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Theory and Methodology

Goal programming for decision making: An overview of the current state-of-the-art

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Abstract

There have been significant advances in the theory of goal programming (GP) in recent years, particularly in the area of intelligent modelling and solution analysis. The intention of this paper is to provide an overview of these developments, to detail and assess the current state-of-the-art in the subject, and to highlight areas which seem promising for future research. Modelling techniques such as detection and restoration of pareto efficiency, normalisation, redundancy checking, and non-standard utility function modelling are overviewed. The connection between GP and other multi-objective-programming techniques as well as a utility interpretation of GP are examined. The rationality of ranking Multi-Criteria Decision Making techniques, and of placing GP in such a ranking, is discussed.

Keywords: Goal programming; Decision analysis; Modelling; Utility theory

1. Introduction

Goal Programming (GP) is a multi-objective programming technique. The ethos of GP lies in the Simonan [50] concept of satisfying of objectives. Simon conjectures that in today's complex organisations the decision makers (DMs) do not try to maximise a well defined utility function. In fact the conflicts of interest and the incompleteness of available information make it almost impossible to build a reliable mathematical representation of the DMs' preferences. On the contrary, within this kind of decision environment the DMs try and achieve a set of goals (or targets) as closely as possible. Although GP was not originally conceived within a satisfying philosophy it still provides a good framework in which to implement this kind of philosophy.

The roots of GP lie in a paper by Charnes et al. in 1955 [6] in which they deal with executive compensation methods. A more explicit definition is given by Charnes and Cooper [7] in 1961 in which the term GP is first used. Until the middle of the 1970s, GP applications reported in the literature were rather scarce. Since that time, and chiefly

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due to seminal works by Lee [30] and Ignizio [22], an impressive boom of GP applications and technical improvements have arisen. It can be said that GP has been, and still is, the most widely used multi-criteria decision making technique [44]. Although Schniederjans [48] has detected a decline in the life cycle of GP with regard to theoretical developments, the number of cases along with the range of fields to which GP has been, and is still is, applied is impressive, as shown by recent surveys by Romero [44,45], Schniederjans [47], and Tamiz et al. [54].

GP models can be classified into two major subsets. In the first type the unwanted deviations are assigned weights according to their relative importance to the DM and minimised as an Archimedian sum. This is known as weighted GP(WGP). The algebraic formulation of a WGP is given as:

$$\begin{aligned} \min \quad & z = \sum_{i=1}^{k} (u_i n_i + v_i p_i) \\ \text{s.t.} \quad & f_i(\mathbf{x}) + n_i - p_i = b_i, \quad i = 1 \ldots Q, \quad \mathbf{x} \in C_s, \end{aligned}$$

where $f_i(\mathbf{x})$ is a linear function (objective) of $\mathbf{x}$, and b_i the target value for that objective. n_i and p_i represent the negative and positive deviations from this target value. u_i and v_i are the respective positive weights attached to these deviations in the achievement function z. These weights take the value zero if the minimisation of the corresponding deviational variable is unimportant to the DM. C_s is an optional set of hard constraints as found in linear programming (LP).

In the other major subset of GP the deviational variables are assigned into a number of priority levels and minimised in a lexicographic sense. A lexicographic minimisation being defined as a sequential minimisation of each priority whilst maintaining the minimal values reached by all higher priority level minimisations. This is known as lexicographic GP(LGP), as introduced and chiefly developed by Ijiri [23], Lee [30], and Ignizio [22].

The algebraic representation of an LGP is given as:

$$\begin{aligned} \text{Lex min} \quad & \mathbf{a} = (g_1(\mathbf{n},\mathbf{p}), g_2(\mathbf{n},\mathbf{p}), \ldots\ldots, g_L(\mathbf{n},\mathbf{p})) \\ \text{s.t.} \quad & f_i(\mathbf{x}) + n_i - p_i = b_i, \quad i = 1, \ldots, Q. \end{aligned}$$

This model has L priority levels, and Q objectives. $\mathbf{a}$ is an ordered vector of these L priority levels. n_i and p_i are deviational variables which represent the under and over achievement of the ith goal, respectively. $\mathbf{x}$ is the set of decision variables to be determined. Any 'LP' style hard constraints are placed, by convention, in the first priority level. A standard 'g' (within priority level) function is given by

$$g_l(\mathbf{n},\mathbf{p}) = u_{l_1} n_1 + \cdots + u_{l_q} n_q + v_{l_1} p_1 + \cdots + v_{l_Q} p_Q,$$

where u and v represent inter-priority level weights, as in weighted GP, a zero weight is given to any deviational variable whose minimisation is unimportant.

