Rewards from Savings
Debt Aversion
and Mental Accounts

George M Korniotis†

Department of Finance
University of Notre Dame

October 21, 2004

Abstract

This paper analyzes the process behind the formation of mental accounts by an individual. The model is inspired by the Behavioral Life-Cycle Hypothesis of Shefrin and Thaler. It extends the traditional utility function to accommodate debt aversion. Then the model produces a consumption function with differentiated marginal propensities to consume out of different forms of wealth, which is the main prediction of mental accounting.

A class of quadratic utility functions is used that provides closed form solutions of the consumption function. Mental accounting then arise when the marginal utility of consumption increases with income and the individual is averse to debt. The assumptions are in line with the work of Thaler and Prelec and Loewenstein.

Next, a specific utility function is chosen to demonstrate that the model explains empirical puzzles of individual and aggregate consumption. At the microeconomic level, the consumption of the behavioral consumer tracks income, and it drops at the time of retirement. At the macroeconomic level, the consumption of the behavioral representative agent is not orthogonal to past income innovations, and its volatility is smaller than the volatility of income. Finally, when debt aversion is coupled with either precautionary savings or liquidity constraints, the individual still exhibits a strong tendency to finance consumption mainly out of current income.

* I would like to thank Robert Shiller for all his guidance at all steps of this project. The paper was also benefited from comments by Stefan Krieger, Alexander Michaelides, Ben Polak, Ricky Lam, David Laibson, Ioannis Serafides, and Amil Dasgupt. Finally, special thanks goes to the participants of the Prospectus Workshop in Macroeconomics at the Department of Economics at Yale University. All the remaining errors are mine.

†E-mail: Korniotis.1@nd.edu. Mail: University of Notre Dame, Department of Finance, 102 Mendoza College of Business, Notre Dame, IN 46556. Telephone: (574) 631-9322.
1 Introduction

One of the first theories of consumption is the certainty-equivalence Life-Cycle Hypothesis (LC) model. This framework predicts that optimal consumption is a fraction of the expected present discounted value of life-time wealth. The theory also asserts that the marginal propensity to consume out of different forms of individual and aggregate wealth is the same. This paper proposes a variation to the LC paradigm that gives rise to differentiated propensities to consume out of different forms of wealth.

The existing literature suggests that the benchmark LC model cannot capture a series of empirical regularities on both the aggregate and individual level. Starting from the microeconomic literature, Courant et al. (1986), show that consumption tends to follow the same hump-shaped pattern as the age-earning profile. Similarly, Hall and Mishkin (1982) find that individual consumption tracks income more than what the Life-Cycle model predicts—consumption is hypersensitive to income.

Another issue is whether individual save enough for their retirement and whether the retired run down their savings as their life expectancy decreases. Mirer (1972), Davies (1981) and Bernheim (1985) argue that the retired continue to save and this is due to strong bequest motives. However, Hurd (1987), does not detect any evidence of strong bequest motives. He finds that retired households do dissave but at a lower rate than what the Life-Cycle Model predicts. Also, Venti and Wise (1989) points out that retired households voluntarily maintain the equity in their homes. In a more recent paper, Banks, Blundell and Tanner (1998) document a dip in consumption at the time of retirement. This drop is not driven by expenses related to the job they held.

On the macroeconomic level, Flavin (1981) and Deaton (1987) assume that there is a representative agent that can capture the average behavior of the income. Then, they investigate whether the behavior of aggregate consumption can be captured by the life-cycle model. Flavin (1981) finds that aggregate consumption changes are not orthogonal to past aggregate income changes. Contrary to the prediction of the life-cycle model, consumption is sensitive to past realizations of income. Deaton (1987) investigates the volatility of consumption and income changes. In the date he finds that consumption growth is smoother than income growth, contrary to what the LC model predicts. Moreover, Carroll and Summers (1991) argue that the growth rate of aggregate consumption is close to the growth rate of aggregate income within a span of few years. This is evidence of hypersensitivity of aggregate consumption on aggregate income.

Naturally, the base-line life-cycle model has been extended to improve its empirical performance. The certainty equivalence model utilizes a quadratic utility which ignores precautionary savings. Precautionary savings arises when the third derivative of the utility function exists. Nevertheless, precautionary saving alone do not explain all the aspect of the data. On the microeconomic level, Dynan (1993) finds that away from the certainty equivalence
world, the volatility in the rate of growth of consumption is important in determining its expected growth rate. However, she can not find such a relationship empirically—a phenomenon coined as the missing precautionary saving effect. On the macroeconomic level, the most consistent feature of the data is the decline in national saving rates and again precautionary motives do not seem to tell the whole story. See Carroll, (1992).

The model with precautionary savings is then enriched to include liquidity constraints. See the work of Deaton (1991), Zeldes (1989) and Carroll (1992). Restrictions to borrowing is a reasonable assumption especially for young individuals who have not accumulated a substantial stock of savings. This model however cannot fully incorporate the consumption discontinuities at retirement, the patience heterogeneity with respect to age, income and wealth, the gap between saving intentions and saving actions as well as the high accumulation of households in illiquid assets. Laibson (1998) and Laibson, Repetto and Tobacman (1998) show that the hyperbolic discounting model can accommodate these stylized facts.

Another alternative is the Behavioral Life Cycle (BLC) model of Shefrin and Thaler (1985). Its main feature is mental accounting: the marginal propensities to consume out of different forms of wealth is different. Call this phenomenon differentiated marginal propensities to consume (DMPC). The evidence on the existence of mental accounts at the individual level is vast. See Thaler (1999) for an extensive overview of the mental accounting literature. However, there is little research on the reasons behind the formation of mental accounts. Henderson and Peterson note that most discussions of mental accounting have focused on the consequences of framing decisions in this manner rather than on the processes underlying mental accounting (1992, page 92). Hirst, Joyce and Schadewald add that “little is known about the processes that underlie mental accounting” (1994, page 136).

The goal of the current project is therefore twofold. First, necessary and sufficient conditions for the DMPC result are given. They require that as income increases, the marginal utility of consumption increases. Also, the individual has to be averse to debt, and enjoy a reward from a positive saving flow. Second, it is shown that the behavioral model does not contradict most of the empirical evidence on the macro and micro level. The analysis is restricted in the class of linear-quadratic-strictly concave (LQSC) utility functions that dependent on income. This class induces analytical results that clarify how individual behavior changes under debt aversion.

The rest of the paper is organized as follows. Section 2 presents the behavioral foundations of the model. It also provides the necessary and sufficient conditions for the DMPC result within the class of quadratic utility functions. In Section 3 a particular member of the LQSC class is chosen for the analysis that follows. In Section 4 the compliance of the behavioral model with individual data is analyzed. In Section 5 the macroeconomic implications of the behavioral model are analyzed. Section 6 discusses some remarks and extensions. Finally, the Appendix includes the mathematical details of the paper.
The Behavioral Microeconomic Model

In a simple consumption model the individual takes two actions every time period: she consumes and she saves. Assume that the individual pays attention to all her actions and she derives utility from everything she does. The utility function should accordingly include, not only consumption, but savings as well. If the individual spends more than her current income—\((Y_t - C_t) < 0\)—she suffers a utility loss. Conversely, when she manages to save something out of her current income—\((Y_t - C_t) > 0\)—she enjoys a utility gain. Such behavior can be attributed to a debt aversion—the individual does not enjoy the process of borrowing. A way to capture this story is to extend the utility function and allow for a utility reward (punishment) from a positive (negative) flow of savings. Then, purchasing a good has two dimensions: its acquisition provides provides the utility of consuming it, the transaction of buying it provides a reward or a punishment depending on how it influences savings. The importance of debt aversion on consumption behavior is highlighted by Prelec and Loewenstein (1998). Further, Thaler (1985) supports the acquisition and transaction dimensions when purchasing a good.

**Debt Aversion.** “[A] strong empirical regularity in the discounting surveys is that the discount rate for gains is much greater than for losses. People are quite anxious to receive a positive reward, especially a small one, but are less anxious to postpone a loss. Part of this preference comes from a simple ‘debt aversion’. Many people pay off mortgages and student loans quicker than they have to, even when the rate they are paying is less than they earn on safe investments”. See Thaler (1992), page 100.

Prelec and Loewenstein (1998) proposed a model that utilizes debt aversion, together with other behavioral elements. They argue that “when people make purchases, they often experience an immediate pain of paying, which can undermine the pleasure derived from consumption” (1998, page 4). Thus the utility process should be the sum of the happiness from consuming, and the grief from paying. Their suggestion is followed in the present study. The utility function captures the “pain” of consuming by the difference between income and the cost of consumption.

In discussing debt aversion, Prelec and Loewenstein argue that “there should be a strong tendency to accelerate payments for items whose utility declines over time” (1998, page 13). Individuals tend to borrow to buy goods that can be used while future repayments are made. Presently the model includes a perishable good that cannot be stored, and the “acceleration of payment” notion is translated into an increased tendency to finance consumption from current income.

Finally, Prelec and Loewenstein adopt a purchase criterion that “predicts a dislike of fully planned borrowing from future income for present consumption” (1998, page 15). This is reasonable since “there is evidence that young persons with temporarily low incomes, such as those educating themselves for lucrative careers, fail to

**Transaction Utility Theory.** In his 1985 paper, Thaler proposed the transaction utility theory where the evaluation of transactions involves the acquisition utility and the transaction utility. When the consumer is buying a good, she not only derives utility from consuming (acquisition utility), while the process of buying can provide a negative or positive value (transaction utility). Namely, there are two sources of utility from the action of purchasing a consumption unit. Thaler’s analysis does not contradict the debt-aversion framework of Prelec and Loewenstein (1998). Both papers stress that the process of buying a good has two dimensions: acquisition (consumption) and transaction (debt aversion).

**Behavioral Utility.** Given the work of Thaler (1985) and Prelec and Loewenstein (1998), the utility function comprises two components:

\[ U(C; Y) = U_1(C) + U_2(C; Y). \]

The first one, \( U_1(C) \), is the standard utility from consuming goods (similar to acquisition utility). The second one, \( U_2(C; Y) \), is the utility from the transaction of buying goods with respect to income (similar to transaction utility). The utility function is restricted in the class of linear-quadratic-strictly concave (LQSC) utility functions that satisfy the following assumptions:

\[
\begin{align*}
\frac{\partial \text{LQSC}(C_t, Y_t)}{\partial C_t} &= L_1(C_t, Y_t) > 0, \\
\frac{\partial^2 \text{LQSC}(C_t, Y_t)}{\partial C_t^2} &= -\eta < 0, \\
\frac{\partial \text{LQSC}(C_t, Y_t)}{\partial Y_t} &= L_2(C_t, Y_t), \\
\frac{\partial \text{LQSC}(C_t, Y_t)}{\partial Y_t \partial C_t} &= \theta,
\end{align*}
\]

where \( L_1 \) are \( L_2 \) are linear functions and \( \eta \) and \( \theta \) are constants. Any member of the LQSC family has to be a meaningful utility function. Therefore, the marginal utility of consumption is positive, \( L_1 > 0 \), and it increases at a decreasing rate, \(-\eta < 0\).

Next, the behavior of an individual within the LQSC class is investigated. The individual lives in a simple economy where there is only one perishable good in infinite supply. In every period she receive income, \( Y \), which is uncertain. She can use her income either to consume the good or to increase her assets. The only available asset is a saving account. She earns an interest rate \( r \) on her savings. The interest rate is constant and risk-free. In what follows, a theorem demonstrates the necessary and sufficient conditions for differentiated marginal propensities to consume out of different forms of wealth. It shows that if \( \theta \) is positive and smaller than \( \eta \), then the individual exhibit differentiated propensities to consume. The restriction \( \theta < \eta \) gives rise to debt aversion.
**Theorem:** An individual chooses consumption by maximizing the expected present discounted value of her life-time utility:

$$\max_{\{C_t\}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t \times LQSC(C_t, Y_t) \text{ subject to } A_{t+1} = (1+r)(A_t + Y_t - C_t) \text{ and } \beta(1+r) = 1,$$

where $\beta$ is her discount factor, $r$ is a risk-free time-invariant interest rate, $C$ is her consumption level and $Y$ is her income, which is uncertain. On her optimal consumption path

$$0 < \frac{\partial C_t}{\partial F_t} < \frac{\partial C_t}{\partial A_t} < \frac{\partial C_t}{\partial Y_t} < 1 \text{ where } F_t = \sum_{\tau=1}^{\infty} \frac{\mathbb{E}_t Y_{t+\tau}}{(1+r)^\tau}$$

holds, iff $\theta > 0$ and the debt aversion condition,

$$\theta = \frac{\partial LQSC(C_t, Y_t)}{\partial Y_t \partial C_t} < \left| \frac{\partial^2 LQSC(C_t, Y_t)}{\partial C_t^2} \right| = \eta,$$

holds.

See the Appendix for the proof\(^1\).

