

J. Mounn talk 2 Fact Alg

Def: Fix $n \in \mathbb{N}$, Mfld_n is a n -obj n -mfd's M . Mon: $M \xrightarrow{f} N$
 & open embedding. Sym mon structure coming from disjoint union

Note: $M \sqcup M \xrightarrow{\nabla} M$ is not in Mfld_n .

Def: $\text{Disk}_n \subset \text{Mfld}_n$ full subcat on obj's: $\bigsqcup_I \mathbb{R}^n$, $|I| < \infty$

Disk_n/M cat of embedded disks in M .

Remark: $\text{Disk}_n/M \cong$ Open submfd of M differ to unions of \mathbb{R}^n 's.

Note: $M \xrightarrow{f} N$ embedding $\rightsquigarrow \text{Disk}_n/M \rightarrow \text{Disk}_n/N$.

Def: For $M \in \text{Mfld}_n$ $\mathcal{J}_M \subset \text{Disk}_n/M$ w morph $\bigsqcup_I \mathbb{R}^n \xrightarrow{f} \bigsqcup_J \mathbb{R}^n$, f integer eqn

i.e., $\pi_0 f: I \rightarrow J$ is a bijection and all objects.

Note: \mathcal{J}_M and Disk_n/M have natural filtrations, coming from $\#$ of disks.

universal property, kinda gadget

very concrete objects

Wonderful Thm [L, AF, ...]: $|\mathcal{J}_M| = \bigsqcup_{k \geq 0} \text{Conf}_k(M) / \Sigma_k$

This should be seen by the fact and degeneracies of NSM should allow you to wiggle embeddings...

Talk 2

The set of Embeddings have a natural topology, so Disk_n and Mfld_n are topologically enriched.

So denote $\underline{\text{Disk}}_n$ and $\underline{\text{Mfld}}_n$ to be their ass ∞ -cats. (Top-enriched, quasi-c ∞ , complete seg ∞ , dependent on your needs).

For each $M \in \text{Mfld}_n$: $\text{Disk}_{n/M} \rightarrow \underline{\text{Disk}}_{n/M}$ (This trick, in general, allows us to move old school constructions to the Jammy world)

Under this functor, maps of \mathcal{J}_M goes to equivalences.

Thm: The natural functor $\text{Disk}_{n/M}[\mathcal{J}_M^{-1}] \rightarrow \underline{\text{Disk}}_{n/M}$ is an equiv
 \uparrow simplicial localization, aka, Dwyer, Rezk.

In particular it is final.

\uparrow ex of final functors $\Delta \xrightarrow{\sim} \Delta \times \Delta$ and $\Delta^{\text{inj}} \hookrightarrow \Delta$
 which tells us that realization = fat realization.

So again, this is going to allow us to use old categorical tricks in the Jammy setting.

Remark: $(\underline{\text{Disk}}_{n/M})^{\text{core}} \xrightarrow{\sim} \coprod_n \text{Cont}_k(M) / \mathbb{Z}_k$
 \uparrow Maximal subgroup...

Recall: Data of a com alg in \mathcal{C}^{\otimes} can be determined as a \otimes -functor
 $\text{FinSets}_* \xrightarrow{\mathbb{A}} \mathcal{C}^{\otimes}$

Talk 2

$A(x) = K_{\mathbb{C}} \otimes \text{unit of } \mathcal{C}$, $A(\ast \rightarrow \ast) = K_{\mathbb{C}} \rightarrow A(\ast) =: A$ unit map

$A(\begin{smallmatrix} \circ \\ \downarrow \\ \circ \end{smallmatrix}) = A \otimes A \rightarrow A$ multiplication, and $A(\begin{smallmatrix} \circ & \circ \\ \downarrow & \downarrow \\ \circ & \circ \end{smallmatrix})$ encodes commutativity

