications) are hence of great importance.

Just what those effects might be, however, seem rather dependent on the techniques and model used. Early empirical evidence gathered in the 1990s surprisingly pointed to small or unimportant growth effects of the AIDS epidemic. Regressing growth rates on AIDS incidence for a cross-section of countries, Bloom and Mahal (1997) found small or insignificant coefficients. The World Bank report Confronting AIDS: Public Priorities in a Global Epidemic (1997) came to a similar conclusion and argues that growth effects of the AIDS crisis are likely to be small.

In contrast, many of the theoretical contributions that concentrate on the potential long-run (future) effects predict that the AIDS crisis will have a much larger impact. Kambou, Devarajan and Over (1992), for example, used a simulation model of the economy of Cameroon and conclude that AIDS can cut the rate of GDP growth by up to 50%. Cuddington and Hancock (1994) used a Solow type growth model to study the growth effects of AIDS in Malawi. This model was extended in Cuddington and Hancock (1995) to allow a dualistic labor surplus economy and in Cuddington, Hancock and Rogers (1994) to include demographic and epidemic equations that can be used to study how AIDS related policies impact the
macroeconomy. Similar conclusions concerning sizeable aggregate effects have been reported by Arndt and Lewis (2000). The potential for large effects of the AIDS epidemic on macroeconomic outcomes is confirmed in more disaggregated frameworks in Arndt and Lewis (2001) and Arndt (2003).

There are several channels through which the AIDS epidemic can potentially influence economic growth:

1. AIDS decreases life expectancy and hence the incentive to invest.
2. The AIDS epidemic creates a huge number of orphans. In all likelihood human capital accumulation by orphans faces more obstacles than if parents were present.
3. Potentially large medical costs associated with treatment of AIDS patients can divert public resources from productivity enhancing public expenditures such as education or infrastructure investments.
4. Firms might be reluctant to hire and invest in training of workers if there is a high likelihood of workers dying because of AIDS.
5. To the extent that AIDS is more prevalent among highly skilled workers and that highly skilled labor is complementary to capital, the AIDS epidemic can influence the rate of return on investment and, in turn, savings and capital accumulation.

In this paper we concentrate on the first two channels, viz. the effect of the drop in life expectancy on investment and the large generation of orphans produced by AIDS. In a recent paper, Ferreira and Pessoa (2003) consider how the AIDS-induced decrease in life expectancy decreases the incentive to invest in human capital. They find that in the face of an AIDS epidemic schooling time can decline by half; the potential for large growth effects as a result is obvious.
The large generation of AIDS orphans and their potential impact on African economies has received much less attention. In some countries HIV/AIDS infection rates now exceed 20% of the adult population; in Swaziland, for example, the adult infection rate for 2001 was over 33% (UNAIDS/WHO Epidemiological Fact Sheet). With no cure being available now (or likely to be in the near future), and treatments used in the developed world being prohibitively costly, adults facing early deaths are leaving behind large numbers of orphans. According to some estimates there are already over 13 million orphans due to AIDS. By 2010 the number of orphans is expected to increase to levels exceeding 25 million (see Hunter and Williamson (2002)). For Mozambique, Arndt (2003) estimates that by 2010 roughly every fourth child will have lost at least one parent.

Orphans typically have fewer opportunities to obtain human capital than if parents were alive. Available evidence points to significant reallocations of resources within families where AIDS is present. Often, upon the death of a parent, children’s time is allocated away from schooling to work in order to supplement family income. According to UNAIDS (2000) children are often pulled from school when a parent contracts AIDS. Moreover, the presence of AIDS in a family requires additional expenditures both in terms of time and in terms of financial expenditures for the (medical) care of the AIDS patient, so that fewer resources are available for schooling and other investments.

In section II we present a model in which the AIDS epidemic can influence long-run growth. This model is an OLG economy where individuals live potentially for three periods: youth, adulthood and old age. In each period a fraction of the adult population contracts AIDS, which implies certain death before the third period, old age. There are two augmentable factors in the model, physical capital (which is accumulated in the standard fashion from savings of the
young) and human capital. Our approach differs from earlier theoretical research on this issue (e.g. Kambou, Devarajan and Over, 1992, and Cuddington and Hancock, 1994) inasmuch as earlier papers in this vein did not explicitly model intertemporal choice.

The accumulation of human capital follows Lucas (1988) in the sense that the two inputs in future human capital are time and parental human capital. Each child is endowed with one unit of time, which can be allocated to learning or to working so that family income may be supplemented. Parents make time allocation decisions for their children; hence each individual makes decisions only in the two periods adulthood and old age.

If there is AIDS in the family, the parents also decide how to allocate family income between standard consumption goods such as food, clothing and shelter and medical expenses for the AIDS patient. The related medical expenditures are modeled as utility enhancing goods only. The definition of equilibrium for this simple version of model is given in section III; we derive the equilibrium solution in section IV.