The condition $\theta < \eta$ is important because it implies that the marginal utility of consumption depends on consumption and savings. Denote the difference $(\eta - \theta)$ by $\lambda$, $\lambda$ being positive. Then, the marginal utility of consumption $L_1$ can be expressed as follows:

$$L_1 = \text{constant} - \eta C + \theta Y,$$

$$= \text{constant} - \lambda C + \theta (Y - C),$$

where the difference of income from consumption is the new flow in the savings account. Given that $\theta$ is positive, individuals value positive savings or dislike negative savings. Therefore, the condition $\theta < \eta$ within the $LQSC$ class gives rise to debt aversive behavior.

The theorem indicates that when $\theta > 0$ and $\theta < \eta$ the individuals form three mental accounts. Let us say that income increases. The marginal utility of every additional unit of consumption increases giving rise to a utility-driven incentive to consume more while income is high. Since every additional unit of consumption provide more happiness, why is the consumer not borrowing to consume even more? The reason lies in the condition $\theta < \eta$. Consuming a lot, decreases the additional happiness of each consumption unit. The drop in the marginal

\(^1\)Results similar to the ones presented in this section can be proved for an arbitrary utility function of the form $U(C, Y - C)$ such that $U$ is strictly increasing and strictly concave with respect to both its arguments. It is also required that $\frac{\partial U}{\partial Y} > 0$, $\frac{\partial^2 U}{\partial C^2} > 0$, $\frac{\partial^2 U}{\partial Y \partial C} > 0$ and $\Delta C_t > 0$. The last assumption is not that compelling, especially for modeling individual behavior.
utility, $\eta$, is higher than the marginal gain of consuming more when income is large, $\theta$, and therefore consumption is restricted from growing arbitrarily large.

Would this individual want to save a lot when her income is high? The answer is no. In the future she can use this extra savings to increase consumption. However, each unit of consumption, not consumed while income is high and consumed in the future, adds relatively less to her life-time utility. Everything else constant, she does have a major incentive to save. On the other hand, would she borrow from her future income to increase consumption today? Again the answer is no because under the condition $\theta < \eta$ the individual values savings. Balancing her motives to consume a lot, when income is high, and save to avoid future debt, she will finance current consumption from current income while channeling a small amount in her savings account.

### 3 A Consumption Function

**Utility Function.** In the previous section two necessary and sufficient conditions for the formation of mental accounts are provided. I now choose one member of the LSCQ family to derive a closed form solution for consumption. This consumption function is used to investigate the compatibility of the model with individual and aggregate data. The utility is a quadratic function of income and consumption:

$$U(C; Y) = U_1(C) + U_2(C; Y) = [aC - bC^2] + [c(Y - C) - d(Y - C)^2].$$

(2)

where $a$, $b$, $c$ and $d$ are positive constants with $a$ greater than $c$. The $U_1$ acquisition, the $U_2$ transaction, and the $U$ total utility functions have to be strictly increasing and strictly concave. Therefore, the following assumptions are made:

$$c > 2d(Y - C), \text{ and}$$

$$a + 2dY > c - 2(b + d)C,$$

for all values of $C$ and $Y$. See the Appendix.

Viewing the behavioral utility function (2) from the perspective of the theorem in Section 2, one infers that marginal utility increases with income,

$$\frac{\partial^2 \left( aC - bC^2 + c(Y - C) - d(Y - C)^2 \right)}{\partial C \partial Y} = 2d > 0,$$
and the debt aversion condition holds:

\[ 2d = \frac{\partial^2 \left( aC - bC^2 + c(Y - C) - d(Y - C)^2 \right)}{\partial Y \partial C} < \left\| \frac{\partial^2 \left( aC - bC^2 + c(Y - C) - d(Y - C)^2 \right)}{\partial C^2} \right\| = 2(b + d). \]

The **Consumption Choice.** Under the behavioral utility (2), consumption depends on income \( Y \), assets \( A \) and future income \( F \):

\[
C_t = \frac{r}{1 + r} A_t + \frac{r}{1 + r} \left[ 1 + \frac{d}{(b + d) r} \right] Y_t + \frac{r}{1 + r} \frac{b}{b + d} F_t, \tag{3}
\]

where \( F_t \) is the presented discounted value of expected income: \( \mathbb{E}_t \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1 + r)^i} \).

Then, the marginal propensities to consume are the following:

- **MPC out of** \( Y_t \): \( \frac{r}{1 + r} \left[ 1 + \frac{d}{(b + d) r} \right] \)
- **MPC out of** \( A_t \): \( \frac{r}{1 + r} \), and
- **MPC out of** \( F_t \): \( \frac{r}{1 + r} \frac{b}{b + d} \).

As expected, the **MPC out of** current income is larger than the **MPC out of** current assets, and the latter is larger than the **MPC out of** future income:

\[
\frac{r}{1 + r} \left[ 1 + \frac{d}{(b + d) r} \right] > \frac{r}{1 + r} > \frac{r}{1 + r} \frac{b}{b + d} = \frac{r}{1 + r} \left[ 1 - \frac{d}{b + d} \right].
\]

The prediction of the **BLC** is satisfied and mental accounting arises from debt aversion. The behavioral utility (2) provides an additional incentive to consume from current income. The behavioral consumer does not want to smooth consumption as much as a **LC** consumer does because the former enjoys consumption more when income is higher. However, given that the behavioral consumer appreciates savings she is reluctant to borrow and she always channels a small amount of funds into her saving account. At the end, her consumption mainly tracks income and it is partially financed by savings, while she is very reluctant to borrow from her future earnings.

**Moments of Utility Process.** The optimization of the consumer’s problem shows that she prefers to spend the most out of her current income. Then, when income is high, consumption should be hide too. It is in her best interest to choose consumption so that is positively correlated with income. This is true because the covariance of income and consumption influences her decisions.
The consumer chooses consumption as if she is maximizing the present discounted value of expected utility.

Expected utility can be expressed in terms of conditional moments:

\[
E_0 \left[ aC - bC^2 + c(Y - C) - d(Y - C)^2 \right] = (a - c) E_0 (C) - (b + d) [E_0 (C)]^2 \\
- (b + d) \text{Var}_0 (C) \\
+ 2d E_0 (Y \times C) + \text{constant},
\]

where \( E_0 (\cdot) \) and \( \text{Var}_0 (\cdot) \) respectively denote the expectation and variance given the information at time zero. The constant term includes the moments of income: \( c E_0 (Y) - d [E_0 (Y)]^2 - \text{Var}_0 (Y) \). Rewriting the utility process in terms of moments reveals that the decisions of the consumer are affected by three terms. First, she wants her consumption stream to maximize the concave function of expected consumption:

\[
(a - c) E_0 (C) - (b + d) [E_0 (C)]^2,
\]

which is part of her expected utility. However, she dislikes a volatile consumption path. For example, an 1% variance in consumption decreases expected utility by \((b + d)\)%.

Finally, the consumer has to consider the covariance of consumption with income, which is captured by the \( E_0 (Y \times C) \) term. Since \( d \) is positive, expected utility increases when the correlation between income and consumption is positive. When income is positive, she has an incentive to coordinate her spending with income to achieve a positive correlation. This correlation cannot be equal to one because when income is negative or zero, she has to finance consumption from savings.

The effect of the \( 2d E_0 (Y \times C) \) term is absent from the traditional life-cycle model. For the \( LC \) consumer the timing of income and consumption is irrelevant. The \( LC \) consumer finds the least variable consumption stream that maximizes expected consumption given life-time income. The \( BLC \) individual finds the least variable consumption stream, which most of the time is positively correlated with income, and maximizes expected consumption given life-time income.

**Behavioral and Life-Cycle Consumer.** The \( BLC \) consumer is inclined to spend more out of current income than out of assets and future income. Why does her behavior differ from the \( LC \) consumer? The first observation is that the \( MPC \) out of assets for the \( LC \) and the \( BLC \) consumers turns out to be the same. For both consumers assets are treated in the same way—they are the means of transferring wealth in the future for smoothing consumption. In the case of future wealth however, the \( LC \) consumer tends to consume more out of it than the \( BLC \) one. The \( BLC \) consumer prefers to use current income to finance current consumptions and she tends not to finance consumption by borrowing from the future earnings.
The major difference between the two individuals is with respect to the MPC out of current income. For the LC consumer it is equals to \( \{r/ (1 + r)\} \) and for the BLC it is equal to \( \{r/ (1 + r) + d/ [(b + d) r]\} \). The MPC for the BLC consumer is comprised by two elements. The first one– \( r/ (1 + r) \)– is the same one as the LC consumer. The second one models the increased inclination of the individual to consume out of her current income. However, this MPC is not equal to one: the consumer does save something in every period.

**Example: Constant Income.** The differences between the behavioral and the traditional consumer are further clarified through a simple example. Assume that income is constant throughout time, i.e. \( Y_t = Y \) for every \( t \). The LC and the BLC consumers then behave in the same way by consuming \( Y \) in every period\(^2\). Their choices are the same because none of them has to save or borrow to achieve her goals. Let us say that at time \( t \) both consumers experience an unexpected positive shock in their current income, \( \Delta Y > 0 \).

The optimal reaction for the LC consumer is to allocate the income windfall over her whole lifetime horizon. The BLC consumer behaves differently because she faces different trade-offs. If she consumes \( \Delta Y \) today and continues to consume \( Y \) for the rest of her life, the increase in her total utility\(^3\) will be equal to:

\[
\Delta TU = [a - 2b(Y + \Delta Y)] \Delta Y,
\]

where \( [a - 2b(Y + \Delta Y)] \) is positive because the marginal utility of consumption is always positive. The expression simply reflects the additional utility from consuming more at period \( t \). Her other option is to save the additional income today and enjoy the rewards of saving. At a future date \( s > t \), she can use her new savings. She will the enjoy higher consumption while feeling guilty for running down her saving stock. Given that all other consumption levels stayed at \( Y \), the increase in her utility\(^4\) is

\[
\Delta TU = [a - 2b(Y + \Delta Y)] \Delta Y - 4d(\Delta Y)^2.
\]

The change in welfare from consuming \( \Delta Y \) now is larger than the change in welfare from consuming \( \Delta Y \) in the future by \( 4d(\Delta Y)^2 \), which represents the guilt of financing consumption from savings. The behavioral consumer

\(^2\)The first order conditions for the BLC consumer under the scenario of constant income dictate that \( C_t = C_{t+1} \). This condition doesn’t necessarily mean that \( C_t \) needs to be equal to \( Y \). To actually see that \( C_t = Y \) is the optimal policy the total utility under \( P_1 = (C_t = Y)_{t=0}^{\infty} \) is compared to the total utility under \( P_2 = ((C_s = Y)_{s=0,s \neq t}, C_t = Y - \varepsilon) \) where \( P_s \) signifies the policy schedule \( x \). In the second case the consumer is giving up some consumption.

Total Utility under \( P_1 \):
\[
TU_1 = (aY - bY^2) \sum_{s=0}^{\infty} \beta^s = \frac{aY - bY^2}{1 - \beta}
\]

The question is, if it would pay to cut back consumption by \( \varepsilon \) and enjoy the extra utility from savings. The answer is no because the change in total utility is equal to \( (2bY - a + c) \) which is negative due to the strict positivity of the marginal utility.

\(^3\)The total differentials is evaluated at evaluated at \( C_t = Y + \Delta Y \).

\(^4\)The total differentials is evaluated at evaluated at \( C_t = Y, C_s = Y + \Delta Y, Y_t = Y + \Delta Y \) and \( Y_s = Y \).
is therefore inclined to spend more of the income windfall today compared to the $LC$ individual.

When income decreases a similar story applies. The $LC$ consumer optimally smooths the negative shock decreasing consumption by a bit in all future periods. The optimal choice for the $BLC$ consumer is again shaped by the two extreme options she faces. She can either decrease consumption today and keep all future consumption levels the same, or borrow money today, keep the current consumption at the old level and repay the loan at some future period $s$. It pays to absorb the negative shock today than in the future, since borrowing is costly. She prefers to lower consumption today and then return to her old higher consumption level in the future. This behavior is the result of debt aversion.

**Related Work.** The behavioral policy function (3) is almost identical to equation (8) of the “excess-sensitivity” model of Flavin (1993) where Flavin’s $\beta$ is equal to $d/ [(b + d) r]$. The new model traces Flavin’s marginal propensity to consume out of transitory income, $\beta$, back to utility fundamentals: $\beta$ measures the relative importance of savings to consumption with respect to marginal utility.

Levin (1998) estimated a series of linear consumption functions like (3) by using data from Longitudinal Retirement History Survey. He finds the following results: “First, spending seems to be very sensitive to changes in income but much less sensitive to changes in wealth. Second, close examination of the relation of wealth to consumption reveals a pattern in which individuals treat assets as not being fungible ... Finally, the amount spend on particular goods seems to depend not only on the individual’s total resources but also on how these resources are split between different assets”. See Levin (1998) page 82. However, he did not supply an analytical framework for the linear consumption functions he estimated. The present model lays out such a framework.

