Rational Homotopy Part II
Model Category and DGA Exercises

1. For a simplicial set X and a commutative ring k, check that the normalized cochain algebra $C^*(X;k)$ defined in 7.1 is in fact a differential graded algebra.

2. Give an example of a simplicial set X (or associated topological space) and a commutative ring k for which $C^*(X;k)$ is not commutative as a differential graded algebra. Give an example of a simplicial set X (or associated topological space) for which $C^*(X;\mathbb{Z})$ is not commutative.

3. (Goerss and Schemmerhorn) Prove that the category of nonnegatively graded cochain complexes of R-modules, Ch_*R, can be given a model category structure with weak equivalences being maps that induce isomorphisms on cohomology, fibrations being surjections in positive degree, and cofibrations being maps that in nonnegative degree are injections with a surjective cokernel.

4. (Goerss and Schemmerhorn) Let $f : R \to S$ be a homomorphism of commutative rings, and let res_f be the restriction of scalars map from S-modules to R-modules. Show that $S \otimes_R - : Ch_*R \rightleftarrows Ch_*S : res_f$ is a Quillen equivalence iff $R = S$.

5. (Campbell) If $f : \mathcal{A}^* \to \mathcal{B}^*$ is a map of differential graded algebras, show that there is a long exact sequence in cohomology

$$... \to H^n(\mathcal{A}^*) \to H^n(\mathcal{B}^*) \to H^{n+1}(\mathcal{A}^*, \mathcal{B}^*) \to H^{n+1}(\mathcal{A}^*) \to ...$$