Problems

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1. Show that $(S^i)^{\wedge 2}_{h\Sigma_2} \simeq \Sigma^i(\mathbb{R}P^\infty/\mathbb{R}P^{i-1})$.

2. Let X be an E_∞-algebra, and let $e_i \in H_i(\mathbb{R}P^\infty)$ be a generator. Show that the diagonal map $B\Sigma_{2+}\wedge X \to E\Sigma_{2+}\wedge_{\Sigma_2} X^{\wedge 2}$, composed with the structure map for X takes $e_i \otimes x \in H_*(\mathbb{R}P^\infty)\otimes H_*(X) \simeq H_*B\Sigma_{2+}\wedge X^{\wedge 2}$ to $Q^i(x) \in H_*(X)$.

3. Calculate $H_*((S^i)^{\wedge k}_{h\Sigma_k};\mathbb{Q})$, conclude that the only rational Dyer-Lashof operations are $x \mapsto x^k$ for all k.

4. Calculate $H_*((S^i)^{\wedge p}_{h\Sigma_p};\mathbb{F}_p)$ for odd primes p, conclude that we have odd primary Dyer-Lashof operations $\beta^\epsilon Q^i : H_*(X) \mapsto H_{*+2(p-1)i-\epsilon}(X)$ for $\epsilon = 0, 1$, and X an E_∞ space.

5. Show that $(X^{\wedge|G|}_{hG})^{\wedge|G'|}_{hG'} \simeq X^{\wedge|G|\cdot|G'|}_{hG'\wr G}$ for any finite groups G, G' with G acting on $X^{|G|}$ by permuting the copies.

6. (a) Use the fact that there is a model for E_{n+1}-algebra, $\mathcal{E}_{n+1}$, with $\mathcal{E}_{n+1}(2) = E\Sigma_2^{(n)}$, where $()^{(n)}$ is the n'th skeleta, to define the operation

   $$\lambda_n : H_i(X;\mathbb{F}_2)\otimes H_j(X;\mathbb{F}_2) \to H_{i+j+n}(X;\mathbb{F}_2)$$

   for X an E_{n+1}-algebra, such that $\lambda_0(x,y) = xy - yx$, and $\lambda_n = 0$ if X is an E_{n+2}-algebra.

   (b) Show that X an E_{n+1}-algebra, show that we have Dyer-Lashof operations $Q^i : H_s(X;\mathbb{F}_2) \to H_{s+i}(X;\mathbb{F}_2)$ for $i - s < n$.

7. Use the Snaith Splitting $\Sigma^\infty\Omega^\infty\Sigma^\infty(X) \simeq \Sigma^\infty \bigvee_i X^{\wedge i}_{h\Sigma_i}$ to give a complete calculation of $H_*(\Omega^\infty\Sigma^\infty(X);\mathbb{F}_2)$ for X a finite CW-complex.

8. Show that $(E \wedge X)^{\wedge_E k}_{h\Sigma_k} \simeq E \wedge X^{\wedge k}_{h\Sigma_k}$ for any structured, commutative, ring spectrum E and any spectrum X. Use this to conclude that given $\alpha \in E_i(X)$ we get a map $\mathcal{P}(\alpha) : E_*(S^{ki}_{h\Sigma_k}) \to E_*(X^{\wedge k}_{h\Sigma_k})$.