## 4 Microeconomic Facts

The goal of this section and of Section 5 is to demonstrate that the behavioral model does not contradict the stylized facts in consumption data. In this section, I investigate the compliance of the model with individual consumption data.

### 4.1 Hypersensitivity to Income

One weakness of the life-cycle model lies in its prediction that consumption should not track income. Hall and Mishkin (1982) showed however that individual consumption does track income.

In the current setting, the behavioral consumers spends more out of their current income. They are reluctant to pursue extensive saving or borrowing to smooth consumption and to disassociate it from income fluctuations.
This point was first made by Shefrin and Thaler when introducing the behavioral life-cycle hypothesis. See Shefrin and Thaler (1988) pages 629-633.

4.2 Retirement Income

Individuals in general experience a significant drop in their income when they retire. The life-cycle hypothesis envisions a consumer that foresees the expected decrease in income at the time of retirement. She should therefore save enough, and her after-retirement consumption should not drop. However, the sudden decrease in income is also coupled by a large drop in the consumption levels of the retirees, even if the retirement date can be forecasted pretty accurately. Banks, Blundell and Tanner (1998) document the consumption fall when the household head retires. They also find that this behavior cannot be captured by a forward-looking consumption-smoothing model, which accounts for expected demographic changes and mortality risk.

The behavioral model can shed light on the observed drop in consumption at retirement. The BLC consumer prefers to consume more when income is high, say before retirement. She therefore chooses to save less than the LC consumer and she experiences a decline in her consumption at the time of retirement. This situation is illustrated in a example with no uncertainty.

Assume that there is an LC and a BLC consumer facing a finite horizon \([0, T]\). They also receive the same income \(Y\) for every \(t \in [0, T_R]\), where \(T_R\) is the retirement period. When they retire their income disappears. Both consumers solve the following problem:

\[
\max_{\{C_t\}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t U_t \quad \text{subject to} \quad A_{t+1} = (1+r)(A_t + Y_t - C_t) \quad \text{and} \quad \beta(1+r) = 1,
\]

where \(U_t = aC_t - bC_t^2\) for the LC consumer, and \(U_t = aC_t - bC_t^2 + c(Y_t - C_t) - d(Y_t - C_t)^2\) for the BLC one.

The optimal consumption policy for the LC individual is obvious: she smooths her consumption and in every period she spends an equal fraction of the presented discounted value of her life-time income. In particular, her consumption level is always equal to:

\[
C_{LC} = \frac{1}{\sum_{t=0}^{T} (1+r)^t} \sum_{t=0}^{T_R} \frac{Y}{(1+r)^t} = \frac{r(1+r)^T}{(1+r)^{T+1} - 1} \frac{(1+r)^{T_R} - 1}{r(1+r)^{T_R-1}} Y = \left\{ \frac{(1+r)^{T-T_R-1}}{(1+r)^{T+1}-1} \right\} Y \quad (4)
\]

Note how the LC consumer spends the same amount before and after retirement.

The choices for the BLC consumer are different because savings enter in her utility process. In particular, her
Euler equations at all time periods, but the time of retirement, dictate that she should smooth consumption:

\[ C_t = C_{t+1} \quad \text{for} \ t \in \{0, ..., T_{R-2}, T_R, ..., T\}. \]

However, at the time of retirement the Euler equation predict an one time drop in consumption:

\[ C_{T_{R-1}} = \frac{d}{b + d} Y + C_{T_R}. \]

We derive the same conclusion, when we calculate her consumption before and after retirement. Before retirement her consumption is equal to:

\[ C_{BLC} = \frac{d}{b + d} Y + \left[ 1 - \frac{d}{b + d} \right] \frac{(1 + r)^{T_{R-1}}}{(1 + r)^{T_{R-1}} - 1} (1 + r)^{T_{R-1}} - 1 \].

After she retires her consumption will drops by \( \frac{d}{b + d} Y \) and becomes equal to:

\[ C_{BLC,R} = \left[ 1 - \frac{d}{b + d} \right] (1 + r)^{T_{R-1}} - 1 \frac{(1 + r)^{T_{R-1}}}{(1 + r)^{T_{R-1}} - 1} Y. \]

The life-time welfare of the behavioral consumer increases, when her consumption tracks income. Consequently, she chooses to consume more before retirement when her income is positive.

5 Macroeconomic Implications

5.1 Consumption and Income Innovations

We now turn to the time-series properties of aggregate consumption. To move from the microeconomic to the macroeconomic model, it is assumed that there is a representative agent who captures the behavior of the average individual. Then, the micro model (2) is appropriate for analyzing the aggregate per capita consumption.

The discussion first presents the predictions of the \( LC \) model. This part draws on the analysis in Blanchard and Fischer (1989). Next, the predictions of the \( BLC \) model are investigated. It is demonstrated that the behavioral model is compatible with the stylized facts of aggregate consumption data.

5.1.1 The Life-Cycle Model

Flavin (1981) and Deaton (1987) documented the \textit{excess-sensitivity} and \textit{excess-smoothness} puzzles within the certainty equivalence framework of the life-cycle model. The \( LC \) consumer behaves as if she is optimizing her
life-time welfare under her budget constraints:

$$\max_{\{ C_t \}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (aC_t - bC_t^2) \quad \text{subject to} \quad A_{t+1} = (1+r)(A_t + Y_t - C_t),$$

(5)

where $\beta (1 + r) = 1$. Her consumption is a constant fraction of her total wealth:

$$C_t = \frac{r}{1+r} \left[ A_t + Y_t + \mathbb{E}_t \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1+r)^i} \right].$$

(6)

See the Appendix for details. Her optimal behavior does not differentiate between different forms of wealth. Her marginal propensities to consume out of current income, current assets, and expected future income are all the same. She therefore smooths consumption by saving when her income level is high, and finances consumption from her saving stock when her income level is low.

The model also projects that changes in consumption should only be related to the new information in current income. At each point in time, the $LC$ consumer takes her decision given all the available information. The information regarding future income, which is contained in past income realizations, is already incorporated in her decision process. Therefore, the new information originate from the difference between the realization of current income and the expectation of current income. Call this difference income innovation. This prediction is formalized after defining the stochastic process of income.

**Income Process.** The evolution of consumption through time depends on how the individual is revising her expectations with respect to income:

$$\Delta C_t = \frac{r}{1+r} \sum_{i=0}^{\infty} \left[ \frac{(E_t - E_{t-1}) Y_{t+i}}{(1+r)^i} \right].$$

(7)

where $\Delta C_t$ is the change in consumption, and $(E_t - E_{t-1}) Y_{t+i}$ is her revision of future income given the new information that is revealed in period $t$. The term $(E_t - E_{t-1}) Y_{t+i}$ is referred to as income innovation. To obtain a closed-form solution for the change in consumption, the researcher needs a model for income that will pin down the income expectations.

**Stationary Income.** Flavin (1981) argues that aggregate income is a stationary process. To illustrate her argument, assume that income obeys a first order autoregression with $\rho$ being the coefficient on lagged income, $Y_t = \rho Y_{t-1} + \varepsilon_t$. Then the change in consumption of an $LC$ consumer is equal to a fraction of the current income
innovation $\varepsilon_t$:

$$
\Delta C_t = \frac{r}{1 + r - \rho} \varepsilon_t. 
$$

(8)

See the Appendix for the proof the result. The $LC$ consumer smooths unexpected changes in income as they are captures by the income error term $\varepsilon_t$. She pays no attention to past income innovations since the news they carry have already been incorporates in her decision. Consumption only reacts to new information, which is revealed through $\varepsilon_t$. However, Flavin (1981) documents that changes in per capita U.S. consumption are empirically responsive to past income changes, which approximate past income innovations. Hence, current consumption changes are sensitive to past income changes, contradicting the prediction of the life-cycle model. Flavin (1981) calls this phenomenon the excess-sensitivity of consumption.

**Non-Stationary Income.** Another empirical puzzle is the excess-smoothness of consumption, which is documented by Deaton (1987). Deaton finds aggregate income to be non-stationary, and shows how the life-cycle model predicts that current consumption changes should be more volatile than current income changes. However, this contradict what we observe in the data. To demonstrate his argument, assume that the level of income is non-stationary, $Y_t = (1 + \rho) Y_{t-1} - \rho Y_{t-2} + \varepsilon_t$, while its first differences are stationary. The change in income follows a first order autoregressive process with the $\rho$ being the parameter on the lagged first difference $\Delta Y_t = \rho \Delta Y_{t-1} + \varepsilon_t$. In this scenario, the $LC$ model projects that consumption should react to income innovations as follows:

$$
\Delta C_t = \frac{1 + r}{1 + r - \rho} \varepsilon_t. 
$$

(9)

See the Appendix for the proof the result. If $\rho > 0$, there is no consumption smoothing since the fraction $(1 + r) / (1 + r - \rho)$ is greater than one. Given that income is not stationary, the consumer realizes that all its revisions are permanent. For example, if current income increases by 10%, then all future income levels are expected to increase by 10%. She therefore borrows from the future and increase today’s consumption by more than 10%.

Furthermore, the variance of consumption changes, $\sigma^2_{\Delta C}$, is equal to $[(1 + r) / (1 + r - \rho)]^2 \sigma^2_\varepsilon$ which is larger than the variance of the income innovation $\sigma^2_\varepsilon$. This prediction violates the stylized fact that the observed volatility of aggregate consumption changes is smaller than the observed volatility of aggregate income changes. See Deaton (1987), and Campbell and Deaton (1989). He calls this phenomenon the excess-smoothness puzzle of consumption changes to current income changes\(^5\).

\(^5\)Flavin, (1981) and Deaton, (1987), treated the two puzzles separately. Campbell and Deaton latter stressed that they should be investigated together (1989). Excess-sensitivity deals with the relationship of current consumption changes to
5.1.2 The Behavioral Model

The compatibility of the behavioral model with the empirical observations of Flavin (1981), and Deaton (1987) is now investigated. Why should past income innovations matter for the behavioral consumer? In the BLC framework income influences, not only the consumer’s life-time resources, but also her marginal utility. Since consumption choices are controlled by the behavior of marginal utilities, income plays a major role in decision making. The path of the income process is shaped by the income innovations, which influence marginal utilities. Hence, consumption changes are not orthogonal to past income innovations.

Variation in consumption of the behavioral consumer should not to be more volatile than variations in income. Recall that the marginal utilities of consumption ($MU_C$) and savings ($MU_{Y-C}$) depend on income:

$$MU_C = (a - c) - 2(b + d)C + 2dY,$$

$$MU_{Y-C} = c - 2d(Y - C).$$

A positive change in income increases the marginal utility of consumption, while decreasing the marginal utility of savings. Assume that income permanently increases by 10%. The individual can either consume the entire income windfall now, or she can borrow and increase consumption by more than 10%. This is the choice of the LC consumer. The behavioral consumer increases consumption, since the positive change in income makes each consumption unit more desirable. Nevertheless, if her consumption surpasses her income, the marginal utility of savings increases, savings become attractive, and her incentive to increase consumption diminishes. Such a mechanism is absent from the LC model. Consequently, the final consumption choice of the BLC individual will be smaller than the consumption choice of an LC individual.

The different reaction of consumption in the behavioral model manifests itself when one calculates the consumption forecast error for the BLC consumer:

$$\Delta C_t = \frac{d}{b + d} \Delta Y_t + \frac{b}{b + d} \frac{r}{1 + r} \sum_{i=0}^{\infty} \frac{1}{(1 + r)^i} (E_t - E_{t-1}) Y_{t+i}. $$

The above relationship shows that changes in income, affected by past income innovations, affect consumption changes directly, reconciling the excess-sensitivity puzzle. Also, the impact of income revisions ($E_t - E_{t-1}) Y_{t+i}$ past income changes and excess-smoothness has to do with the variance of current consumption changes with respect to the variance of current income changes. But, if the income process exhibits intertemporal correlation then current income is correlated with past income realizations and in extend the variance of current income is also influenced by past income realizations. Furthermore, they argue that it is reasonable to expect that the consumers have a richer information set than the econometrician. In this case, the consumers form their expectations using the richer information set and a way for the econometrician to extract this information is through the saving ratio. “Hence, provided that the lagged saving ratio has predictive power for the change in labor income, the orthogonality condition and the condition for smoothness [that they derive] are identical” (Campbell and Deaton 1989, page 366).
is mitigated by the fraction $b/(b + d)$, which helps the model to reconcile the excess-smoothness puzzle.