Although LGP and WGP are the most widely used GP variants (Tamiz et al. [54] show that around 64% of GP applications reported in the literature use LGP and 21% WGP) there are other GP variants. Among these are MINMAX GP, introduced by Flavell in 1976 [14], where the maximum deviation is minimised, and its close relative Fuzzy GP introduced by Zimmermann [8] in 1978.

This overview of the current state-of-the-art in GP attempts to stimulate OR practitioners in the right use of this satisfying approach by means of increasing the clarity of the philosophy underlying the GP formulation. It is important to note that this paper is concerned with GP as a tool for decision making. Consequently, important uses of GP in other areas such as regression analysis or discriminant analysis are not considered here. Similarly, closely related fields, such as data envelopment analysis are not included either. The reason for these omissions is the amount and scope of the application of GP means all these subjects could not receive the review space they deserve in a single paper.

The remainder of this paper is divided into six sections. Section 2 gives an overview of the current state-of-the-art regarding GP modelling, detailing approaches to issues such as pareto efficiency, normalisation, and redundancy checking. Section 3 discusses connections between GP and other multi-objective programming techniques such as compromise programming (CP) and the reference point method (RPM). Section 4 gives links between GP and utility function theory. Section 5 discusses the feasibility of ranking MCDM methods. Section 6 reviews other recent developments in the field of GP. Finally, Section 7 summarises and points towards directions for future research.

2. An overview of current modelling techniques

An area now recognised as being of equal importance as the actual solving of a goal programme is that of accurate modelling and solution analysis. There are various modelling pitfalls that should be avoided, particularly by those unfamiliar in working in a multi-objective environment, or those forming goal programmes by the conversion of previously single objective LP models. This section endeavours to highlight the pitfalls, and gives current methods for the avoidance and resolution of any resulting errors.

2.1. Pareto efficiency considerations

One major criticism of GP in the past has concerned the area of pareto efficiency. In any multiple objective problem, a solution is said to be pareto inefficient (or dominated) if the achieved level of any one objective can be improved without worsening the achieved level of any other objective [44]. The standard GP formulation can produce inefficient solutions if the target values are set too pessimistically. This fact led some authors to argue against the use of GP [28] and led to various GP variants and extensions, such as the method of Hannan [20] and the RPM discussed in Section 3 [62]. The problem of restoring pareto efficiency to an inefficient GP has now been solved. The basis for all restoration techniques lies in the pioneering work by Hannan [20]. This section gives the state-of-the-art regarding accurate pareto inefficiency detection, isolation, and restoration techniques.

2.1.1. Pareto state of an objective

A recent work by Tamiz and Jones [57] subdivides each objective in a GP model into one of three pareto states.

- *Efficient*: Achieved value of the objective cannot be improved without worsening the level of another objective.
- *Inefficient*: Achieved value of the objective can be improved without worsening the level of another objective.
- *Unbounded*: Achieved value of the objective can be infinitely improved without worsening the level of another objective.

From the definitions it can be seen that the efficient and inefficient subsets exclusively and exhaustively form the objective set, and the unbounded objectives are a further subset of the inefficient subset. If any objective is inefficient, then the entire model is inefficient. Likewise if any objective is unbounded, then the entire model is unbounded, a likely indication of modelling error.

This classification scheme provides a logical means of isolating pareto inefficiency and unboundedness in a model. A series of tests are given by Tamiz and Jones [57] which classify the objectives into their pareto states in a computationally efficient manner. This method requires only degenerate simplex iterations, and thus allows detection whilst remaining at the original solution point, from where restoration can begin if necessary.

2.1.2. Restoration of pareto efficiency

Given that one or more objectives are found to be pareto efficient, the remaining task is to project the inefficient solution onto the efficient boundary in a manner satisfactory to the DM. Before any restoration-type movement can take place, each objective must be safeguarded against degradation. This can be easily achieved by the placing of upper or lower bounds on the deviational variables:

$$\mathrm{NWV}_j \geqslant \mathrm{NWV}_j^*, \qquad \mathrm{WV}_j \leqslant \mathrm{WV}_j^*,$$

where NWV_j and WV_j are the non-weighted and weighted deviational variables for the jth objective respectively, which take the values of NWV_j^* and WV_j^* at the inefficient solution point.

