Exercises:

1) In $\mathcal{C}at_\infty$, construct an explicit lift of

$$\begin{tikzcd} \Lambda^2_2 \arrow[r, "F"] \arrow[d] & \mathcal{M}fld_1^{fr} \\ \Delta^2 \arrow[ur] \end{tikzcd}$$

Where:

$$\Lambda^2_2 = \operatorname{colim}\left(\Delta^1 \xleftarrow{t} * \xrightarrow{t} \Delta^1 \right)$$

$$\wr$$

$$\begin{tikzcd} \Lambda^2_2 \arrow[r, "\check{F}"'] \arrow[rr, bend left, "F"] & \mathcal{M}fld_1^{fr} \arrow[r] & \mathcal{M}fld_1^{fr} \end{tikzcd}$$

with $\check{F} =$
$$\begin{tikzcd} (-1,1) \arrow[r, "\mathrm{Id}"] \arrow[d, "\mathrm{Id}_{(-1,1)} \sqcup \varnothing"] & (-1,1) \\ (-1,1) \sqcup (-1,1) \end{tikzcd}$$

Can one do this in $\mathcal{M}fld_1^{fr}$?
Maybe via zig-zags?

2) If one takes the definition of $\mathcal{D}isk_1^{fr}$, & deletes every reference to framings, one obtains

$$\mathcal{D}isk_1 \in Cat_\infty^\otimes$$

From the data of a symm. monoidal functor:

$$\mathcal{D}isk_1^\sqcup \xrightarrow{A} Vect_K^\otimes$$

Construct an algebra with an anti-involution.

3) Use functoriality to construct:

$$A \longrightarrow \int_{S^1} A \simeq HH_*(A)$$

Construct a factorization of this map:

$$A \longrightarrow A/_{[A,A]} \longrightarrow HH_*(A)$$

where $A/_{[A,A]} := \mathrm{colim}\left(A^{\otimes 2} \underset{\mu\circ\sigma}{\overset{\mu}{\rightrightarrows}} A \right)$