**Stationary Income Process.** As in Section 5.5.1, assume that aggregate income follows an autoregressive process of order 1. Then the variation in consumption becomes:

$$
\Delta C_t = \frac{d}{b + d} \sum_{j=0}^{\infty} \rho^j \Delta \varepsilon_{t-j} + \frac{b}{b + d} \frac{1}{1 + r - \rho} \varepsilon_t
$$

$$
= \left[ \frac{d}{b + d} + \frac{b}{b + d} \frac{1}{1 + r - \rho} \right] \varepsilon_t - \frac{d}{d + b} \varepsilon_{t-1} + \frac{d}{b + d} \sum_{j=1}^{\infty} \rho^j \Delta \varepsilon_{t-j}.
$$

(10)

The relationship indicates that consumption adjusts to all changes of past income innovations as it should according to Campbell and Deaton (1989, page 358). Furthermore, the adjustment with respect to the today’s innovation is larger than that of the traditional $LC$ model because:

$$\left[ \frac{d}{b + d} + \frac{b}{b + d} \frac{1}{1 + r - \rho} \right] > \frac{r}{1 + r - \rho}.$$

Also, the volatility of consumption changes takes the following form:

$$\text{Var}(\Delta C_t) = \left[ \left( \frac{d}{b + d} \right)^2 \frac{2}{1 - \rho^2} + \left( \frac{b}{b + d} \frac{1 + r}{1 + r - \rho} \right)^2 \right] \sigma^2_{\varepsilon},$$

which is smaller than the variance of the income changes. See Appendix. Hence, under the AR(1) assumption for income, the predicted change in consumption reacts more to current income innovations, it is not orthogonal to past income innovations, and it is less volatile than income changes. These predictions conform with the empirical behavior of aggregate consumption.

**Non-Stationary Income Process.** When the first differences of aggregate income are stationary, the consumption change becomes:

$$
\Delta C_t = \frac{d}{b + d} \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} + \frac{b}{b + d} \frac{1 + r}{1 + r - \rho} \varepsilon_t
$$

$$
= \left[ \frac{d}{b + d} + \frac{b}{b + d} \frac{1 + r}{1 + r - \rho} \right] \varepsilon_t + \frac{d}{b + d} \sum_{j=1}^{\infty} \rho^j \varepsilon_{t-j}.
$$

As in the case of stationarity, all the past income innovations affect consumption variation. Furthermore, the predicted impact of the current innovation $\varepsilon_t$ is smaller than the one in the $LC$ model since

$$\frac{1 + r}{1 + r - \rho} > \left( \frac{d}{b + d} + \frac{b}{b + d} \frac{1 + r}{1 + r - \rho} \right).$$
In the behavioral model, even if an income change is permanent, the individual will borrow less than the LC consumer she dislikes being in debt. In addition, the variance of the consumption change becomes:

$$\text{Var}(\Delta C_t) = \left[ \left( \frac{d}{b + d} \right)^2 \frac{1}{1 - \rho^2} + \left( \frac{b}{b + d} \right)^2 \left( \frac{1 + r}{1 + \rho} \right)^2 \right] \sigma^2_{\varepsilon}.$$ 

If $2(b/d)$ is greater than $\rho/(1 - \rho^2)$, the volatility of consumption changes is smaller than the respective volatility in the life-cycle model. See the Appendix. More importantly, when the first differences in income are positively correlated, and the preference parameter $d$ is greater than $b$, the variance of income changes is larger than the variance of consumption changes. See the Appendix. Hence, the behavioral model provides a set of parameter values that explains excess-smoothness, even if aggregate income is not stationary. To summarize, under non-stationarity of the income process, consumption changes are predicted not to over-react to current income innovations, not to be orthogonal to past income innovations, while being less volatile than income changes. Once more, the behavioral paradigm produces predictions that are compatible with the empirical behavior of aggregate consumption.

5.2 The Saving Stock

The dynamic impact of debt aversion in the behavioral utility (2) is translated into inclination to consume more from current income, which induces the differentiated MPC outcome (2). On the optimal consumption path, if a dollar of income becomes part of the saving stock, the consumer will be less inclined to use it to finance current consumption. When she deposits a dollar amount in the saving stock today, she knows that it will become painful to spend it for consumption in the future.

The reluctance to consume from the saving stock, and the inclination to spend more out of current income are naturally reflected in the saving choices. In general, the saving stock at period $t$, $S_t$, is defined as the present value of assets plus income minus consumption, $S_t = [r/(1 + r)] A_t + Y_t - C_t$. See equation 4 in Campbell and Deaton (1989). Using this definition, the saving stocks of the life-cycle consumer is equal to:

$$S_{LC_t} = \frac{1}{1 + r} Y_t - \frac{r}{1 + r} E_t \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1 + r)^i},$$  \hspace{1cm} (11)$$

while the saving stock for the behavioral consumer is equal to:

$$S_{BLC_t} = \left[ 1 - \frac{r}{1 + r} + \frac{d}{(b + d)(1 + r)} \right] Y_t - \frac{r}{1 + r} \frac{b}{b + d} E_t \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1 + r)^i}. \hspace{1cm} (12)$$

Both expressions contain expectations of future income that are calculated under a stationary and a non-stationary
income process. See the Appendix.

In the case of stationarity, \( Y_t = \rho Y_{t-1} + \varepsilon_t \), one can show that in each period the saving stock is determined by current income:

\[
SLC_t = \left[ \frac{1}{1 + r} - \frac{r}{1 + r} \frac{\rho}{1 + r - \rho} \right] Y_t = \frac{1 - \rho}{1 + r - \rho} Y_t, \text{ and } SBL_t = \frac{b}{b + d} \frac{1 - \rho}{1 + r - \rho} Y_t = \frac{b}{b + d} SLC_t.
\]

Clearly the BLC consumer saves the least because financing consumption from the saving stock is painful. She knows that she likes to save, but she also understands that it will be hard to withdraw funds from her savings account. She therefore decides not to save that much. When income has stationary first differences, \( Y_t = (1 + \rho) Y_{t-1} - \rho Y_{t-2} + \varepsilon_t \), similar results hold:

\[
SLC_t = -\frac{\rho}{1 + r - \rho} \Delta Y_t, \text{ and } SBL_t = \frac{b}{b + d} \left[ -\frac{\rho}{1 + r - \rho} \Delta Y_t \right] = \frac{b}{b + d} SLC_t.
\]

Under non-stationarity, any change in income is permanent. Say that income changes by 1%. The LC consumer knows that the favorable windfall will last forever, and therefore she borrows today making her saving stock negative and equal to \( \rho/(1 + r - \rho) \). The behavioral consumer will also borrow. Her loan though is smaller and the amount she owes is a fraction of the amount that the LC consumer owes.

6 Remarks and Extensions

I now turn to a series of remarks and extensions of the basic behavioral model. First, I compare it to the work of Shefrin and Thaler (1988). Next, it is shown that a model where the utility from savings depends on consumption can also produce mental accounts. Further, the importance of the saving term can be made to diminish as time goes by, inducing time-dependence in the MPC’s. The behavioral model is then compared to models with allow utility from wealth.

Two major extensions of the life-cycle paradigm are precautionary savings and liquidity constraints. It is demonstrated that when debt aversion is coupled with either precautionary savings or liquidity constraints, the individual still has a high motivation to finance current spending mainly from current consumption. Finally, it is argued that the behavior of the BLC model remains distinct even if the model is compared to the Campbell and Mankiw (1989) world of rule-of-thumb and LC consumers.
6.1 Shefrin and Thaler

The formulation of the behavioral utility (2) is a simplification of what Shefrin and Thaler suggest in their 1988 paper. In their model the utility functions $U_1$ and $U_2$ have kinks, while here they are smooth and concave. Nevertheless, the behavioral utility (2) is related to their work. In their paper they argue that the individual has a coexisting dual preference structure, the doer and the planner. The doer is only concerned with the present and wants to consume as much as possible today. In my case the doer’s utility could be $U_1(C_t) = aC_t - bC_t^2$.

The planner, on the other hand, cares about the future and worries that if the doer consumes too much each period, there will not be enough savings to finance future consumption. The planner is thus interested in the saving flow at each period and wants to constrain the current consumption of the doer. To do so, the latter exerts willpower on the doer by punishing negative saving flows. As Shefrin and Thaler note (1988, page 612) “The psychic cost of using will power ... may be thought of as a negative sensation (corresponding roughly to guilt) which diminishes the positive sensations associated with $[U_1(C_t)]$.” Willpower depends on current consumption $C_t$, and on the set of feasible consumption choices approximated by the level of current income, $Y_t$. This paper models willpower as a quadratic function

$$U_2(C_t, Y_t) = c(Y_t - C_t) - d(Y_t - C_t)^2.$$ 

Consequently, the cumulative utility function of the doer, $U(C_t, Y_t)$, is equal to the sum of $U_1(C_t)$ and $U_2(C_t, Y_t)$.

6.2 Another Utility Function

The mental accounting result is not unique to the utility function (2). Another function from the LQSC family that produces the DMPC prediction is the following:

$$U(C_t; Y_t) = aC_t - bC_t^2 + d(Y_t - C_t)C_t.$$  (13)

The impact of the debt aversion term, $(Y_t - C_t)$, now depends on the level of consumption. When the individual consistently faces low income, she inevitable has low consumption, and the rewards from positive saving flows become less important. Under (13), her Euler equation takes the form of:

$$E_t(C_{t+1}) = C_t + \frac{d}{2(b + d)} [E_t(Y_{t+1}) - Y_t],$$
and her optimal consumption behaves as follows:

\[ C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \left[ 1 + \frac{d}{2(b+d)r} \right] Y_t + \frac{r}{1+r} \left[ 1 - \frac{b}{2(b+d)} \right] \sum_{\tau=1}^{\infty} E_t Y_{t+\tau} \left( 1 + \frac{r}{1+r} \right)^{\tau}. \]

The ranking of the new MPC’s is as expected; the MPC out of future income is higher than the MPC out of asset, and the MPC out of assets is higher than the MPC of income.

### 6.3 Wealth in the Utility Function

The paper investigates the impact of extending the utility function to include utility from saving flows. It then connects the differentiated marginal propensities to consume result with debt aversion. If an individual dislikes debt, it could imply the she derives satisfaction from owning a large savings account. It should not be surprising then, that including wealth in the utility function will also yield mental accounting effects.

Kuznitz (2000) notes that with wealth in the utility function, it is not enough for the consumer to know that she will be rich, she wants to feel rich. He then shows that when utility from wealth is allowed, the consumer reacts differently to income changes depending on their timing: as future income changes get closer, their impact on current consumption becomes stronger.

Wealth in the utility function is also advocated by Carroll (1998). He focuses on the behavior of the rich, and he concludes that it can be explained by two possibilities. Either the rich consider accumulation of wealth as an end in itself, or unspent wealth provide a flow of services, such as power of social statues, which have the identical effect on behavior as though wealth were intrinsically desirable.

### 6.4 Significance of the Savings Term

A potential weakness of the behavioral utility function (2) is that the importance of the savings term remains constant through out the life of the individual. Savings are a way to transfer consumption in the future, and as the individual is getting older, they should became less significance. As retirement draws near, the saving term in the utility function should not matter. This observation is compatible with Shefrin and Thaler who note the “willpower effort becomes less costly as retirement draws near” (1988, page 613). To deal with this shortcoming, the utility function is modified and the savings term is forced to vanish as time tends to infinity:

\[ U(C_t; Y_t) = aC_t - bC_t^2 + (1 - \delta) \delta^t \left[ c(Y_t - C_t) - d(Y_t - C_t)^2 \right], \]  

(14)
where $0 < \delta < 1$. Note that savings term is multiplied by $(1 - \delta) \delta^t$ to make its long-run contribution equal to one. The optimal consumption is similar to (3). Its new feature is the time dependence of the MPC’s:

$$C_t = \left[ \sum_{j=1}^{\infty} \frac{b + d \delta^t}{(1 + r)^j (b + d \delta^{j+1})} \right]^{-1} \times \left[ \frac{c}{2} \sum_{j=1}^{\infty} \frac{\delta^t (\delta^j - 1)}{(1 + r)^j} + A_t + \delta^t \left( \frac{1 + r}{r} \right) Y_t + \sum_{j=0}^{\infty} \frac{1 - \delta^{t+1} (1 - \delta) d E_t Y_{t+j}}{(1 + r)^j} \right]. \tag{15}$$

Consequently, as time goes to infinity the individual becomes a life-cycle consumer because the significance of the debt aversion term $\left[c(Y_t - C_t) - d(Y_t - C_t)^2\right]$ disappears.