With these bounds in place, any improvement in any objective will be an acceptable Pareto move towards the efficient frontier. There may be many solutions on the efficient frontier that are possible results of the restoration process. A good restoration method selects the point amongst this solution set that is optimal with respect to the

DMs preferences. There are several possible restoration methods, and these are outlined below.

- *Straight restoration [20,36,44]:* in this method an unweighted sum of the unweighted deviational variables corresponding to the inefficient objectives is maximised. This is achieved by the addition of an extra priority level to the achievement function of an LGP, or the conversion of a WGP into an LGP with two priority levels. For example, the achievement function

  $$\min \mathbf{a} = [(n_1), (2n_2 + 5p_3)]$$

  becomes (assuming all three objectives are inefficient),

  $$\min \mathbf{a} = [(n_1), (2n_2 + 5p_3), (-p_1 - p_2 - n_3)].$$

  This method will ensure a pareto efficient solution is reached providing the model is not pareto unbounded. However, the individual weights on the deviational variables are not taken into account.
- *Preference based restoration [57]:* in this method a sum of the unwanted deviational variables is again penalised. In this case however, the weights and/or priority levels used for the weighted deviational variables are transferred to the unweighted variables. For example, the achievement function

  $$\min \mathbf{a} = [(n_1), (2n_2 + 5p_3)],$$

  now becomes (again assuming all objectives are inefficient)

  $$\min \mathbf{a} = [(n_1), (2n_2 + 5p_3), (-p_1), (-2p_2 - 5n_3)].$$

  This method ensures continuity of preference both below and above the target values, thus ensuring a pareto efficient point is found in accordance with the DM's original preferences.
- *Interactive restoration [57]:* in this method the set of inefficient objectives are presented to the DM who then chooses the one they would most like to see improved. This process continues iteratively until an efficient solution is produced. The interactive method allows for the fact that the DM's preferences below and above the target may not be continuous. This is a frequent occurrence in GP models, where some goal levels represent standards to be met (legal requirements, environmental standards, etc.) whilst others represent DM desires (low cost, high profit, etc.). This method has the advantage of involving the DM in the restoration process, and is thus subject to the discussions regarding multi-objective interactive methods [60]. Fortunately, the number of interactive iterations required to restore efficiency is usually low [57].

2.2. Normalisation techniques

A major issue of debate within the GP community has concerned the use of normalisation techniques to overcome incommensurability. Incommensurability in a WGP, or within a priority level of an LGP, occurs when deviational variables measured in different units are summed up directly. This simple summation will cause an unintentional bias towards the objectives with a larger magnitude. This bias may lead to erroneous or misleading results.

One suggestion to overcome this difficulty is to divide each objective through by a constant pertaining to that objective. This ensures that all objectives have roughly the same magnitude. Such a constant is known as a *normalisation constant*. This leads to the revised algebraic format for a WGP:

$$\min \quad z = \sum_{i=1}^{Q} \left(\frac{u_i n_i + v_i p_i}{k_i} \right) \tag{1}$$

$$\text{s.t.} \quad f_i(\mathbf{x}) + n_i - p_i = b_i, \quad i = 1 \ldots Q, \tag{2}$$

$$x \in C_s,$$

where k_i is the normalisation constant for the ith objective.

There are several different normalisation methods, each with its own normalisation constant. The attributes of each are given below.

- *Percentage normalisation [44]:* Here the normalisation constant is the target value divided by hundred: $N_i = b_i/100$. This ensures that all deviations are measured on a percentage scale. This method restores meaning to the optimal achievement function value, which now measures the total percentage sum of deviations from goals. It does however, require accurate setting of the target values and is not applicable to models in which any objective has a target value of zero.

- *Euclidean normalisation [12,58]:* The normalisation constant in this method is the Euclidean norm of the technical coefficients in the objective: $N_i = \sqrt{\sum_j a_{ij}^2}$. This method is computationally robust and has the advantage of reconciling the L_2 distance of deviation with the L_1 GP formulation. It does not however restore significant meaning to the final achievement function value.
- *Summation normalisation [24]:* Here the normalisation constant is the absolute value of the technical coefficients in the objective: $N_i = \sum_j \mid a_{ij} \mid$. This method has a larger divisor than the Euclidean method and is found to be better when scaling problems that are badly incommensurable. It has the same robustness as the Euclidean method, but also does not restore meaning to the achievement function value.
- *Zero–one normalisation [36]:* The normalisation constant in this method is equal to the distance between the target value and the worst possible value for the relevant deviational variable within the feasible set defined by the hard constraints in the model. Note that in this method the normalisation constant is associated with the penalised deviational variable rather than the objective. Thus, $k_i^p = p_i^{\max}$ and $k_i^n = n_i^{\max}$, where $n_i^{\max}$ and $p_i^{\max}$ are the worst possible values of n_i, p_i within the feasible set. This scales all deviations onto a scale between zero (target) and one (worst possible). An alternative is to set the value zero to equal the minimal possible deviation within the feasible set, giving $k_i^p = p_i^{\max} - p_i^{\min}$ and $k_i^n = n_i^{\max} - n_i^{\min}$. This eliminates distortions due to unrealistically set target values. The zero–one method restores meaning to the final achievement function value, being a measure of non-achievement of targets. It is however, the least computationally stable of the methods [24], requiring the existence of a closed feasible set with regard to each objective and no degenerate (zero ranged) objectives. The computational time needed is significantly greater than the other methods due to the necessity to compute the worst values of each deviational variable.