In section V, we present and solve a modified version of our model in which AIDS related medical expenditures can enhance labor productivity of the AIDS infected worker. This modification is motivated by the lower labor productivity exhibited by AIDS infected workers (see, for example, Guinness and Alban (2000)) and the effect that medications may have upon their productivity. This modification also allows us to better study the effects of public and international subsidies of AIDS medications, as well as the effects of international laws that control the distribution of generic-version drugs that sell at a fraction of the standard price. The idea behind such policy recommendations is to make AIDS patients well enough to live more or less normal and productive lives, which would include being better able to care for their children (who themselves could perhaps return to school).
In section VI we solve both versions of the model numerically and present the main results. We calibrate this model to sub-Saharan economies and run the model through some potential AIDS epidemics scenarios. Finally, section VII provides our conclusions.

II. The Simple Model

Each individual in the economy belongs to a dynasty indexed by $\omega \in \Omega$, where $\Omega$ spans the entire population in each generation. We normalize the size of the young population each period to one. Each individual of each generation $t$ lives for, at most, three periods. In the first period of life (hereafter “youth”) the individual either works or learns. Learning, here, is either formal schooling or receiving informal education from parents. What matters is that learning is associated with an opportunity cost, foregone wage income.

In the second period of life (“adulthood”), the individual inelastically supplies labor to the firm. At the beginning of adulthood an individual learns if he has HIV/AIDS; he faces a probability $\pi_t$ of remaining healthy and a probability of $1-\pi_t$ of being infected with HIV/AIDS. HIV/AIDS infection implies a loss of health, which lowers labor income in the second period of life. Infection also implies certain death at the end of period two. Our modeling of mortality risk as purely exogenous is similar to the way it is modeled in Pecchenino and Utendorf (1999).

As a consequence, only individuals who remain uninfected save a portion of their income for the third period (“old age”). In contrast, an individual who is ill does not save at all in the second period; but in addition to the “usual” consumption expenses, an HIV/AIDS infected individual faces medical expenditures. We think of these medical expenses only as those associated only with HIV/AIDS.

In our model, adults make all schooling/working decisions for the family, including their children. We assume that effective labor of HIV/AIDS infected individuals is a fraction $\Psi<1$ of
effective labor of healthy individuals. We also assume that effective labor of children is a fraction $\Delta < 1$ of effective labor of the healthy adults.

There is a government that taxes labor income of all adults. In our model labor income of children is not taxed; this assumption captures children working in the household or in the informal sector. The tax revenue is used to finance government consumption or investment in some non-productive kind of capital. This tax is introduced to mirror one of the many distortions in African economies described in Collier and Gunning (1999). Our interpretation of government investment in non-productive capital is consistent with Devarajan, Easterly and Pack (2000). The only other item in the government budget is a potential subsidy to AIDS related health care. We assume that the government’s budget constraint is balanced in each period.

The preferences of a healthy individual are given by
\[ \alpha_1 \ln c_t + \alpha_2 \ln f_t + \alpha_3 \ln c_{t+1} + \alpha_4 \ln h_{t+1}, \]
where $c_t$ is the individual’s own consumption in adulthood, $f_t$ is the consumption of his children, $c_{t+1}$ is the individual’s consumption in old age, and $h_{t+1}$ is his children’s human capital. We have introduced this type of intergenerational altruism into the model to ensure that investment in children’s human capital is positive.

An individual who is HIV/AIDS infected knows that he will not survive to period three, and thus does not derive utility from old-age consumption. However, HIV infection does bring about medical expenses, which enter the individual’s preferences. Thus, a HIV positive individual’s preference function is
\[ \alpha_1^{1/\rho} \ln [c_t^\rho + \theta m_t^\rho] + \alpha_2 \ln f_t + \alpha_4 \ln h_{t+1}, \]
where $m_t$ is AIDS related medical expenditures and where $\rho$ and $\theta$ are preference parameters, with $\rho$ measuring the elasticity of substitution and $\theta$ the share of medical expenses in the budget.
The technology is described by a standard Cobb-Douglas production function

\[ Y_t = A K_t^\alpha L_t^{1-\alpha}, \]

where \( Y_t \) is aggregate output, \( K_t \) is the aggregate capital stock and \( L_t \) is the aggregate stock of effective labor in the economy.

In this model there are three different kinds of labor: labor by healthy adults; labor by HIV/AIDS infected adults; and labor by children. We assume that all three types of labor are perfect substitutes so that effective labor is simply the weighted average of the three kinds of labor, which can be written as

\[ L_t = \{\pi_t [1 + (1-n_t^h)\Delta] + (1-\pi_t) [\Psi + (1-n_t^s)\Delta]\} H_t, \]

where \( H_t \) represents aggregate human capital at time \( t \) and \( n \) represents the fraction of the children’s time that is devoted to learning, both when parents are healthy \((n_t^h)\) and when parents are sick \((n_t^s)\).