### 6.5 Precautionary Savings

Dynan (1993) compared the life-cycle model to a model with precautionary savings. She solves the maximization problem of the individual under a constant-relative-risk-aversion (CRRA) utility function:

$$U(C_t) = \frac{(C_t)^{1-\rho}}{1-\rho}. \tag{16}$$

where $\rho$ is the degree of relative risk aversion. With $\beta(1 + r) = 1$, the first order conditions of optimizing life-time welfare is equal to the following expression:

$$1 = E_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^{\rho} \right]. \tag{17}$$

Substituting $C_{t+1}^{\rho}$ with its second order Taylor expansion around $C_t^6$, simplifies the first order condition (17) to the subsequent expression:

$$E_t \frac{\Delta C_{t+1}}{C_t} = (\rho + 1) E_t \left( \frac{\Delta C_{t+1}}{C_t} \right)^2, \tag{18}$$

where “$\Delta$” is the first difference operator. The Euler equation prescribes that the expected consumption growth should depend on its expected variance, which captures precautionary savings. The level of the parameter $(\rho + 1)$ measures how important precautionary savings are. Dynan estimates the above relationship with household data. She finds the parameter $(\rho + 1)$ to be statistically insignificant, and she concludes that the risk involved in the conditional variance of consumption does not influence the expected growth in consumption. She called this

---

The second order Taylor approximation is of the following form:

$$C_{t+1}^{\rho} \approx C_t^{\rho} - \rho \frac{\Delta C_{t+1}}{C_t^{\rho+1}} + \rho (\rho + 1) \frac{(\Delta C_{t+1})^2}{C_t^{\rho+2}}.$$
empirical finding the missing precautionary savings effect.

Can the behavioral model explain part of the missing precautionary saving effect? To investigate such a possibility the CRRA utility is altered to include a debt aversion term:

\[ U(C_t; Y_t) = a \frac{(C_t)^{1-\rho}}{1-\rho} + c (Y_t - C_t) - d (Y_t - C_t)^2. \]  

The parameter \( a \) models the temptation to consume today. With \( \beta(1 + r) = 1 \), the Euler equation of utility maximization under (19) becomes:

\[ 1 = \mathbb{E}_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^\rho + \frac{2d}{a} C_t^\rho \Delta Y_{t+1} - \frac{2d}{a} C_t^\rho \Delta C_{t+1} \right], \]

where \( \Delta X_{t+1} = X_{t+1} - X_t \), \( X = \{ C, Y \} \). A Taylor approximation demonstrates that expected consumption growth depends on the conditional variance of consumption and on the expected level of income:

\[ \mathbb{E}_t \frac{\Delta C_{t+1}}{C_t} = \left[ \frac{\rho (\rho + 1)}{\rho + \frac{2d}{a} C_t^\rho + 1} \right] \mathbb{E}_t \left( \frac{\Delta C_{t+1}}{C_t} \right)^2 + \left[ \frac{2d}{a} C_t^\rho \right] \mathbb{E}_t \Delta Y_{t+1}. \]  

The conditional variance of consumption captures the effect of precautionary savings. In a CRRA model, the consumer is inclined to build a buffer saving stock to tackle future misfortunes that make her consumption more volatile. However, a BLC consumer understands that as soon as a dollar becomes part of the saving stock, it becomes difficult to spend it. Hence, as long as consumption is large enough, the term in front of \( \mathbb{E}_t (\Delta C_{t+1}/C_t)^2 \), will be small enough and her precautionary motive is reduced. This represents a possible explanation for Dynan’s missing precautionary saving effect.

The other term on the right-hand-side of (20) captures the impact of predictable movements in income, \( \mathbb{E}_t \Delta Y_{t+1} \), on predictable changes in consumption, \( \mathbb{E}_t \Delta C_{t+1} \). An expected increase in income makes each consumption unit more desirable. The consumer also likes savings and therefore a part of the future increase in income will be channelled to her savings account.

### 6.6 Cash-In-Advance and Liquidity Constraints

The behavioral model implies that the propensity to consume depends on the source of wealth, and it is the highest in the case of income. However, such a result arises in a traditional life-cycle model when the consumer is facing cash-in-advance liquidity constraints. If the consumer cannot borrow, she has to finance consumption mainly from current income. Next it is investigated whether one can distinguish between the traditional LC with cash-in-advance liquidity constraints, and the BLC model with cash-in-advance liquidity constraints.
To solve models with liquidity constraints and income uncertainty, one needs to resort to numerical methods. See Deaton (1991). To obtain analytical results, I assume that there is no income uncertainty, and I use the model proposed by Helpman (1981). Helpman demonstrate how to obtain closed-form solutions with liquidity constraints without income uncertainty.

Helpman assumes that the consumer dwells in a monetary economy. In this economy the only asset is money, \( m \), which is the sole means to purchasing goods. There is only one good and its price is set to one. Furthermore, in each period the individual receives income equal to \( y_t \).

The timing of the model is as follows. The individual starts her life with a given amount of money holdings. In period 1 she can spend part of it on goods. The amount of money that she has not spent is transferred to the subsequent period. At the end of period 1 she receives income which cannot be used to buy goods in period 1. The amount of money that has not been spent during period 1 plus the income received at the end of period 1 determine her money holdings at the beginning of period 2. Then the process repeats itself.

In real terms the individual considers the following maximization problem:

\[
\max_{\{C_t\}} \quad E_0 \sum_{t=1}^{\infty} \beta^t U(C_t) \quad \text{subject to} \quad m_{t+1} = m_t + y_t - C_t, \quad m_1 = m, \quad C_t \leq m_t,
\]

where \( C \) is consumption, \( m \) is real money holdings and \( y \) is real income. At the steady state she wants to equalize consumption with income. She will be locked in this steady state when she manages to accumulate a \( y \) level of real money holdings. Then, she will enter each period with money holdings equal to \( y \), that finance the consumption of the period. At the end of the period she receives her income \( y \), which is used for next period’s consumption.

How do the life-cycle and the behavioral consumer act in this monetary world? The optimal consumption of the liquidity constrained life-cycle consumer under the utility \( U(C_t) = (aC_t - bC_t^2) \) is:

\[ C(m) = m, \text{ for } 0 < m \leq \mu_1, \text{ and } \]

\[ C(m, y) = \left[ 1 - \frac{T \beta^T}{\sum_{\tau=1}^{T} \beta^{1-\tau}} \right] \frac{a}{2b} m + \frac{m}{\sum_{\tau=1}^{T} \beta^{1-\tau}} \frac{T-1}{\sum_{\tau=1}^{T} \beta^{1-\tau}} y, \text{ for } \mu_{T-1} < m \leq \mu_T, \quad T = \{2, 3, \ldots\}, \]

where \( C(m) \) is consumption at the steady state and \( C(m, y) \) is consumption off the steady state. The auxiliary numbers, \( \{\mu_0, \mu_1, \ldots, \mu_T, \ldots\} \), model how many periods away from the steady state the individual is. If the assets \( m \) belong to the interval \( (\mu_{T-1}, \mu_T] \), then the individual will arrive at the steady state consumption level in \( T \).
periods. The off-the-steady-state consumption is a piecewise linear function with the marginal propensity to consume out of income being larger than the MPC out of assets. This prediction fits the mental accounting framework.

Moving to the constrained behavioral consumer, one finds her optimal consumption to be equal to:

\[
C(m) = m, \text{ for } 0 < m \leq \mu_1, \text{ and}
\]

\[
C(m, y) = \left[1 - \frac{T}{\sum_{\tau=1}^{T} \beta^{1-\tau}} \right] \frac{a-c}{2(b+d)} + \frac{m}{\sum_{\tau=1}^{T} \beta^{1-\tau}} + \frac{1}{\sum_{\tau=1}^{T} \beta^{1-\tau}} \left[d \sum_{\tau=1}^{T} \beta^{1-\tau} + bT \right] - 1 \right] y,
\]

for \( \mu_{T-1} < m \leq \mu_T, T = \{2, 3, \ldots\} \). As before, the auxiliary numbers, \( \{\mu_0, \mu_1, \ldots, \mu_T, \ldots\} \), model how many periods away from the steady state the individual is. The off-the-steady-state consumption, \( C(m, y) \), is again a piecewise linear consumption with differentiated MPC out of assets and income. However, the kinks in the policy function of the behavioral consumer are located at different points compared to life-cycle consumer. Furthermore, the behavioral consumer has the largest MPC out of income:

\[
\frac{1}{b+d} \left(d \sum_{\tau=1}^{T} \beta^{1-\tau} + bT \right) - T > \frac{d}{b+d} \left(\sum_{\tau=1}^{T} \tau - T \right) > 0.
\]

Consequently, debt aversion once again made the consumption of the behavioral consumer track income more closely. This result is also present when one compares the constrained LC with the unconstrained BLC.

### 6.7 Representative Agent

The macroeconomic implications of the behavioral model were analyzed under the representative agent assumption. Such a paradigm overlooks decision differences between individuals who face heterogeneous income streams. Are the predictions of the behavioral model similar to the predictions of an LC model with heterogeneous agents?

A simple heterogeneous agent model was proposed by Campbell and Mankiw (1989). They assumed that there

\[
\mu_1 = \beta y + (1-\beta) \frac{a}{b}, \quad \mu_{T-1} = y \left(\sum_{\tau=1}^{T-1} \beta^{\tau-T} \right) + \frac{a}{2b} \sum_{\tau=1}^{T-1} (1-\beta^{\tau}), \quad \mu_T = y \left(\sum_{\tau=1}^{T} \beta^{\tau-T+1} \right) + \frac{a}{2b} \sum_{\tau=1}^{T} (1-\beta^{\tau}).
\]

\[
\mu_1 = \frac{d + \beta b}{b+d} y + \frac{1-\beta}{2(b+d)} (a-c), \quad \mu_{T-1} = y \frac{b}{b+d} \left(\sum_{\tau=1}^{T-1} \beta^{\tau-T} \right) + \frac{a-c}{2(b+d)} \sum_{\tau=1}^{T-1} (1-\beta^{\tau}),
\]

\[
\mu_T = y \frac{b}{b+d} \left(\sum_{\tau=1}^{T} \beta^{\tau-T+1} \right) + \frac{a-c}{2(b+d)} \sum_{\tau=1}^{T} (1-\beta^{\tau}).
\]
are two groups of consumers. The individuals in the first group are the rule-of-thumb consumers who constitute λ fraction of the population and they set consumption equal to their income, i.e. \( C_{1t} = Y_{1t} \). The individuals in the second group are fully rational considering the same intertemporal problem as the life-cycle consumers. Their consumption function is therefore identical to equation (6). Per capita aggregate consumption is obtained by aggregating the consumption choices of the two groups:

\[
C_t = \lambda C_{1t} + (1 - \lambda) C_{2t} = \lambda Y_{1t} + (1 - \lambda) \frac{r}{1 + r} \left[ A_{2t} + E_t \sum_{i=0}^{\infty} \frac{Y_{2,t+i}}{(1 + r)^t} \right] = (1 - \lambda) \left[ \frac{r}{1 + r} \right] \left[ A_{2t} + E_t \sum_{i=1}^{\infty} \frac{Y_{2,t+i}}{(1 + r)^t} \right] + \left[ \lambda Y_{1t} + \frac{(1 - \lambda) r}{1 + r} Y_{2t} \right].
\]

This aggregate per capita consumption function is different from the one of the representative BLC consumer. The Campbell-Mankiw model predicts that the MPC out of assets and future income are the same, which does not conform with the predictions of a mental accounting model.

### 7 Conclusion

This paper proposes a consumption model based on the behavioral life-cycle hypothesis of Shefrin and Thaler (1988). The model is appealing because it provides a behavioral foundation to mental account. It shows that if people enjoy consumption more when income is high, and they dislike debt, then their propensity to consume out of different forms of wealth is different. When income is high, the consumer feels less guilty for spending and she increases her consumption. However, she is averse to debt and she does not want her consumption be higher than her income. Even if she likes to consumes a lot, she always wants to channel some of her resources in her saving account. Ultimately, she is prone to spend more out of current income than out of her assets, while being very reluctant to finance consumption from future earnings. This proposition is proved within the class of linear-quadratic-strictly concave utility functions that dependent on income.

The properties of the \( LQSC \) class follow the work of Thaler (1985), and Prelec and Loewenstein (1998). Thaler stresses that the action of buying goods is painful, and it should be captured by the transaction utility. Prelec and Loewenstein explored how people behave under debt aversion. Synthesizing their work, the quadratic utility is extended to include a reward (punishment) from a positive (negative) saving flow.

Next, a particular member of the \( LQSC \) class is chosen where total utility is equal to the sum of the utility of consuming a good, plus the reward for buying goods that cost less than current income. Under this function, individual choices are described by a linear consumption function, which Levin (1998) finds to be supported by
the data. The model also traces Flavin’s marginal propensity to consume out of transitory income back to utility fundamentals: it measures the relative importance of savings to consumption with respect to marginal utility. See Flavin (1993).

The behavioral model contributes to the explanation of empirical puzzles in the microeconomic and macroeconomic literature. The life-cycle model has difficulty in explaining why individual consumption tracks income so closely, and why consumption drops at the time of retirement. See Mishkin (1982), and Banks, Blundell and Tanner (1998) respectively. In the behavioral model consumption varies with income because each consumption unit is more desirable when income is positive. Then the decline of consumption at the time of retirement income is perfectly consistent with the behavioral model.