The idea of obtaining a measure of incommensurability is discussed by Jones [24] who gives hybrid algorithm which applies no normalisation, Euclidean normalisation, or summation normalisation dependent on the level of incommensurability found.

2.3. The selection of preferential weights

Weights within a GP context are introduced with for the following double purpose:

1. To normalise the goals in the model.
2. To indicate the DM's preferences with respect to each goal.

The first purpose of normalisation is detailed in the Section 2.2. Regarding the second purpose of DM preferences, there are several methods for specifying the corresponding weight values in GP, as detailed by Ringuest [43]. Two promising methods for use within a GP context are detailed below.

Firstly, Gass [16] explains how a worthwhile link can be established between the Analytical Hierarchy Process (AHP) [46] and GP. In fact, the weights derived from the pairwise comparison of AHP can be incorporated directly into a WGP model. Gass [17] also states that in some cases the normalising weight is simply part of the whole weight that is absorbed by the AHP weight determination.

Secondly, the setting of preferential weights can be approached through an interactive MCDM method. An example of this kind of approach is given by Lara and Romero [29] where preferential weights are elicited and incorporated into a GP model by resorting to the interactive MCDM method of Zionts and Wallenius [4].

2.4. Naive prioritization and redundancy checking

All algorithms for the solution of LGP models are characterised by the following idea:

If the LP problem corresponding to the ith priority level has no alternate solutions then goals placed in priorities lower than the ith one become redundant, i.e. they become "ornaments" for the LGP model

Amador and Romero [1] empirically show that the redundancy of goals is not only a theoretical possibility but a practical problem. In fact these authors look for redundant goals in a number of LGP applications reported in the literature. In all but one of the applications, at least one of the priority levels is redundant. Moreover, about 50% of the cases have two or more redundant priority levels. Finally, in terms of aggregated results, at least a quarter of the goals are redundant. These results clearly show the practical importance of the topic of redundancy in LGP models.

Some possible causes of redundant goals include:
1. An excessive number of priority levels, especially in comparison to the number of goals.
2. The fixing of targets equal to or close to the ideal values.
3. The inclusion of many two-sided goals in the achievement function (a two-sided goal is one in which both deviational variables are penalised).

Although Amador and Romero offer a simple procedure for the identification of redundant goals, the important area of redundancy analysis is still open. In fact, resolution of redundancy is a more difficult task than identification, requiring both re-inspection of the model and re-setting of target values. The development of a systematic means of carrying out such a task so as to eliminate redundancy provides an interesting area for future research, with possible connections to the field of interactive algorithms (as detailed in Section 6) and intelligent GP systems [24]. It is interesting to note that the intelligent GP system (GPSYS) is able to automatically detect redundant goals within an LGP model [24].

3. Connections between GP and other MCDM techniques

Within the MCDM profession it is a common practice to present different approaches in a disconnected way, giving the impression that each approach is completely autonomous. However, this is not the case. In fact, as will be shown in this section, there are important links between GP and other MCDM distance function methods. Firstly it is shown how the following general distance function model can be viewed as a single root for several MCDM approaches:

$$\min \quad \left[\sum_{i=1}^{Q} w_i^p \left|\frac{b_i - f_i(x)}{k_i}\right|^p\right]^{1/p} \tag{3}$$
$$\text{s.t.} \quad x \in C_s,$$

where p is the metric, w_i are Archimedian or non-preemptive weights and k_i are normalising constants (e.g. the difference between ideal and nadir values).