Human capital of each child is produced using human capital of the parent and the child’s own time as an input. For simplicity we assume, that the learning technology is linear in the time input. This production function is given by

\[ h_{t+1} = B^i n_t h_t, \]

where \( B^i \) takes on the value \( B^H \) when the parent is healthy and \( B^S \) when the parent is sick. We assume \( B^H \geq B^S \), as a child’s return to schooling will be lower if he has no parent present to devote time to his development. In our base case scenario we let \( B^H = B^S = B \) and then we conduct sensitivity analysis for the case \( B^H > B^S \).

**III. Equilibrium**

In this section, we state the household’s maximization problems and we define a competitive equilibrium for our economy. A healthy middle-aged adult solves the problem
max \{c_t, f_t, c_t+1, n_t\} \quad \alpha_1 \ln c_t + \alpha_2 \ln f_t + \alpha_3 \ln c_{t+1} + \alpha_4 \ln h_{t+1}

s.t. \quad c_t + f_t + s_t = (1-\tau_t) w_t h_t + (1 - n_t) w_t h_t \Delta
\quad c_{t+1} = (1 + r_{t+1}) s_t
\quad h_{t+1} = B n_t h_t,

where \(w_t\) is the wage rate per effective unit of labor, \(r_{t+1}\) is the net real interest rate and \(\tau\) is the labor income tax rate imposed by the government. We will refer to this as problem PH.

An HIV/AIDS infected person solves the problem

max \{c_t, f_t, m_t, n_t\} \quad \alpha^1 / \rho \ln [c_t^\rho + \theta m_t^\rho] + \alpha_2 \ln f_t + \alpha_4 \ln h_{t+1}

s.t. \quad c_t + f_t + \sigma_t p_t m_t = (1-\tau_t) w_t h_t \Psi + (1-n_t) w_t h_t \Delta
\quad h_{t+1} = B n_t h_t,

where \(p_t\) is the pre-subsidy price of medical care and \(\sigma_t\) expresses the subsidy rate so that \(\sigma_t p_t\) is the post-subsidy price of medical care relative to consumption. We will refer to this as problem PS.

We are now in a position to define an equilibrium for this economy.

**Definition:** A competitive equilibrium for this economy is a collection of sequences of distributions of individual household decisions \(\{c^h_t, c^h_{t+1}, f^h_t, n^h_t\}_{t=0}^\infty\) when healthy, \(\{c^s_t, f^s_t, m^s_t, n^s_t\}_{t=0}^\infty\) when sick, sequences of aggregate stocks of physical capital \(\{K_t\}_{t=0}^\infty\) and effective labor \(\{L_t\}\) and sequences of prices \(\{w_t, r_{t+1}, p_t\}_{t=0}^\infty\) satisfying

(i) the decisions \(\{c^h_t, c^h_{t+1}, f^h_t, n^h_t\}\) solve problem PH,
(ii) the decisions \(\{c^s_t, f^s_t, m^s_t, n^s_t\}\) solve problem PS,
(iii) capital markets clear so that
\[ K_{t+1} = \int_{\Omega} s_t(\omega) \, d\mu(\omega), \]

(iv) human capital follows the law of motion

\[ h_{t+1} = B^s n_t^s h_t \quad \text{or} \quad h_{t+1} = B^h n_t^h h_t \]

depending on health status,

(v) factor markets are competitive so that factor prices are determined by

\[ w_t = (1-\alpha) \frac{Y_t}{L_t}, \quad r_t = \alpha \frac{Y_t}{K_t} \]

(vi) the government budget constraint is satisfied every period, that is

\[ G_t + (1-\sigma_t) p_t M_t = (\pi_t + (1-\pi_t) \Psi) \tau_t w_t H. \]

Note that only factor prices are determined endogenously in the model; the price of medical care is taken to be exogenous. What we have in mind is that the price of the AIDS medications is determined in a world market and the small African country takes this price as given. Alternatively we can think of a constant returns to scale production function which transforms one unit of output into \( \frac{1}{p_t} \) units of medical services. If medical services are supplied in a competitive market the price of medical services will be equal to its marginal cost \( p_t \).