At the macroeconomic level the life-cycle model can not reconcile why consumption changes are affected by past income innovations, and why consumption growth is smoother than income growth. In the behavioral model consumption growth depends not only on the revisions of future income, but also on income growth. Income growth is connected to past income innovations, which then influence consumption. In addition, the behavioral model includes a debt aversion mechanism that restricts consumption from become greater than income. With this mechanism in place, consumption growth can be less volatile than income growth, even when income is non-stationary.

The baseline model is not the only one that can induce mental accounting. It is shown that a model where the utility from savings depends on consumption can have the same result. The current model also has similar predictions to models with utility from wealth. Further, one can make the importance of the saving term diminish as individuals grow older. Such a model is more realistic as people do run down part their savings after retirement.

Two major extensions of the life-cycle paradigm are precautionary savings and liquidity constraints. It is demonstrated that when debt aversion is coupled with either precautionary savings or liquidity constraints, the behavioral consumer still wants to spend a large portion of her income in every period. Finally, the macroeconomic implications are obtained under the representative agent assumption, which can produce misleading results. However, the behavior of the $BLC$ model remains distinct even if the model is compared to the Campbell and Mankiw world of rule-of-thumb and $LC$ consumers. See Campbell and Mankiw (1989).
References


A APPENDIX

The appendix includes the mathematical details for the Life-Cycle and Behavioral Life-Cycle models. First, the proof of the theorem in Section 2 is presented. Second, the results for the Life-Cycle model are explained and finally the results for the behavioral model are investigated.

A.1 LSCQ and DMPC

**Theorem:** An individual chooses consumption by maximizing the expected present discounted value of her life-time utility:

\[
\max_{\{C_t\}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t \times LQSC(C_t, Y_t) \quad \text{subject to} \quad A_{t+1} = (1+r)(A_t + Y_t - C_t) \quad \text{and} \quad \beta(1+r) = 1,
\]

where \(\beta\) is her discount factor, \(r\) is a risk-free time-invariant interest rate, \(C\) is her consumption level and \(Y\) is her income, which is uncertain. On her optimal consumption path

\[
0 < \frac{\partial C_t}{\partial F_t} < \frac{\partial C_t}{\partial A_t} < \frac{\partial C_t}{\partial Y_t} < 1
\]

holds, iff \(\theta > 0\) and the debt aversion condition,

\[
\theta = \frac{\partial LQSC(C_t, Y_t)}{\partial Y_t \partial C_t} < \left| \frac{\partial^2 LQSC(C_t, Y_t)}{\partial C_t^2} \right| = \eta,
\]

holds.

**Proof**

In the LQSC class, \(L_1\) is a linear function of its arguments, say \(L_1(C, Y) = a - \eta C + \theta Y\). Then, combining the Euler equation with the law of iterated expectations, one obtains the following relationship:

\[
\mathbb{E}_t(-\eta C_{t+s} + \theta Y_{t+s}) = -\eta C_t + \theta Y_t,
\]

\[
\mathbb{E}_t C_{t+s} = C_t - \frac{\theta}{\eta} Y_t + \frac{\theta}{\eta} \mathbb{E}_t Y_{t+s}.
\]

Substitute the above relationship in the intertemporal budget constraint:

\[
C_t + \sum_{\tau=1}^{\infty} \frac{\mathbb{E}_t C_{t+\tau}}{(1+r)^\tau} = A_t + Y_t + \sum_{\tau=1}^{\infty} \frac{\mathbb{E}_t Y_{t+\tau}}{(1+r)^\tau},
\]

\[
C_t + \sum_{\tau=1}^{\infty} \frac{C_t - \frac{\theta}{\eta} Y_t + \frac{\theta}{\eta} \mathbb{E}_t Y_{t+s}}{(1+r)^\tau} = A_t + Y_t + \sum_{\tau=1}^{\infty} \frac{\mathbb{E}_t Y_{t+\tau}}{(1+r)^\tau},
\]

and obtain her consumption function:

\[
C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \left[1 + \frac{\theta}{\eta} \right] Y_t + \frac{r}{1+r} \left[1 - \frac{\theta}{\eta} \right] \sum_{\tau=1}^{\infty} \frac{\mathbb{E}_t Y_{t+\tau}}{(1+r)^\tau}.
\]

Therefore, a positive \(\theta\) guarantees that \(\frac{\partial C_t}{\partial F_t} < \frac{\partial C_t}{\partial A_t} < \frac{\partial C_t}{\partial Y_t}\). Also, by combining \(\theta < \eta\) one shows that the MPC out of
future income is positive and the MPC out of current income is less than 1:

\[
\frac{\partial C_t}{\partial F_t} = \frac{r}{1 + r} \left[ 1 - \frac{\theta}{\eta} \right] > 0, \quad \text{and} \quad 1 - \frac{\partial C_t}{\partial Y_t} = 1 - \frac{r}{1 + r} \left[ 1 + \frac{\theta}{r\eta} \right] = \frac{\eta - \theta}{(1 + r)\eta} > 0.
\]

This proves the necessary part of the theorem.

Now assume that the DMPC result holds. Then the MPC out of assets should be smaller than the MPC out of current income, i.e.

\[
\frac{r}{1 + r} < \frac{r}{1 + r} \left[ 1 + \frac{\theta}{r\eta} \right].
\]

Given that \( \eta \) is positive, the above inequality holds when \( \theta \) must be positive. Furthermore, given that the MPC out of future income should also be positive, it has to be the case that:

\[
\frac{r}{1 + r} \left[ 1 - \frac{\theta}{\eta} \right] > 0,
\]

which implies that \( \theta \) must be smaller than \( \eta \).

\[\blacksquare\]

A.2 Traditional Life-Cycle Hypothesis Model

A.2.1 LC Consumption Policy Function

In case of the traditional LC model, the individual solves the following optimization problem

\[
\max_{\{C_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (aC_t - bC_t^2) \quad \text{subject to} \quad A_{t+1} = (1 + r)(A_t + Y_t - C_t), \quad \text{and} \quad \beta (1 + r) = 1. \tag{22}
\]

Her Euler equation dictates that consumption is expected to stay constant through her life-time:

\[
\mathbb{E}_t (C_{t+1}) = C_t. \tag{23}
\]

Constancy of consumption stems from the absence of precautionary savings (due to the quadratic utility function), no restrictions to borrowing, and no incentive to intertemporally transfer funds (since \( (1 + r)\beta = 1 \)). The law of iterative expectations extends to Euler equation to cover all time periods:

\[
\mathbb{E}_t (C_{t+\tau}) = C_t \quad \tau = 1, 2, ..., \tag{24}
\]

To obtain the closed-form solution of her consumption function, one uses her intertemporal budget constraint:

\[
\mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \frac{C_{t+\tau}}{(1 + r)^\tau} \right] = A_t + \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \frac{Y_{t+\tau}}{(1 + r)^\tau} \right]. \tag{24}
\]

Combine the the intertemporal budget constraint with the Euler equation, \( \mathbb{E}_t (C_{t+\tau}) = C_t \quad \tau = 1, 2, ..., \) to obtain her optimal consumption level:

\[
C_t = \frac{r}{1 + r} \left[ A_t + \mathbb{E}_t \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1 + r)^i} \right]. \tag{25}
\]
A.2.2 Time Series Properties of LC Consumption

The next step is to uncover the structure of the consumption innovations. Manipulating the consumption function (25) results in the following expression

\[ C_t = \frac{r}{1 + r} \left[ A_t + E_t \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1 + r)^i} \right] \]

\[ = \frac{r}{1 + r} A_t + \frac{r}{1 + r} E_t \left[ \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1 + r)^i} \right] \]

\[ = \frac{r}{1 + r} (1 + r) (A_{t-1} + Y_{t-1} - C_{t-1}) + \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ E_t Y_{t+i} \right] \]

\[ = r (A_{t-1} + Y_{t-1} - C_{t-1}) + \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ E_t Y_{t+i} - E_t Y_{t+i} \right] \]

The last equality was achieved by substituting \( A_{t+1} \) with \( (1 + r) (A_{t-1} + Y_{t-1} - C_{t-1}) \), and by adding and subtracting

\[ \frac{r}{1 + r} \sum_{i=1}^{\infty} \left[ E_{t-1} Y_{t+i} \right] \]

Given the expression for \( C_t \), \( C_{t-1} \) follows a similar expression:

\[ C_{t-1} = \frac{r}{1 + r} \left[ A_{t-1} + E_{t-1} \sum_{i=0}^{\infty} \frac{Y_{t-1+i}}{(1 + r)^i} \right] \]

which can be rewritten as follows:

\[ (1 + r) C_{t-1} = r (A_{t-1} + Y_{t-1}) + \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ E_{t-1} Y_{t+i} \right] \]

\[ r (A_{t-1} + Y_{t-1}) = (1 + r) C_{t-1} - \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ E_{t-1} Y_{t+i} \right] \]

Then, substitute out \( r (A_{t-1} + Y_{t-1}) \) in \( C_t \) with the expression from \( C_{t-1} \):

\[ C_t = -r C_{t-1} + (1 + r) C_{t-1} + \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ (E_t - E_{t-1}) Y_{t+i} \right] \]

Rewrite the above relationship, and find the change in consumption:

\[ \Delta C_t = \frac{r}{1 + r} \sum_{i=0}^{\infty} \left[ (E_t - E_{t-1}) Y_{t+i} \right] \]

Consumption reacts only to unexpected news in terms of the income process captured by the revisions in income expectations \((E_t - E_{t-1}) Y_{t+i}\).

The exact form of \( \Delta C_t \) depends on the process that income is following. In mean deviations, income typically follows a general autoregressive process of the form:

\[ Y_t = \sum_{i=1}^{k} \psi_i Y_{t-k} + \varepsilon_t \]

(26)

where \( \sum_{i=1}^{k} \psi_i \leq 1 \), \( \varepsilon_t \) is an independent and identically distributed process with mean zero and variance \( \sigma^2_\varepsilon \) and \( E_t \varepsilon_{t+1} = 0 \).
Call $\varepsilon_t$ the income innovation at time $t$. See Blanchard and Fischer (1989).

Similar to Flavin (1981), assume that income follows a stationary first order autoregression, i.e. $\psi_1 = \rho$ and $\psi_i = 0$ for $i = 2, \ldots, k$, $Y_t = \rho Y_{t-1} + \varepsilon_t$, $|\rho| < 1$. Then the following results hold:

1. By recursive backwards substitution, income is expressed as an infinite sum of past income innovations weighted by $\rho$:

$$Y_t = \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j}. \quad (27)$$

2. Thus, by the law of iterative expectations, the conditional expectation of $Y_t$ at $t - k$ becomes:

$$\mathbb{E}_{t-k} Y_t = \sum_{j=k}^{\infty} \rho^j \varepsilon_{t-j} \text{ since } \mathbb{E}_{t-k} \varepsilon_w = 0 \text{ for every } w > t - k.$$

3. Given the result (27), the changes in income, $\Delta Y_t$ is equal to the change in income innovations:

$$Y_t - Y_{t-1} = \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} - \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-1-j}$$

$$\Delta Y_t = \sum_{j=0}^{\infty} \rho^j \Delta \varepsilon_{t-j}.$$

The variance of income changes becomes:

$$\text{Var} (\Delta Y_t) = \sum_{j=0}^{\infty} \rho^{2j} \text{Var} (\varepsilon_{t-j} - \varepsilon_{t-j-1}) = 2\sigma^2 \sum_{j=0}^{\infty} \rho^{2j} = \frac{2}{1 - \rho^2} \sigma^2.$$

4. Further, by results (1) and (2) one can calculate the cumulative effect of the revisions in income expectations:

$$\sum_{i=0}^{\infty} \left[ \frac{\mathbb{E}_t Y_{t+i} - \mathbb{E}_{t-1} Y_{t+i}}{(1+r)^i} \right] = (1+r)^0 (Y_t - \mathbb{E}_{t-1} Y_t) + (1+r)^{-1} (\mathbb{E}_t Y_{t+1} - \mathbb{E}_{t-1} Y_{t+1})$$

$$+ (1+r)^{-2} (\mathbb{E}_t Y_{t+2} - \mathbb{E}_{t-1} Y_{t+2}) + \ldots$$

$$= \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} - \sum_{j=1}^{\infty} \rho^j \varepsilon_{t-j} + (1+r)^{-1} \left[ \sum_{j=1}^{\infty} \rho^j \varepsilon_{t-j+1} - \sum_{j=2}^{\infty} \rho^j \varepsilon_{t-j+1} \right]$$

$$+ (1+r)^{-2} \left[ \sum_{j=2}^{\infty} \rho^j \varepsilon_{t-j+2} - \sum_{j=3}^{\infty} \rho^j \varepsilon_{t-j+2} \right] + \ldots$$

$$= \sum_{j=0}^{\infty} \left[ \frac{\rho}{1+r} \right]^j \varepsilon_t = \frac{1}{1 - \frac{\rho}{1+r}} \varepsilon_t = \frac{1 + r}{1 + r - \rho} \varepsilon_t.$$