To obtain a WGP model from Eq. (3) it is only necessary to make $p = 1$ and to introduce a simple change of variables based on positive and negative deviations, as shown by Charnes and Cooper [10]. The change of variables required is $n_i = (1/2)[|b_i - f_i(x)| + (b_i - f_i(x))]$ and $p_i = (1/2)[|b_i - f_i(x)| - (b_i - f_i(x))]$. To generate the WGP model it is only necessary to add n_i and p_i and to subtract p_i from n_i [10]. If in the corresponding WGP model b_i is set equal to b_i^*, where b_i^* is an infeasibility high target or anchor value, then a CP model for the $p = 1$ metric is obtained, as detailed in [44].

If in Eq. (3) we now make $p = \infty$, the maximum deviation is minimised, which leads to the following algebraic model:

$$\min \quad D = \max_{1 \leqslant i \leqslant Q} \left[\frac{w_i}{k_i}[b_i - f_i(x)]\right] \tag{4}$$
$$\text{s.t.} \quad x \in C_s. \tag{5}$$

Since the above function is not smooth, its minimisation is usually performed by solving the equivalent problem [38]

$$\min \quad D$$
$$\text{s.t.} \quad \frac{w_i}{k_i}[b_i - f_i(x)] \leqslant D, \quad i = 1 \ldots Q, \tag{6}$$
$$x \in C_s.$$

By setting $b_i = b_i^*$ in Eq. (6), a CP model for the metric $p = \infty$ is obtained. Moreover, when the targets are fixed at their anchor values, the positive deviational variables become redundant because over-achieving these targets is not possible. (This statement is true in the case “more is better”, in the opposite case (i.e. “less is better”) then the negative deviational variables will take the value zero and the reasoning is symmetric.) Hence for the ith goal we have

$$n_i = b_i^* - f_i(x). \tag{7}$$

By substituting Eq. (7) into Eq. (6) the following MINMAX or Chebyshev GP formulation is obtained:

$$\begin{aligned} \min \quad & D \\ \text{s.t.} \quad & \frac{w_i n_i}{k_i} \leqslant D, \\ & f_i(x) + n_i - p_i = b_i, \quad i = 1 \ldots Q, \\ & x \in C_s. \end{aligned} \tag{8}$$

When b is less than its anchor value, models (4) and (6) can generate non-efficient solutions. One way to guarantee efficiency is suggested by Wierzbicki [62] within the context of what is known as the RPM. The idea of this method is to incorporate into model (4) a small regularization term, such as $\epsilon \sum_{i=1}^{K} (w_i/k_i) f_i(x)$, where ϵ is an arbitrarily small positive number, which forces the efficiency of the resulting solution. The corresponding smooth formulation, which forms the basis of the RPM, is:

$$\begin{aligned} \min \quad & \left\{ D - \epsilon \sum_{i=1}^{Q} \frac{w_i}{k_i} f_i(x) \right\} \\ \text{s.t.} \quad & \frac{w_i}{k_i} [b_i - f_i(x)] \leqslant D, \quad i = 1 \ldots Q, \\ & x \in C_s. \end{aligned} \tag{9}$$

Therefore, within an RPM context, if the reference points are fixed at their anchor values then the regularization term becomes redundant and the model is equivalent to a GP MINMAX or a CP formulation (for the $p = \infty$ metric). Interesting relationships between GP and RPM can be seen in work by Ogryczak [39].

So it can be said that the differences between CP, GP, and RPM are more philosophical than analytical. In fact the CP and RPM approaches have an underlying optimizing philosophy whilst GP has an underlying satisfying philosophy. Technically, the differences among these approaches are minimal. It can be said that CP is in one way less general than GP and RPM since it specifies a particular value for each attribute (the anchor value), although this seems a quite reasonable practice [43]. However, in another way CP is more general than GP and RPM since it considers all metrics. Each GP variant only considers a single metric and the RPM always uses the $p = \infty$ metric. It should also be pointed out that the possibility of minimising both deviational variables (i.e. under as well as over achievements) gives the GP formulations an important feature that CP or RPM cannot easily accommodate.

4. Goal programming and utility functions

4.1. A utility interpretation of GP

In this section the different GP variants will be interpreted in terms of utility. That is, the DM’s preference structure underlying each variant will be examined. Moreover, given the connections between GP and the other distance function methods (CP and RPM) established in Section 3, some ideas about the utility structure underlying these methods will be highlighted.

