Exercises

Jens Jakob Kjaer

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1

Given a functor $F : \mathcal{C} \to \mathcal{D}$ we can define $\mathrm{cr}_n(F) : \mathcal{C}^n \to \mathcal{D}$ by $\mathrm{cr}_n(F)(X_1, \ldots, X_n) := \mathrm{thofib}_{S \subset \{1,\ldots,n\}}(\bigvee_{i \notin S} X_i)$, where thofib is the total homotopy fiber (iteratively applying homotopy fiber of the diagram).

1.1

Write out the diagram defining $\mathrm{cr}_2(F)$

1.2

Use the fact that $D_n(F)(X) \simeq \mathrm{hocolim}_{l_1,\ldots,l_n} \Omega^{l_1+\ldots l_n} \mathrm{cr}_n(F)(\Sigma^{l_1} X, \ldots, \Sigma^{l_n} X)_{h\Sigma_n}$, to calculate $\partial_*(id_{DGL})$ for $* = 1, 2$.

1.3

For $F : DGL \to DGL$ show that $\partial_n(F \circ \Sigma) = \partial_n(F) \wedge S^n$. Work out the Σ_n action.

2

2.1

Given a functor $F : \mathcal{C} \to \mathcal{D}$ we can define

$$T_n(F)(X) = \mathrm{holim}_{\emptyset \neq S \subset \{1,\ldots,n+1\}} F(X \star S)$$

where $X \star Y = X \times I \times Y/(x, 0, y) \sim (x, 0, y'),\ (x, 1, y) \sim (x', 1, y)$, note $X \star \{1\} = CX$ and $X \star \{1, 2\} = \Sigma X$. Use the fact that $P_n(F)(X) = \mathrm{holim}(T_n(F)(X) \to T_n(T_n(F))(X) \to \ldots)$ to compute $P_1(id_{DGL})$.

2.2

Show that the identity on $Ch_{\mathbb{Q}}$ is 1-homogeneous.

3

3.1

Use the fact that $\partial_2(id_{Top_*}) \simeq S^{-1}$ with trivial Σ_2 action, and the operadic structure on $\partial_*(id_{Top_*})$, to define a product:

$$[\ ,\ ] : \pi_k D_n(X) \otimes \pi_l D_m(X) \to \pi_{k+l-1} D_{n+m}(X)$$

Show that $[x, y] = (-1)^{|x| \cdot |y|}[y, x]$.

3.2

Show that $\{Lie(n)\}_{n \in \mathbb{N}_0}$ as defined in the lectures forms an operad in chain complexes, and that algebras over it are exactly DGL.