Hence, the forecast error of consumption $\Delta C_t$ depends only on the current income innovation $\varepsilon_t$:

$$\Delta C_t = \frac{r}{1+r} \sum_{i=0}^{\infty} \left[ \frac{\mathbb{E}_t - \mathbb{E}_{t-1}}{(1+r)^i} \right] Y_{t+i} = \frac{r}{1+r - \rho} \varepsilon_t.$$

The case where the level of income is stationary has been analyzed. Now, we move to the case where the level of income is non-stationary, but its first differences are stationary, i.e. $\psi_1 = (1 + \rho)$, $\psi_2 = -\rho$ and $\psi_i = 0$ for $i = 3, \ldots, k$. Under
stationary first differences the following three expressions hold:

\[ Y_t = (1 + \rho) Y_{t-1} - \rho Y_{t-2} + \varepsilon_t, \]

\[ \Delta Y_t = \rho \Delta Y_{t-1} + \varepsilon_t \Rightarrow \Delta Y_t = \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j}, \]

\[ \text{Var}(\Delta Y_t) = \sum_{j=0}^{\infty} \rho^{2j} \text{Var}(\varepsilon_{t-j}) = \sigma^2 \sum_{j=0}^{\infty} \rho^{2j} = \frac{1}{1 - \rho^2} \sigma^2. \]

In the case of stationary first differences the following results also hold:

5. Backwards recursive substitution yields a moving average representation of the income process:

\[ Y_t = \sum_{j=0}^{\infty} \left[ \sum_{k=0}^{j} \rho^k \right] \varepsilon_{t-j}. \]

6. Therefore, the present discounted value of revisions in income expectations become:

\[
\begin{align*}
\sum_{i=0}^{\infty} \left[ \frac{(E_t - E_{t-1}) Y_{t+i}}{(1+r)^i} \right] &= \sum_{i=0}^{\infty} \left[ \frac{\sum_{j=0}^{i} \rho^j}{(1+r)^i} \right] \varepsilon_t = \sum_{i=0}^{\infty} \left[ \frac{1 - \rho^{i+1}}{1 - \rho} \right] \varepsilon_t \\
&= \frac{\varepsilon_t}{1 - 1/(1+r)} - \rho \sum_{i=0}^{\infty} \left( \frac{\rho}{1+r} \right)^i = \frac{\varepsilon_t}{1 - 1/(1+r)} - \frac{\rho}{1 + \rho/1+r} \\
&= \frac{\varepsilon_t}{1 - \rho (1 + \rho/1+r)} = \frac{1}{1 - \rho/1+r} \varepsilon_t.
\end{align*}
\]

Hence, based on result (6), the change in consumption is equal to:

\[ \Delta C_t = \frac{r}{1+r} \sum_{i=0}^{\infty} \left[ \frac{(E_t - E_{t-1}) Y_{t+i}}{(1+r)^i} \right] = \frac{r}{1+r} \times \frac{1}{1 + \rho/1+r} \varepsilon_t \]

\[ = \frac{r}{1+r} - (1 + \rho/1+r) \varepsilon_t = \frac{r(1+r)}{r^2 + r - \rho r} \varepsilon_t = \left[ \frac{1+r}{1+r - \rho} \right] \varepsilon_t. \]

A.3 Behavioral Life-Cycle Hypothesis Model

In the case of the behavioral specification (2) the individual behaves in accordance to the following optimization problem:

\[
\max_{\{C_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ a C_t - b C_t^2 + c (Y_t - C_t) - d (Y_t - C_t)^2 \right] \quad \text{subject to} \quad A_{t+1} = (1+r)(A_t + Y_t - C_t), \quad \beta (1+r) = 1. \]

Her Euler equation is the following:

\[
\begin{align*}
a - 2b C_t - c + 2d (Y_t - C_t) &= \mathbb{E}_t \left[ a - 2b C_{t+1} - c + 2d (Y_{t+1} - C_{t+1}) \right], \\
b C_t - d (Y_t - C_t) &= \mathbb{E}_t \left[ b C_{t+1} - d (Y_{t+1} - C_{t+1}) \right], \\
[(b + d) C_t - d Y_t] &= \mathbb{E}_t [(b + d) C_{t+1} - d Y_{t+1}].
\end{align*}
\]

Further, the law of iterative expectations dictates that:

\[
[(b + d) C_t - d Y_t] = \mathbb{E}_t [(b + d) C_{t+i} - d Y_{t+i}], i = 1, 2, \ldots
\]

\[
\mathbb{E}_t C_{t+i} = C_t + \frac{d}{b + d} (\mathbb{E}_t Y_{t+i} - Y_t).
\]

34
To uncover the optimal policy function one starts from the intertemporal budget constraint and substitutes out the expectations of future consumption, $E_t C_{t+i}$, using the Euler equation:

$$
\sum_{i=0}^{\infty} \frac{E_t C_{t+i}}{(1+r)^i} = A_t + E_t \left[ \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1+r)^i} \right],
$$

$$
C_t + \sum_{i=1}^{\infty} \frac{E_t C_{t+i}}{(1+r)^i} = A_t + Y_t + E_t \left[ \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1+r)^i} \right],
$$

$$
C_t + \sum_{i=1}^{\infty} \left[ \frac{C_t + \frac{d}{b+d} (E_t Y_{t+i} - Y_t)}{(1+r)^i} \right] = A_t + Y_t + E_t \left[ \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1+r)^i} \right],
$$

$$
C_t \left[ \frac{1}{r} \left( 1 + \frac{1}{b+d} \right) Y_t + \frac{d}{b+d} \sum_{i=0}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i} \right] = A_t + Y_t + E_t \left[ \sum_{i=1}^{\infty} \frac{Y_{t+i}}{(1+r)^i} \right],
$$

$$
C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d) r} \right] Y_t + \frac{r}{1+r} \left( 1 - \frac{d}{b+d} \right) \sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i},
$$

$$
C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d) r} \right] Y_t + \frac{r}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i}.
$$

I now obtain the changes in consumption. Starting from the preceding expression, $A_t$ is substituted out using the one-period budget constraint, $A_t = (1+r) (A_{t-1} + Y_{t-1} - C_{t-1})$:

$$
C_t = \frac{r}{1+r} (A_{t-1} + Y_{t-1} - C_{t-1}) + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d) r} \right] Y_t + \frac{r}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i}.
$$

In the above expression add and subtract the following cumulative income expectation:

$$
\frac{r b}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_{t-1} Y_{t+i}}{(1+r)^i}
$$

to obtain:

$$
C_t = r (A_{t-1} + Y_{t-1}) - r C_{t-1} + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d) r} \right] Y_t + \frac{r}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_{t-1} Y_{t+i}}{(1+r)^i} + \frac{r b}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_t Y_{t+i} - E_{t-1} Y_{t+i}}{(1+r)^i}.
$$

Now consider the policy function of lagged consumption, $C_{t-1}$:

$$
C_{t-1} = \frac{r}{1+r} A_{t-1} + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d) r} \right] Y_{t-1} + \frac{r}{1+r b + d} \sum_{i=1}^{\infty} \frac{E_{t-1} Y_{t+i-1}}{(1+r)^i}.
$$

Note that the present discounted value of income can be rewritten:

$$
\sum_{i=1}^{\infty} \frac{E_{t-1} Y_{t+i-1}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{E_{t-1} Y_{t+i}}{(1+r)^{i+1}},
$$

35
and therefore $C_{t-1}$ is expressed as follows:

$$C_{t-1} = \frac{r}{1+r} A_{t-1} + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d)r} \right] Y_{t-1} + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=0}^{\infty} \left[ \frac{E_{t-1}Y_{t+i}}{(1+r)^{i+1}} \right].$$

Multiply the above through by $(1+r)$ to obtain:

$$(1+r)C_{t-1} = r A_{t-1} + r \left[ 1 + \frac{d}{(b+d)r} \right] Y_{t-1} + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=0}^{\infty} \left[ \frac{E_{t-1}Y_{t+i}}{(1+r)^{i}} \right].$$

The difference $r(A_{t-1} + Y_{t-1})$ is then equal to:

$$r(A_{t-1} + Y_{t-1}) = (1+r)C_{t-1} - \frac{d}{b+d}Y_{t-1} - \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{E_{t-1}Y_{t+i}}{(1+r)^{i}} \right].$$

Substituting $r(A_{t-1} + Y_{t-1})$ back in the expression for $C_t$, one can deduce that:

$$C_t = (1+r)C_{t-1} - \frac{d}{b+d}Y_{t-1} - \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{E_{t-1}Y_{t+i}}{(1+r)^{i}} \right]$$

$$- rC_{t-1} + \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d)r} \right] Y_t + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{E_{t-1}Y_{t+i}}{(1+r)^{i}} \right] + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{(E_t - E_{t-1})Y_{t+i}}{(1+r)^{i}} \right],$$

which gives rise to a relationship for the change in consumption:

$$\Delta C_t = \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d)r} \right] Y_t - \frac{d}{b+d}Y_{t-1} - \frac{r}{1+r} \frac{b}{b+d} E_{t-1}Y_t + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{(E_t - E_{t-1})Y_{t+i}}{(1+r)^{i}} \right].$$

Now, add and subtract $\left[ \frac{r}{1+r} \frac{b}{b+d} Y_t \right]$ to the right-hand-side of the above expression to obtain:

$$\Delta C_t = \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d)r} \right] Y_t + \left[ \frac{r}{1+r} \frac{b}{b+d} Y_t - \frac{r}{1+r} \frac{b}{b+d} Y_t \right]$$

$$- \frac{d}{b+d} Y_{t-1} - \frac{r}{1+r} \frac{b}{b+d} E_{t-1}Y_t + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{(E_t - E_{t-1})Y_{t+i}}{(1+r)^{i}} \right].$$

Rewrite the element $\left[ \frac{r}{1+r} \frac{b}{b+d} Y_t \right]$ as $\left[ \frac{r}{1+r} \frac{b}{b+d} E_tY_t \right]$. Then, combine it with the element $- \frac{r}{1+r} \frac{b}{b+d} Y_t$ to obtain:

$$\Delta C_t = \frac{r}{1+r} \left[ 1 + \frac{d}{(b+d)r} \right] Y_t - \frac{r}{1+r} \frac{b}{b+d} Y_t$$

$$- \frac{d}{b+d} Y_{t-1} + \frac{r}{1+r} \frac{b}{b+d} [E_t - E_{t-1}] Y_t + \frac{r}{1+r} \frac{b}{b+d} \sum_{i=1}^{\infty} \left[ \frac{(E_t - E_{t-1})Y_{t+i}}{(1+r)^{i}} \right].$$
The revision of current income

\[
\left[ \frac{r}{1 + r} \frac{b}{b + d} \mathbf{E}_t - \mathbf{E}_{t-1} \right] Y_t
\]

can be included in the part, which contains all the future expected income revisions

\[
\frac{r}{1 + r} \frac{b}{b + d} \sum_{i=1}^{\infty} \left[ \frac{(\mathbf{E}_t - \mathbf{E}_{t-1}) Y_{t+i}}{(1 + r)^i} \right]
\]
to obtain the following:

\[
\Delta C_t = \frac{r}{1 + r} \left[ 1 + \frac{d}{(b + d) r} \right] Y_t - \frac{r}{1 + r} \frac{b}{b + d} Y_t - \frac{d}{b + d} Y_{t-1} + \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=0}^{\infty} \left[ \frac{(\mathbf{E}_t - \mathbf{E}_{t-1}) Y_{t+i}}{(1 + r)^i} \right]
\]

\[
\Delta C_t = \frac{r}{1 + r} \left[ 1 + \frac{d}{(b + d) r} - \frac{b}{b + d} \right] Y_t - \frac{d}{b + d} Y_{t-1} + \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=0}^{\infty} \left[ \frac{(\mathbf{E}_t - \mathbf{E}_{t-1}) Y_{t+i}}{(1 + r)^i} \right]
\]

\[
\Delta C_t = \frac{r}{1 + r} \left[ \frac{d}{(b + d) r} + \frac{d}{b + d} \right] Y_t - \frac{d}{b + d} Y_{t-1} + \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=0}^{\infty} \left[ (\mathbf{E}_t - \mathbf{E}_{t-1}) Y_{t+i} \right] (1 + r)^i
\]

\[
\Delta C_t = \frac{d}{b + d} \Delta Y_t + \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=0}^{\infty} \left[ (\mathbf{E}_t - \mathbf{E}_{t-1}) Y_{t+i} \right] (1 + r)^i
\]

Under the stationary AR(1) income process, and by result (3), one can show that the change in consumption is equal to:

\[
\Delta C_t = \frac{d}{b + d} \sum_{j=0}^{\infty} \rho^j \Delta \varepsilon_{t-j} + \frac{b}{b + d} \frac{r}{1 + r - \rho} \varepsilon_t,
\]

\[
= \left( \frac{d}{b + d} + \frac{r}{1 + r - \rho} \frac{b}{b + d} \right) \varepsilon_t - \frac{r}{1 + r - \rho} \frac{b}{b + d} \varepsilon_{t-1} + \frac{d}{b + d} \sum_{j=1}^{\infty} \rho^j \Delta \varepsilon_{t-j}.
\]