Starting with LGP: The non-compatibility between utility functions and lexicographic orderings is well known [11]. That is, an LGP model does not optimise the DM’s utility function. In order to assess the effect of this property on the pragmatic value of LGP it is necessary to realise that the reason for this non-compatibility lies in the non-continuity of preferences inherent to lexicographic orderings. In fact, an assumption of non-continuity of preferences implies the impossibility of ordering the DM’s preferences by a monotonic numerical representation or utility function [44].

Therefore, within an LGP context, the worthwhile matter of discussion is not whether to disqualify the lexicographic approach or not because of the commented incompatibility. Rather it is to investigate whether the reality of the problem situation is compatible with the assumption of the continuity of preferences. There may be many scenarios, chiefly within a natural resources management context, where the non-continuity of preferences seems plausible. For instance, let us assume a forestry planning problem where two attributes are considered: timber production and an index measuring the risk of biological collapse of the forest. It is obvious that in this context the acceptance of the continuity of preferences would be unrealistic. Indeed, the assumption of continuity would imply accepting that there is always an increment in the volume of timber produced which compensates an increase in the risk of biological collapse of the forest, no matter how great the value of this index. In this kind of situation a non-compensatory lexicographic model can accurately reflect the reality being modelled.

Concerning WGP: Dyer [13] demonstrates how the underlying utility function of this GP variant is additive and separable. Moreover, if in a WGP model the targets have been set at their anchor values, a linear and additive utility function is maximised. Hence, WGP can be viewed as a specification of an additive utility function, so the assumption of mutual preference independence must hold [43].

With regard to MINMAX GP, first consider the case where all targets are fixed at their anchor values. In this case the solution provided by the MINMAX GP model represents a balanced allocation among the achievements of the different goals [2,3]. Hence the solution generated by this type of GP formulation satisfies the following chain of equalities:

$$\frac{w_1}{k_1}[b_1^* - f_1(x)] = \cdots = \frac{w_i}{k_i}[b_i^* - f_i(x)] = \cdots = \frac{w_Q}{k_Q}[b_Q^* - f_Q(x)]. \tag{10}$$

The utility contours compatible with Eq. (10) are actually made up of straight lines that intersect at right angles at the line defined by the equation $(w_1/k_1)[b_1^* - f_1(x)] = (w_2/k_2)[b_2^* - f_2(x)]$. Algebraically, these contours are represented by the following utility function:

$$U = -\left\{ \max_{i \leqslant 1 \leqslant Q} \left[\frac{w_i}{k_i}(b_i^* - f_i(x)) \right] \right\}. \tag{11}$$

This kind of utility function is usually called a Rawlsian function because of the connections between it and the principles of justice introduced by Rawls [41]. The perfectly equilibrated character (Rawlsian) of a MINMAX GP is rigorously true only when the targets are fixed at their anchor values. In general for any vector of targets (with no anchors) the solution provided by a MINMAX GP model coincides with, or is the nearest possible solution to, a Rawlsian perfectly equilibrated solution. This kind of solution as well as the corresponding utility function shall be called quasi-Rawlsian.

Some authors (e.g. Steuer and Choo [53]) have proposed, with the name of augmented Chebyshev, the following utility function:

$$U = -\left\{ \max_{1 \leqslant i \leqslant Q} \left[\frac{w_i}{k_i}(b_i^* - f_i(x) \right] - \lambda \sum_{i=1}^{Q} \frac{w_i}{k_i} f_i(x) \right\}.$$

This function, which has been used to build interactive methods, is not a pure Rawlsian function. In fact, for $\lambda = \infty$, U becomes an additive and linear utility function. For $\lambda = 0$, U is a pure Rawlsian function. For small values of λ, U can be considered a quasi-Rawlsian function. It is also interesting to note the resemblance between the augmented Chebyshev function and the scalarising achievement function proposed by Wierzbicki as a basis for the reference point methodologies.

Given the links between the GP, CP, and RPM methods established in Section 3, the following intuitive utility interpretation can be given. It is straightforward to see the separable and additive characters of utility functions that underlie a CP model when the metric is not $p = \infty$, as well as their Rawlsian character when $p = \infty$. Moreover, when $p = 1$ the corresponding separable utility function is linear for each single attribute alone. With regard to RPM, if the reference point is fixed at the anchor values the underlying utility function is Rawlsian, but for any other reference point the utility function is quasi-Rawlsian. Table 1 summarises the kind of utility structures that underlie the different multi-criteria distance function methods.

