Operads and Loop Space Machinery Seminar

Week 2: Exercises

J.D. Quigley

(1) Show that the commutative, associative, and endomorphism operads as defined in Example 2.4 and Definition 3.1 of the lecture are operads

(2) Show that the little intervals operad from Example 4.3 (3) is an A_∞ operad

(3) Let G be a finite group (this can be done more generally, but we only need finite here). The *universal space* EG of G is characterized by the property

$$(EG)^H \simeq \begin{cases} pt & H = \{e\}, \\ \emptyset & else. \end{cases}$$

Show that the Barratt-Eccles operad from Example 4.4 (2) with $\mathcal{O}_n = E\Sigma_n$ is an E_∞ operad. What are the structure maps?

(4) Let $\mathcal{O}_n$ be the little n-cubes operad from Example 4.4 (3).
  (a) Show that this is an operad.
  (b) For a based space X, define maps $\theta_{n,j} : \mathcal{O}_n(j) \times (\Omega^n X)^j \to \Omega^n X$ as follows. Let $c = \langle c_1, \ldots, c_j \rangle \in \mathcal{O}_n(j)$ and let $\gamma = (\gamma_1, \ldots, \gamma_j) \in (\Omega^n X)^j$. Define $\theta_{n,j}(c, \gamma)$ to be $\gamma_r c_r^{-1}$ on $c_r(J^n)$ and to be trivial on the complement of the image of c. Show that $\theta_{n,j}$ defines an action θ_n of $\mathcal{O}_n$ on $\Omega^n X$, i.e. an operation $\mathcal{O}_n \to \mathcal{E}_X$ where $\mathcal{E}_X$ is the endomorphism operad of X. [Hint: This is Theorem 5.1 of May's "Geometry of infinite loop spaces"]