Topics in Differential Geometry – Kähler-Einstein metrics

Instructor: Gábor Székelyhidi

The study of Kähler-Einstein metrics lies at the interface between algebraic and complex geometry, and geometric analysis. The existence theory began with Yau's work in the 70s, and during the following decades many new ideas have been introduced, culminating in Chen-Donaldson-Sun's solution of the Yau-Tian-Donaldson conjecture. The goal of this course is to give an overview of some of these developments. A tentative outline of the topics that I intend to cover is as follows, although this may change as the course progresses.

- **Yau's theorem:** After reviewing the basics of Kähler geometry and some results about analysis on manifolds, I will explain the proof of Yau's theorem on the existence of Kähler-Einstein metrics on Kähler manifolds with zero and negative first Chern class.
- **Convergence of Riemannian manifolds:** I will discuss how we can define the limit of a sequence of Riemannian manifolds. When we have good control of the local geometry, we can get smooth limits, while under very weak assumptions we can define Gromov-Hausdorff limits that are just metric spaces.
- **Cheeger-Colding theory:** I will give an overview of Cheeger-Colding's theory of non-collapsed limit spaces of Riemannian manifolds under Ricci curvature bounds.
- **Positive Kähler-Einstein metrics:** I will discuss the work of Chen-Donaldson-Sun on the existence of Kähler-Einstein metrics on manifolds with positive first Chern class. A major step is Donaldson-Sun's theorem that shows that the Gromov-Hausdorff limit of Kähler-Einstein manifolds has the structure of a normal complex variety.

Prerequisites: I will try to cover as much of the background material as possible, but a basic familiarity with differential geometry, PDEs, and a few ideas of algebraic geometry would be very useful.

Grading policy: There will be roughly four or five homework assignments. The grade will be based on effort put into the assignments.