Table 1
Utility structures of different MCDM distance approaches

MCDM technique	Utility structure
WGP	Separable and additive
MINMAX GP	Rawlsian or quasi-Rawlsian
LGP	Non-existance of a utility function (i.e. non-compensatory structure)
CP ($\Pi = 1$)	Separable, additive and linear in each single attribute
CP ($2 \leqslant \Pi < \infty$)	Separable, additive and non-linear in each single attribute (quadratic for $\Pi = 2$, cubic for $\Pi = 3$...)
CP ($\Pi = \infty$)	Rawlsian

4.2. Incorporation of non-standard utility functions

The standard GP formulation allows for only a linear relationship between the unwanted deviation from the target value and the penalty contribution to the achievement function. The gradient of this relationship is given by the weight of the deviational variable. This corresponds to the case of a standard, linear utility function in the region on the penalised side of the target value. However in practice, utility functions may arise which are neither linear or continuous.

This observation led to a series of papers which expanded the flexibility of the basic GP model to cover a widening range of underlying utility functions. Charnes et al. [9] allow for a ranged target value rather than a single point. Kvanli [27] and Can and Houck [5] use piecewise linear increasing utility functions in financial planning and water planning, respectively. Romero [44] gives a description of the theory of this type of utility function (also known as a penalty function). Martel and Aouni [35] give a method for integrating the Promethee type utility functions into the GP model. Most recently, Tamiz and Jones [55,56] give a method of modelling any non-linear discontinuous, monotonically increasing utility function whilst remaining within the GP format. This method breaks the utility function into a series of four basic types of preference change.

- *Increase in preference:* an increase in the per-unit penalty at a point.
- *Decrease in preference:* a decrease in the per-unit penalty at a point.
- *Discontinuity:* a sudden increase in penalty at a point.
- *Non-linearity:* a section in which the underlying utility function is non-linear. This case is modelled as a series of increases and decreases in preference.

Further details of this method and an application to a manufacturing model are given in [55].

5. Feasibility of ranking MCDM techniques

The fast growth of MCDM has generated an impressive number of operational approaches. It can be said that nowadays the analyst is besieged by an enormous amount of seemingly sensible MCDM tools. This proliferation in MCDM tools has led some authors to establish something resembling a ranking of MCDM techniques according to their advantages and disadvantages. In these rankings GP appears in almost the last position! In this section, the practical value of such rankings is assessed.

Among others, Gershon and Duckstein [18,19], Ozernoy [40], and Tecle and Duckstein [61] give the idea that the choice of an MCDM technique is actually a multi-criteria problem in itself. Based on this idea, they seek to develop an algorithmic structure capable of ordering a set of MCDM techniques. For instance Tecle and Duckstein [61] develop an algorithm to rank 15 popular MCDM techniques. This algorithm is underpinned by a multi-programming technique known as composite programming (CTP) which is an extension of CP.

As a result of their research, they find CP, CTP, the method of the displaced ideal, and cooperative game theory to be the "best" techniques. On the contrary, the STEM method, GP, and the surrogate worth trade-off method occupy the bottom of the ranking. Although the authors check the robustness of the algorithm as regards the weights used, they do not check this aspect with regard to other crucial parameters. These parameters include the DM's competence; the complexity of the set of constraints; the number of criteria under consideration, etc.

In our view this procedure, as is common with all similar attempts, has an underlying problem of logical circularity. Indeed, one may wonder why the authors choose CTP to rank the different techniques. Is the ranking robust with respect to the technique chosen? This problem is reminiscent of the old philosophical problem of justifying induction by resorting to the inductive principle. In this case, Tecle and Duckstein [61] justify the superiority of CP and CTP by resorting to CTP (an extension of CP). It is obvious that this kind of result is not exempt of certain circular bias.

It is also important to consider how GP – an approach based on a Simonian philosophy of satisfying; STEM – based on an interactive optimisation philosophy with local preferences; or multi-attribute utility theory (MAUT) – based on a utility philosophy with absolute preferences, can be compared through a mechanistic algorithm? In short, each MCDM technique is based on a philosophy and there is not a single "correct" philosophy.

Even though the efforts mentioned above are well articulated, they assume a philosophy which is not easy to accept: i.e. "it is possible to build an algorithm to rank different MCDM techniques". It is difficult to accept that this interesting problem can be tackled in such a mechanistic way, without attaching decisive importance to the practical features of the decision problem under consideration.