IV. Solving the Model

We can now solve the model. After some algebra, the first order conditions for problem \( P^H \) of a healthy individual yield

\[ (1) \quad c_t^h = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} (1 + \Delta - \tau_t) w_t h_t, \]

\[ (2) \quad f_t^h = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} (1 + \Delta - \tau_t) w_t h_t, \]

\[ (3) \quad s_t = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} (1 + \Delta - \tau_t) w_t h_t, \]
As only the proportion of healthy individuals $\pi_i$ are doing any saving, and health is independent of human capital level, equation (3) implies that

$$K_{t+1} = \frac{\pi_i \alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} (1 + \Delta - \tau_i)w_i H_i.$$  

The first order conditions of problem $P^S$ of an HIV/AIDS infected individual yield

$$c_t^s = \frac{\alpha_i}{\alpha_1 + \alpha_2 + \alpha_4} \cdot \frac{1}{1 + \theta^{1/p} (\sigma_i p_i)^{-1/p}} (\Delta + \Psi(1 - \tau_i))w_i h_i,$$

$$f_t^s = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_4} \cdot (\Delta + \Psi(1 - \tau_i))w_i h_i,$$

$$m_t^s = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_4} \cdot \frac{1}{1 + \theta^{1/p} (\sigma_i p_i)^{-1/p}} (\Delta + \Psi(1 - \tau_i))w_i h_i,$$

implying that

$$p_t m_t^s = \frac{\alpha_i}{\alpha_1 + \alpha_2 + \alpha_4} \cdot \frac{1}{1 + \theta^{1/p} (\sigma_i p_i)^{-1/p}} (\Delta + \Psi(1 - \tau_i))w_i h_i.$$  

Economy-wide spending on medical services then is given by

$$p_i M_i = (1 - \pi_i) \cdot \frac{\alpha_i}{\alpha_1 + \alpha_2 + \alpha_4} \cdot \frac{1}{1 + \theta^{1/p} (\sigma_i p_i)^{-1/p}} (\Delta + \Psi(1 - \tau_i))w_i H_i,$$

and individual schooling time is
Notice that schooling time when parents are infected $n_t^s$ is smaller than schooling time when parents are healthy $n_t^h$ only when $\Psi$, the productivity of ill parents, is sufficiently small. If infected parents were as productive as healthy parents, schooling time with infected parents would be higher than with healthy parents. This is because with healthy parents there are more demands for income from child labor, i.e. consumption for old age. Thus, in order to ensure that $n_t^h > n_t^s$, we assume that $\Psi$ is sufficiently low.

Conditions (5) and (12) give an economy-wide rate of growth of human capital and the corresponding labor input as

$$H_{t+1} = \left( B^H \pi_t, \frac{\alpha_4 (\Delta + \Psi (1 - \tau_i))}{\Delta (\alpha_1 + \alpha_2 + \alpha_4)} + B^S (1 - \pi_t), \frac{\alpha_4 (\Delta + \Psi (1 - \tau_i))}{\Delta (\alpha_1 + \alpha_2 + \alpha_4)} \right) H_t$$

$$L_t = (\pi_t (1 + \Delta (1 - n_t^h)) + (1 - \pi_t) (\Psi + \Delta (1 - n_t^s))) H_t,$$

$$= \left( \pi_t \left( 1 + \frac{\Delta (\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 (1 - \tau_i))}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} \right) + (1 - \pi_t) \left( 1 + \frac{\Delta (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) \Psi (1 - \tau_i)}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} \right) \right) H_t.$$  

The budget-balance condition gives us

$$\pi_t + \Psi (1 - \pi_t) w_t H_t \tau_i = G_t + (1 - \sigma_t) p_t M_t.$$

According to equation (15) any fixed tax rate and any fixed level of (wasteful) government expenditures $G_t$ uniquely determines the subsidy rate for medical care. When we solve the model numerically, we will fix $\tau_i$ and $G_t$ exogenously. Together with the endogenous variables, the government budget constraint determines the subsidy rate $\sigma$.

From the firm’s profit maximization we obtain

$$w_t = (1 - \alpha) Y_t / L_t.$$
Equations (5) and (12) yield solutions for the time allocations $n^t_h$ and $n^t_i$, respectively. Given the time allocations, equation (13) generates the law of motion for aggregate human capital which, with equation (16), determines the real wage $w_t$. The law of motion for physical capital is then given by equation (6). Aggregate medical expenditures are determined by (11) and, finally, given the wasteful government expenditures $G_t$ and the tax rate $\tau$, the government budget constraint determines the subsidy rate $\sigma$.

V. The Modified Model

In the model presented in section II medical expenditures are only utility enhancing. In the version of the model presented here medical expenditures enhance labor productivity of HIV positive workers. All aspects of the model are the same as in section II, except here $\Psi$ is a function of medical expenditures of the HIV positive individual. The function $\Psi(m_t)$ satisfies the following properties:

(i) $\Psi(0) = \Psi_0$

(ii) $\lim_{x \to \infty} \Psi(x) = \Psi_1$, with $0 < \Psi_0 < \Psi_1 < 1$

(iii) $\Psi'(x) > 0, \Psi''(x) < 0$.