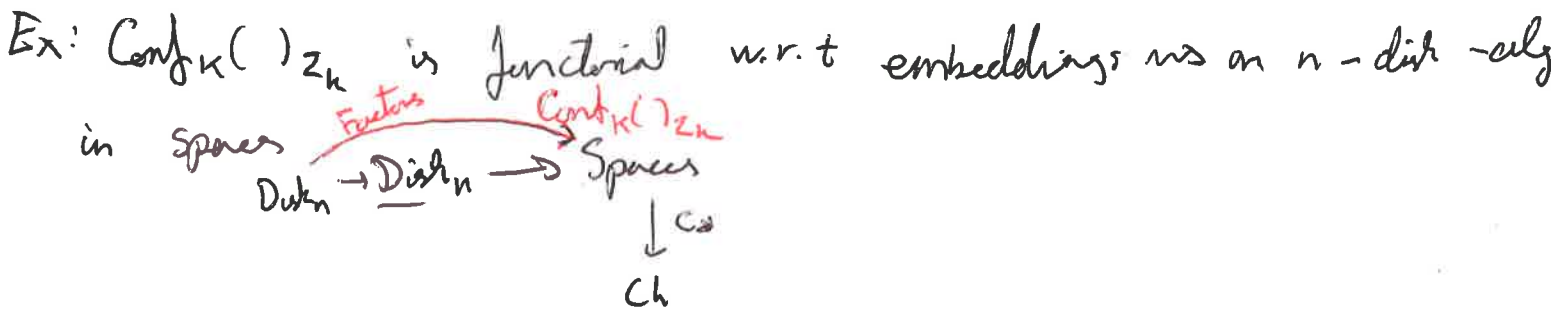
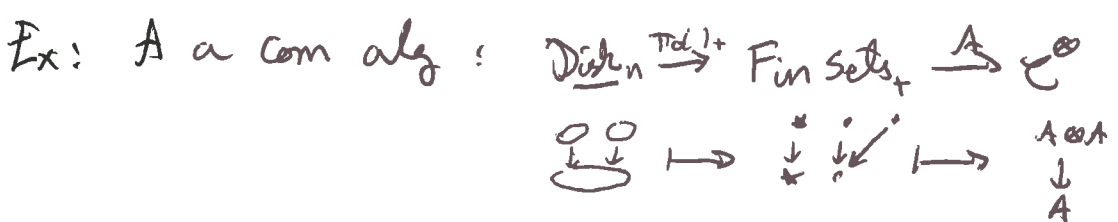
Def: An n -disk alg in $\underline{\mathcal{C}}, A$, is a \otimes -functor $\underline{\text{Disk}}_n \xrightarrow{A} \underline{\mathcal{C}}$.

$A(\emptyset) = K_{\mathbb{C}}$, $A := A(\mathbb{R}^n)$, $A(\emptyset \rightarrow \mathbb{R}^n)$ gives a unit, and

$\text{Emb}(\mathbb{R}^n, \mathbb{R}^n) \rightarrow \mathcal{C}(A, A)$ so by adjunction $\mathcal{O}(n) \otimes A \rightarrow A$ so $\mathcal{O}(n)$ A has a $\mathcal{O}(n)$ -action.

So we add geom-decoration to Emb we get closer to little disks. We kill the $\mathcal{O}(n)$ -action.

The framed E_n -alg is not the same as the framed little disks -alg!



Talk 2

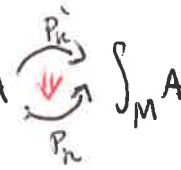
"Spinning" two disks inside a larger disk gives

$$S^{n-1} \rightarrow \text{Emb} \left(\frac{1}{2} \mathbb{R}^n, \mathbb{R}^n \right) \xrightarrow{A} \mathcal{C}(A^{\otimes 2}, A)$$

So assuming $C = Ch$ we get $A^{\otimes 2}[n-1] \rightarrow A$ which makes A into a shifted Lie-alg (Recall Tanyas talk)

Def: A an n -disk-alg, M -Mod. $\int_M^A = \text{Colim}(\underline{\text{Disk}}_n/M \rightarrow \underline{\text{Disk}}_n \xrightarrow{A} \underline{\mathbb{C}})$

Note: $p_R \in \text{Conf}_K(M)/\mathbb{Z}_K \rightsquigarrow \star \xrightarrow{p_R} \underline{\text{Disk}}_n/M \rightsquigarrow A \rightarrow \int_M^A$
↑ use the splitting of the core of $\underline{\text{Disk}}_n/M$ to get this map.

given p_n ^{both} $p'_n \rightsquigarrow A$  \int_M^A