Operads and Loop Space Machinery Seminar

Week 4: Exercises

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(1) Let $\mathcal{O}$ be an operad and let C be its associated monad. Let X be a simplicial space. Show that there is a natural homeomorphism

$$\nu : |C_\bullet X| \to C|X|.$$

(2) Show that the map $\tilde{\gamma}$ from the proof of Theorem 4.2 is well-defined and continuous.

(3) Describe the monadic bar construction $B_\bullet(\Sigma, C_1, S^1)$ where C_1 is the monad associated to some E_1-operad. Is it possible to write a homotopy equivalence between this and $\mathbb{C}P^\infty$?

(4) Let M be a topological monoid, and let BM be its bar construction (as a monoid). Show that the map

$$M \to \Omega BM$$

is group completion.

(5) Let E be a multiplicative cohomology theory and let X be an infinite loop space. Using the E_∞-algebra structure of X, define power operations on $E_*(X)$, i.e. natural transformations $E_*(X) \to E_{*+k}(X)$ for all $k \geq 0$.