In addition the variance of the consumption change is equal to:

\[
\text{Var} (\Delta C_t) = \left( \frac{d}{b + d} \right)^2 \sum_{j=0}^{\infty} \rho^j (2 \sigma_{\varepsilon}^2) + \left( \frac{b}{b + d} \frac{r}{1 + r - \rho} \right)^2 \sigma_{\varepsilon}^2
\]

\[
= \left[ \left( \frac{d}{b + d} \right)^2 \frac{2}{1 - \rho^2} + \left( \frac{b}{b + d} \frac{r}{1 + r - \rho} \right)^2 \right] \sigma_{\varepsilon}^2.
\]
Comparing the variance of the change in consumption to that of the change in income, one finds that:

$$\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t)$$

$$= \left[ \frac{2}{1-\rho^2} - \left( \frac{d}{b+d} \right)^2 \cdot \frac{2}{1-\rho^2} - \left( \frac{b}{b+d} \cdot \frac{1+r}{1+r-\rho} \right)^2 \right] \sigma^2 \epsilon$$

$$= \left[ \frac{2(b+d)^2 (1+r-\rho)^2 - 2d^2 (1+r-\rho)^2 - b^2 r^2 (1-\rho^2)}{(1-\rho^2)(b+d)^2 (1+r-\rho)^2} \right] \sigma^2 \epsilon^2.$$

So, the difference between $\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t)$ becomes:

$$\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t)$$

$$= \left[ \frac{2(b^2 + d^2 + 2bd)(1+r-\rho)^2 - 2d^2 (1+r-\rho)^2 - b^2 r^2 (1-\rho^2)}{(1-\rho^2)(b+d)^2 (1+r-\rho)^2} \right] \sigma^2 \epsilon^2$$

The variance of income changes is larger than the variance of consumption changes because:

$$2b^2 (1+r-\rho)^2 > b^2 r^2 (1-\rho)^2$$ since $0 < r < 1$

$$> b^2 r^2 (1-2\rho + \rho^2)$$

$$= b^2 r^2 (1-2\rho^2 + \rho^2)$$ since $|\rho| < 1$

$$= b^2 r^2 (1-\rho^2) > 0.$$

Under the assumption of stationary first differences, with (28) and (29) one infers that:

$$\Delta C_t = \frac{d}{b+d} \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j} + \frac{b}{b+d} \cdot \frac{1+r}{1+r-\rho} \epsilon_t$$

$$= \left[ \frac{d}{b+d} + \frac{1+r}{1+r-\rho} \cdot \frac{b}{b+d} \right] \epsilon_t + \frac{d}{b+d} \sum_{j=1}^{\infty} \rho^j \epsilon_{t-j}. \tag{36}$$

The corresponding variance of consumption changes takes the following form:

$$\text{Var} (\Delta C_t) = \left( \frac{d}{b+d} \right)^2 \sum_{j=0}^{\infty} \rho^{2j} (\sigma^2 \epsilon^2) \cdot \left( \frac{b}{b+d} \cdot \frac{1+r}{1+r-\rho} \right)^2 \sigma^2 \epsilon^2 = \left[ \left( \frac{d}{b+d} \right)^2 \cdot \frac{1}{1-\rho^2} + \left( \frac{b}{b+d} \cdot \frac{1+r}{1+r-\rho} \right)^2 \right] \sigma^2 \epsilon^2. \tag{37}$$

In addition, the difference in the variances of income and consumption changes becomes:

$$\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t)$$

$$= \left[ \frac{1}{1-\rho^2} - \left( \frac{d}{b+d} \right)^2 \cdot \frac{1}{1-\rho^2} - \left( \frac{b}{b+d} \cdot \frac{1+r}{1+r-\rho} \right)^2 \right] \sigma^2 \epsilon^2$$

$$= \left[ \frac{(b+d)^2 (1+r-\rho)^2 - d^2 (1+r-\rho)^2 - b^2 (1-\rho^2) (1+r)^2}{(1-\rho^2)(b+d)^2 (1+r-\rho)^2} \right] \sigma^2 \epsilon^2.$$
So, the difference between \( \text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t) \) becomes:

\[
\begin{align*}
\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t) &= \left[ \frac{(b^2 + d^2 + 2bd) (1 + r - \rho)^2 - d^2 (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2}{(1 - \rho^2) (b + d)^2 (1 + r - \rho)^2} \right] \sigma^2 \\
&= \left[ \frac{b^2 (1 + r - \rho)^2 + 2bd (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2}{(1 - \rho^2) (b + d)^2 (1 + r - \rho)^2} \right] \sigma^2 \\
&= \left[ \frac{b (b + 2d) (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2}{(1 - \rho^2) (b + d)^2 (1 + r - \rho)^2} \right] \sigma^2.
\end{align*}
\]

In the case of negative correlation in income changes, \(-1 < \rho < 0\), the volatility of income changes is larger than the volatility in consumption changes since:

\[
\begin{align*}
b (b + 2d) (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2 \\
b^2 (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2 + 2bd (1 + r - \rho)^2 \\
> b^2 (1 + r - \rho)^2 - b^2 (1 + r)^2 + 2bd (1 + r - \rho)^2 \\
> b^2 (1 + r - \rho)^2 - b^2 (1 + r)^2 + 2bd (1 + r - \rho)^2 = 2bd (1 + r - \rho)^2 > 0.
\end{align*}
\]

In the case of positive correlation in income changes, \(0 < \rho < 1\), the volatility of income changes is larger than the volatility in consumption changes, under the following necessary condition:

\[
d > \frac{b (1 - \rho^2) (1 + r)^2}{(1 + r - \rho)^2}.
\]

The condition ensures that \(b (b + 2d) (1 + r - \rho)^2 - b^2 (1 - \rho^2) (1 + r)^2\) is positive. Since \((1 - \rho^2) (b + d)^2 (1 + r - \rho)^2\) is always positive, the restriction on \(d\) ensures that the difference in \(\text{Var} (\Delta Y_t) - \text{Var} (\Delta C_t)\) is also positive.

### A.4 The Saving Stock

The saving stock at period \(t\), \(S_t\), is defined as \(S_t = \frac{1}{1 + r} A_t + Y_t - C_t\). See equation in Campbell and Deaton (1989). Savings take the following form for the life-cycle consumer:

\[
SLC_t = \frac{1}{1 + r} Y_t - \frac{r}{1 + r} \sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1 + r)^i},
\]

The saving stock for the behavioral consumer is slightly different:

\[
SBLC_t = \left[ 1 - \frac{r}{1 + r} - \frac{d}{(b + d) (1 + r)} \right] Y_t - \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1 + r)^i}.
\]

Both relationships depend on current income and on the present discounted value of expected income. To obtain closed-form expressions, one has to specify a model for the income process.

First, in the case of stationarity AR(1) income, \(Y_t = \rho Y_{t-1} + \epsilon_t\), one can show that current income is a sufficient statistic for the present discounted value of expected income:

\[
\sum_{i=1}^{\infty} \frac{E_t Y_{t+i}}{(1 + r)^i} = \frac{\sum_{j=0}^{\infty} \rho^j E_t \epsilon_{t+i-j}}{(1 + r)^i} = \frac{\sum_{j=0}^{\infty} \rho^j \sum_{j=0}^{\infty} E_t \epsilon_{t-j}}{(1 + r)^i} = \frac{\rho \sum_{j=0}^{\infty} E_t \epsilon_{t-j}}{(1 + r)^i},
\]

because \(E_t \epsilon_{t+w} = 0\) for every \(w\) greater than zero.

\[
\sum_{i=1}^{\infty} \frac{\rho^i}{(1 + r)^i} Y_t = \frac{\rho}{1 + r - \rho} Y_t, \quad \text{since} \quad |\rho| < 1.
\]
Therefore, the saving stock for the LC consumer under AR(1) income stationarity is equal to:

\[ SLC_t = \left( \frac{1}{1+r} - \frac{r}{1+r} \frac{1}{1 + r - \rho} \right) Y_t = \frac{1}{1+r} - \rho Y_t. \]

The saving stock for the BLC consumer under AR(1) income stationarity is equal to:

\[ SBLC_t = \left[ 1 - \frac{r}{1+r} - \frac{d}{b+d}(1+r) \frac{r}{1+r b+d 1 + r - \rho} \right] Y_t = \left[ \frac{b}{b+d} \frac{1}{1+r} \right] Y_t = \frac{b}{b+d} SLC_t. \]

Second, when income has stationary first differences, \( \Delta Y_t = \rho \Delta Y_{t-1} + \varepsilon_t \), its levels follows a non-stationary AR(2) process and can be rewritten in the following form:

\[ Y_t = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \rho^k \varepsilon_{t-i-j-k} + \rho^i Y_t. \]

Then the conditional expectation at time \( t \) of the income income level at period \( t + i \) is equal to:

\[ Y_{t+i} = \left( \sum_{k=0}^{i} \rho^k \right) \left( \sum_{j=0}^{\infty} \varepsilon_{t-j} \right) + \rho^i Y_t, \]

because \( \mathbb{E}_t \varepsilon_{t+w} = 0 \) for every \( w \) greater than zero. Consequently, the savings of the life-cycle consumer become as follows:

\[ SLC_t = \frac{1}{1+r} Y_t - \frac{r}{1+r} \left[ \left( \sum_{k=0}^{\infty} \sum_{i=0}^{k} \rho^i \right) \times \left( \sum_{j=0}^{\infty} \varepsilon_{t-j} \right) + \sum_{i=1}^{\infty} \left( \frac{\rho}{1+r} \right)^i Y_t \right] \]

\[ = \left[ \frac{1}{1+r} - \frac{r \rho}{(1+r) (1+r - \rho)} \right] Y_t - \frac{r}{1+r} \left( \sum_{j=0}^{\infty} \varepsilon_{t-j} \right) \times \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} \left( \frac{\rho}{1+r} \right)^i \]

\[ = \frac{1}{1+r - \rho} Y_t - \frac{r}{1+r} \frac{1}{1-r} \left( \sum_{j=0}^{\infty} \varepsilon_{t-j} \right) \times \sum_{i=1}^{\infty} \left[ \frac{1}{(1+r)^i} - \left( \frac{\rho}{1+r} \right)^i \right] \]

\[ = \frac{1}{1+r - \rho} \left[ (1-\rho) Y_t - \left( \sum_{i=0}^{\infty} \varepsilon_{t-i} \right) \right] = \frac{1}{1+r - \rho} [(1-\rho) Y_t - (Y_t - \rho Y_{t-1})] \]

\[ = -\frac{\rho}{1+r - \rho} \Delta Y_t. \]

The last step of the above derivation is based on the following observation

\[ Y_t = \sum_{j=0}^{\infty} \left( \sum_{k=0}^{j} \rho^k \right) \varepsilon_{t-j} = \rho Y_{t-1} + \sum_{j=0}^{\infty} \varepsilon_{t-j}. \]
For the case of the BLC consumer one can show $\subset$ 

\[
SBLC_t = \left[ 1 - \frac{r}{1 + r} - \frac{d}{(b + d)(1 + r)} \right] Y_t - \frac{r}{1 + r} \frac{b}{b + d} \sum_{i=1}^{\infty} \left[ \frac{E_t Y_{t+i}}{(1 + r)^i} \right] \\
= \left[ 1 - \frac{r}{1 + r} - \frac{d}{(b + d)(1 + r)} \right] Y_t - \frac{r}{1 + r} \frac{b}{b + d} \left[ \left( \sum_{i=1}^{\infty} \frac{\sum_{k=0}^{i-1} \rho^k}{(1 + r)^i} \right) \times \left( \sum_{i=0}^{\infty} \varepsilon_{t-i} \right) + \sum_{i=1}^{\infty} \left( \frac{\rho}{1 + r} \right)^i Y_t \right] \\
= \left[ 1 - \frac{r}{1 + r} - \frac{d}{(b + d)(1 + r)} - \frac{r}{1 + r} \frac{b}{b + d} \frac{\rho}{1 + r - \rho} \right] Y_t \\
- \frac{r}{1 + r} \frac{b}{b + d} \frac{1}{1 - \rho} \left[ \frac{1}{r} - \frac{\rho}{1 + r - \rho} \right] \left( \sum_{i=0}^{\infty} \varepsilon_{t-i} \right) \\
= \frac{b}{b + d} \left[ \frac{1}{1 + r - \rho} \right] \left( 1 - \rho \right) Y_t - \left( \sum_{i=0}^{\infty} \varepsilon_{t-i} \right) \right] = \frac{b}{b + d} \left[ \frac{-\rho}{1 + r - \rho} \Delta Y_t \right] \\
= \frac{b}{b + d} SLC_t.
\]