The precise functional form we use for $\Psi$ in our experiments is

$$\Psi(m_t) = \Psi_1 - (\Psi_1 - \Psi_0) \frac{\beta}{m_t + \beta}$$

where $\beta > \theta$; it can be shown that this function satisfies all the above conditions.

The maximization problems solved by the healthy individual and the firm, the government budget constraint and the human capital accumulation formation are exactly as in
section II. The HIV positive individual’s problem does differ, however; he now solves the problem

$$\max_{c_i, f_i, m_i, n_i} \alpha_i \ln c_i + \alpha_2 \ln f_i + \alpha_4 \ln h_{i+1},$$

s.t.  \( c_i + f_i + \sigma_i p_i m_i = w_i h_i [\Psi(m_i)(1 - \tau_i) + \Delta(1 - n_i)] \)

\[ h_{i+1} = Bn_i h_i \]

The first order conditions for an interior solution are

(17) \[ \frac{\alpha_i}{c_i} = \lambda_i \]

(18) \[ \frac{\alpha_2}{c_i} = \hat{\lambda}_i \]

(19) \[ \frac{\alpha_4}{n_i} = \lambda_i \Delta w_i h_i \]

(20) \[ \sigma_i p_i = w_i h_i (1 - \tau_i) \beta (\Psi_i - \Psi_0) (m_i + \beta)^{-2} \]

where \( \lambda_i \) is the Lagrange multiplier. We solve equation (20) for \( m_i \) to get

(21) \[ m_i = \left[ \frac{w_i h_i (1 - \tau_i) \beta (\Psi_i - \Psi_0)}{\sigma_i p_i} \right]^{-\frac{1}{2}} - \beta \]

With (21) in hand we can write labor productivity as

(22) \[ \Psi(m_i) = \Psi_i - (\Psi_i - \Psi_0)^{\frac{1}{2}} \beta^{\frac{1}{2}} \left[ \frac{w_i h_i (1 - \tau_i)}{\sigma_i p_i} \right]^{-\frac{1}{2}}. \]

Using (17)-(20), (22) and the budget constraint we can solve for the equilibrium schooling time of a child whose parent is HIV positive. This yields

(23) \[ n_i = \frac{\alpha_i}{\alpha_i + \alpha_2 + \alpha_4 \Delta w_i h_i} \left[ \beta \sigma p_i - 2 \sigma p_i \beta (\Psi_i - \Psi_0)^{\frac{1}{2}} (w_i h_i (1 - \tau_i))^{\frac{1}{2}} + w_i h_i (1 - \tau_i) \Psi_i + \Delta \right]. \]
Notice that in this version of the model schooling time when parents are infected is a function of the parental stock of human capital. This aspect of the solution complicates the equilibrium dynamics, since the equations of motion for \( H, K \) and \( L \) can no longer be calculated as linear functions of the initial level of human capital. Given the expression in equation (23) and a hypothetical date \( x \) when the epidemic starts, for each period \( t \) there are \( 2^{t-x} \) individual values of \( \{n^t, h\} \) depending on the entire health history of all ancestors. In our numerical exercises for this version of the model we consider scenarios where the AIDS crisis persists for up to two generations. Thus, we need keep track of four different types at most.

The aggregate values of all variables are obtained as the sum of all individual values; where all probable individual values are calculated based on the solutions of \( P^H \) and \( P^S \) depending on the entire past health history. Although it is not practical to show all our calculations, as a manner of illustration, we show the aggregation of human capital after AIDS strikes at time \( t \) for periods \( t+1 \) and \( t+2 \) in the following equations:

\[
H_{t+1} = \int_{\text{Sick-Parents}} h_{t+1} + \int_{\text{Healthy-Parents}} h_{t+1} = h_i n^{t}_{(s)} B(1 - \pi) + h_i n^{t}_{(s)} B \pi
\]

\[
H_{t+2} = \int_{\text{Sick-Parents}} h_{t+1} + \int_{\text{Healthy-Parents}} h_{t+1} + \int_{\text{Sick-Grand-Parents}} h_{t+1} + \int_{\text{Healthy-Grand-Parents}} h_{t+1} = h_i n^{t}_{(s)} B^2 n^{t+1}_{(s)} (1 - \pi)^2 + h_i n^h B^2 n^{t}_{(s)} \pi (1 - \pi) + h_i n^h B^2 n^{t+1}_{(s)} \pi (1 - \pi) + h_i (n^h B)^2 \pi^2
\]

where \( n^t \) is expressed as a function of time, since it depends on the value of \( h_i \).

All aspects of this modified model, except for the aggregation mechanisms, are identical to the one in section II and can be solved in the same fashion as before.