A different attitude of philosophy towards the problem consists in accepting that the relative advantages and disadvantages among the MCDM approaches will depend largely on the characteristics of the problem situation. Within this philosophy GP again appears as a flexible and pragmatic MCDM methodology which is the most suitable for application to many decisional contexts. Indeed, for a decision model with many criteria, say, for example seven, and a complex constraint set (several hundred constraints and decision variables) it is only tractable by the formulation of a GP model.In fact, a problem of this size can have several thousand extreme points [52] and hence it is unrealistic to even try to obtain a good approximation of the efficient set through MOP techniques.

6. Other recent advances in Goal Programming

The advancement of the theory and practice of GP is maintaining a steady rate. This section briefly reviews the many other advances in the field of GP not detailed in the above sections.

- *Interactive GP:* The area of interactive algorithms has provided a major aid in improving the flexibility of the GP model and allowing DMs to become involved in the solution process and thus find a set of target values and weights that produce the best solution according to their preferences. Popular GP interactive algorithms include the methods of Spronk [51], Nakayama [37], Hwang and Masud [36], and an adapted version of the Zionts and Wallenius MOP method [4] by Lara and Romero [29]. More recent GP interactive algorithms are given by Reeves and Hedin [42] and Jones [24]. In addition, Gardiner and Steuer [15] incorporate GP interactive algorithms into their unified MOP interactive algorithm.
- *Teaching of GP:* With many students of today becoming the OR practitioners of tomorrow, the accurate teaching of the concepts behind GP is an important field in terms of accurate modelling. Papers by Lee and Kim [31,32] and Shim and Chin [49] give modern approaches to, and analysis of, this issue. In addition, Ignizio and Cavalier [21] provide an excellent, modern, textbook for tutorial purposes.
- *GP for infeasibility analysis:* An interesting recent new area in which GP has been found to be of use is that of the analysis and resolution of infeasibility in linear programmes. The initial comparison between the GP formulation and the initial stage of the LP simplex method (finding a feasible solution) is first remarked upon by Charnes and Cooper [7]. This concept is more

fully developed and applied to modern day infeasibility analysis methods by Tamiz et al. [59].

- *GP and its interface with artificial intelligence:* The use of GP for the analysis of, and use in, issues regarding the field of artificial intelligence, provides another field of development for GP. Ignizio and Cavalier [21] explain the rudiments of this new hybrid field. Relevant work on this issue is given by Love and Lam [34].
- *GP in combination with other management science techniques:* A successful refinement to the use of GP is its combination with other management science techniques in order to produce more accurate GP models. Of particular importance in this area is the work by Gass [16,17] regarding the use of the AHP to produce accurate weights (see Section 2.3). The work of Khorramshahgol [25,26] incorporates GP into a decision support system by combining it with the Delphi method for the purposes of preferential weight estimation.
- *Stochastic GP:* The case where the parameters of the GP model (goal values, technical coefficients, achievement function weights) are not known with certainty is termed stochastic GP. Liu [33] presents a method for solving stochastic GP based on genetic algorithms. The area of stochastic GP is closely related to fuzzy set theory, hence the term fuzzy GP is used.
- *Non-linear GP:* The majority of GP applications are directed towards the linear case, that is GP's with a linear achievement function, goals, and constraints. However GP is not limited to the linear case. There are some applications that utilise non-linear GP. This is particularly true in the application area of Engineering. A special case of non-linear GP is fractional GP, where the achieved value of the objective consists of a linear function divided by another linear function [44].

7. Conclusion and future research trends

Several implications for the modelling and analysis of future GP models and for direction for new GP research can be deduced from the findings of this paper. These are enumerated below:

1. GP is a pragmatic and flexible methodology especially capable of addressing complex decision variable problems where several objectives as well as many variables and constraints are involved.
2. The choice of GP variant has generally been conducted in a mechanistic way. However, the right variant should be chosen so as to be coherent with the DM's structure of preferences. Extra flexibility in this area can be given through the use of non-standard preference curve modelling techniques.
3. Careful analysis should be conducted both before and after solving the problem to ensure that modelling pitfalls are avoided. Modern approaches to techniques such as normalisation and pareto efficiency detection and restoration can be used for this purpose.
4. The reliance on a single GP variant is not, in general, justified. In fact, in most real-life cases the best modelling practice should include several variants.
5. There is an excessive use of LGP in the literature. LGP models are only valid for decision models with discontinuous preferences. If used, LGP models should not include an excessive number of priority levels because of problems with redundancy.

It is hoped that the analysis in this paper gives a clear overview of the area of GP as a decision support tool. It documents the recent advances in this area, and should lead DMs into a clearer understanding of the pitfalls to be avoided, and benefits gained from, the use of GP to provide solutions to the real-world multi-objective problems they are facing.

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