VI. Calibration and Computational Experiments

For our simulations, we chose parameters for both models roughly corresponding to values typical in sub-Saharan nations. We assume first that prior to about 1980 AIDS infection
rates were negligible. We then chose parameters so that the growth rate of real per capita GDP would be 2%. We let the economy run for 9 periods until a balanced growth path was reached. We then assume that the AIDS epidemic begins in period 9. We consider four different scenarios for the AIDS crisis differing principally by the length of time and depth of the crises. The parameter values we use for all simulations are contained in table 1.

<table>
<thead>
<tr>
<th>Preference parameters</th>
<th>Taxes</th>
<th>Technology parameters</th>
<th>Health productivity parameters</th>
<th>Relative productivity parameters</th>
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<td>$\frac{\alpha}{\tau} = 0.2$</td>
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Table 1. Parameter values for calibration

We assume $\alpha_1 = 1, \alpha_2 = 0.4, \alpha_3 = 0.3$ and $\alpha_4 = 0.2$. In both models, we chose the preference parameters $\theta, \beta$ and $\rho$ to ensure private health care expenditures on AIDS would be small. According to the WHO (2002) private expenditure on health are roughly between 20 and 60% of all health care expenditures, which, in turn, are usually less than 10 percent of GDP. We therefore pick $\theta, \beta$ and $\rho$ so that private AIDS-related health care expenditures are no more than twenty percent of the income of the sick individuals.
Our parameter value for capital’s share of income is standard (see Gollin (2002)). The choices of parameters \( A, B^H \) and \( B^S \) ensure a pre-AIDS annual growth rate of real per capita income of 2%. According to Lebergott (1964) wage income from child labor relative to adult labor is about 0.15. Since our results might be sensitive to changes in this parameter value, we conducted sensitivity analysis letting this parameter vary from .05 to 0.3. In the simple model, for the relative productivity of AIDS infected workers we chose \( \Psi = 0.5 \). This roughly corresponds to AIDS related productivity losses reported in Guinness and Alban (2000), who find losses in agricultural output of up to 60%. Similarly, for the modified model we chose \( \Psi_1 = 1 \) and \( \Psi_0 = 0.5 \), so that productivity could not be greater than one or smaller than 0.5 (depending on the amount of medical expenses).

First Model

Given the uncertainty about the length and the propagation of the current AIDS epidemic, we analyzed four different plausible scenarios in our numerical simulations of the simple model:

- **Scenario 1:** In period 9 the AIDS epidemic decreases survival probability \( \pi \) to 80%. AIDS is present for one period only.

- **Scenario 2:** Starting in period 9 the AIDS epidemic decreases survival probability \( \pi \) to 80% forever.

- **Scenario 3:** Starting in period 9 the AIDS epidemic decreases survival probability for two generations to 80%. Starting with period 11, the survival probability returns to 100%.

- **Scenario 4:** Starting in period 9 the AIDS epidemic arrives and survival rates fall gradually. In particular the survival rates are \( \pi_9 = 95\% \), \( \pi_{10} = 90\% \), \( \pi_{11} = 85\% \) and \( \pi_{12} = 80\% \). After period 12 the survival probability is again 100%.
Figures 1 and 2 illustrate our results from the benchmark model in each scenario. In scenario 1, at the onset of the AIDS epidemic, the drop in survival probability to 80% reduces annual real growth from 2% to 1.79%. This is also true for the other scenarios. In all scenarios where AIDS is temporary, the economy returns to 2% annual real growth eventually. When AIDS is permanent (scenario 2), economic growth returns to the long run level of 1.8% annually.

In scenario 3, the AIDS epidemic lasts for two generations. In generations two and three after the onset of the epidemic income levels are about 17% lower than in the AIDS free case. Notice also that in this scenario per capita incomes permanently remain about 13% below potential. In scenario 4, where AIDS infection rates rise gradually, the initial impact on income is smaller, but the long run impact is larger.

In our sensitivity analysis, presented in Tables 2 through 4, we use only scenario 1. In the baseline case, we assumed that training of orphans and non-orphans is equally productive, i.e. \( B^H = B^S \). Perhaps this is an unrealistic assumption, so in table 2 we report how robust our results are to changes in this assumption.

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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( B^S = B^H )</td>
<td>100</td>
<td>93.66</td>
<td>90.15</td>
<td>92.65</td>
<td>93.42</td>
<td>93.65</td>
<td>93.75</td>
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<tr>
<td>( B^S = 0.95 B^H )</td>
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<td>93.66</td>
<td>89.69</td>
<td>92.04</td>
<td>92.75</td>
<td>92.97</td>
<td>93.06</td>
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<tr>
<td>( B^S = 0.90 B^H )</td>
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<td>93.66</td>
<td>89.22</td>
<td>91.42</td>
<td>92.09</td>
<td>92.29</td>
<td>92.37</td>
</tr>
<tr>
<td>( B^S = 0.75 B^H )</td>
<td>100</td>
<td>93.66</td>
<td>87.82</td>
<td>89.56</td>
<td>90.09</td>
<td>90.24</td>
<td>90.31</td>
</tr>
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</table>

Table 2. Income levels relative to no-AIDS scenario (in %) with varying relative efficiency of orphan education

When \( B^S = B^H \), AIDS lowers income levels by about seven percent in most periods. Assuming that the education technology for orphans is 25% less efficient than for non-orphans generates an additional income loss nearer to ten percent, a fairly minor change of on the order of three percentage points.
Table 3. Varying relative productivity of AIDS infected workers

In table 3 we illustrate how robust our results are to changes in the relative labor productivity of AIDS infected workers. Even if infected workers’ productivity is only 30 percent of healthy workers’ productivity aggregate income levels only drop by approximately ten percent (as opposed to seven in the baseline case).

Table 4. Varying relative productivity of child labor

Finally, from table 4 one can observe the effects of changing the children’s labor productivity $\Delta$ on the drop in output caused by AIDS. Changes in $\Delta$ (four and six fold increases) also have little effect on the level effects of AIDS, except for very low values of $\Delta$ that are relatively unlikely to be relevant for underdeveloped economies.

**Modified model**

When the modifications allowed in our alternative model are introduced, the results are significantly different than those obtained for the benchmark model. Here we only consider two different AIDS scenarios to illustrate the comparison: one where AIDS lasts for one generation and one where AIDS lasts for two generations.
Table 5 shows how the macroeconomic effects of AIDS can be much larger when labor productivity is dependent on medical expenditures. Upon impact AIDS decreases aggregate income by 27%, almost 20% more than in the model where health treatment only enters the utility function. When AIDS lasts two generations, average income is reduced by more than half of the no-AIDS scenario for the second generation.

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<thead>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>Model 1, One-Period AIDS</td>
<td>100</td>
<td>93.66</td>
<td>90.15</td>
<td>92.65</td>
<td>93.42</td>
<td>93.65</td>
<td>93.75</td>
</tr>
<tr>
<td>Model 2, One-Period AIDS</td>
<td>100</td>
<td>72.85</td>
<td>95.55</td>
<td>96.76</td>
<td>97.12</td>
<td>97.23</td>
<td>97.28</td>
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<tr>
<td>Model 1, Two-Period AIDS</td>
<td>100</td>
<td>93.66</td>
<td>84.44</td>
<td>83.53</td>
<td>86.56</td>
<td>87.49</td>
<td>87.89</td>
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<td>Model 2, Two-Period AIDS</td>
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<td>72.85</td>
<td>45.34</td>
<td>98.45</td>
<td>96.51</td>
<td>95.94</td>
<td>95.69</td>
</tr>
</tbody>
</table>

Table 5. Aggregate effects when AIDS decreases labor productivity

*Policy analysis*

In these models the price of medication $p$ can be thought of as a policy instrument; the government might negotiate a lower price of AIDS medicines with foreign pharmaceutical companies. We are thus interested to see how the effects of the AIDS epidemic on income levels change when the price of medicines is varied. The results are contained in table 6. We only consider scenario 1 (AIDS in period 1 only).

<table>
<thead>
<tr>
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<tr>
<td>$p = 0.5$</td>
<td>34.54</td>
<td>100</td>
<td>71.99</td>
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<td>98.35</td>
<td>98.79</td>
<td>98.93</td>
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<td>$p = 0.75$</td>
<td>25.68</td>
<td>100</td>
<td>72.46</td>
<td>96.12</td>
<td>97.44</td>
<td>97.84</td>
<td>97.96</td>
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<td>20.50</td>
<td>100</td>
<td>72.85</td>
<td>95.55</td>
<td>96.76</td>
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<td>97.23</td>
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<tr>
<td>$p = 1.5$</td>
<td>14.71</td>
<td>100</td>
<td>73.51</td>
<td>94.72</td>
<td>95.77</td>
<td>96.09</td>
<td>96.19</td>
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<tr>
<td>$p = 2$</td>
<td>11.28</td>
<td>100</td>
<td>74.04</td>
<td>94.14</td>
<td>95.11</td>
<td>95.40</td>
<td>95.49</td>
</tr>
</tbody>
</table>

Table 6. Varying the price of AIDS medications

We varied the price of medicine by a factor of four ($p=0.5$ to $p=2$), changing the ratio of health care expenditures to income of infected households by a factor of three. The corresponding changes in aggregative income levels, however, are modest. In period 2, for
example a doubling of the price of medicines from $p = 1$ to $p = 2$ decreases average income by roughly 1.5 percent.

Notice also that the impact of price changes is different for the short run than for the long run; lower prices reduce income in the short run and increase it in the long run. This result arises because medicines and child labor act as substitutes in the model; that is, they both have one role only, to provide income. Lower medicine prices make child labor less necessary, and raise adult labor effort as adult productivity rises. However, if the decrease in effective child labor is greater than the increase in sick adults’ productivity, then income would diminish in the period of infection and a direct relation would exist in that period between medicine prices and income. After the infection period, since cheaper prices lead to higher human capital investment, a negative relation exists between medicine prices and income.

In Table 7 we study how changes in government subsidies for medical care influence the aggregate impact of the AIDS epidemic. (Again, we only consider scenario 1.) For this purpose we let $\gamma$ denote the fraction of total tax revenue allocated to health care subsidies so that $(1 - \gamma)$ is the fraction of tax revenues allocated to wasteful government spending. We let $\gamma$ vary by a factor of ten.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>fraction of income allocated to health care</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>No AIDS</td>
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<td>100</td>
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<td>$\gamma = 1%$</td>
<td>16.13</td>
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<td>73.32</td>
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<td>96.36</td>
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<td>$\gamma = 2.5%$</td>
<td>17.85</td>
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<td>73.13</td>
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<td>96.67</td>
<td>96.77</td>
<td>96.82</td>
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<td>$\gamma = 5%$</td>
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<td>72.85</td>
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<td>96.76</td>
<td>97.12</td>
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<tr>
<td>$\gamma = 7.5%$</td>
<td>23.26</td>
<td>100</td>
<td>72.63</td>
<td>95.86</td>
<td>97.13</td>
<td>97.52</td>
<td>97.63</td>
<td>97.68</td>
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<tr>
<td>$\gamma = 10%$</td>
<td>25.89</td>
<td>100</td>
<td>72.45</td>
<td>96.14</td>
<td>97.47</td>
<td>97.87</td>
<td>97.99</td>
<td>98.04</td>
</tr>
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</table>

Table 7. Varying the fraction of government revenue allocated to health care subsidies
Changes in income levels brought about by changes in $\gamma$ seem rather modest. In period 2, even an increase in the subsidy by a factor of 10 raises average incomes only by about 1.2%, even though health care in infected families rises by over 50%.

We also ran sensitivity analysis on the parameter $\beta$ in the health production function. This parameter influences the fraction of family income allocated to medical care; this relationship is highly non-linear. In table 8 we illustrate how changes in the parameter $\beta$ impact our results.

<table>
<thead>
<tr>
<th>Fraction of income allocated to health care</th>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$\beta = 1$</td>
<td>13.15</td>
<td>100</td>
<td>71.24</td>
<td>98.25</td>
<td>100.04</td>
<td>100.58</td>
<td>100.74</td>
</tr>
<tr>
<td>$\beta = 2.5$</td>
<td>17.27</td>
<td>100</td>
<td>71.99</td>
<td>96.87</td>
<td>98.35</td>
<td>98.79</td>
<td>98.93</td>
</tr>
<tr>
<td>$\beta = 5$</td>
<td>20.50</td>
<td>100</td>
<td>72.85</td>
<td>95.55</td>
<td>96.76</td>
<td>97.12</td>
<td>97.23</td>
</tr>
<tr>
<td>$\beta = 10$</td>
<td>22.56</td>
<td>100</td>
<td>74.04</td>
<td>94.14</td>
<td>95.11</td>
<td>95.40</td>
<td>95.49</td>
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<tr>
<td>$\beta = 50$</td>
<td>13.69</td>
<td>100</td>
<td>76.59</td>
<td>92.54</td>
<td>93.41</td>
<td>93.68</td>
<td>93.76</td>
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</table>

Table 8. Varying the health production function parameter $\beta$

Notice that the variation in the parameter $\beta$ is quite large but that the associated change in medical expenditure is modest. The effects of changing $\beta$ on levels of income during and after the AIDS crises are small. Increasing $\beta$ by a factor of 10 from one to 10 decreases income in period 2 by only 4 percent. In other periods the effects are of similar magnitude.

VII. Conclusion

This paper complements a small literature that uses dynamic general equilibrium models to study the long-run macroeconomic impact of AIDS epidemics. It confirms prior findings from dynamic general equilibrium models and simulation exercises that the macroeconomic consequences of the AIDS epidemic can be quite large. This paper also shows, more
controversially from a policy standpoint, that public policies such as changing subsidies for AIDS related medical care have relatively small growth effects.

The answer in this paper of course depends upon the specific assumptions made. In this paper we have only studied ex ante homogeneous individuals. It would be interesting to study a model which heterogeneous individual since there is some evidence that AIDS incidence varies by socioeconomic class. A model with heterogeneous individuals would allow more analysis of the effect of changing the price of AIDS medications since currently only the very wealthy can afford those medications at prices which are common in the developed economies. A second interesting extension of the model would be to allow the birthrate to be endogenous and depend on AIDS. Such an extension would require a model of endogenous fertility that is at least roughly consistent with the observations. We leave these kinds of investigation for the future.
References


