Exercises

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Exercise 1. *H^d_∞ ring spectra and graded ring spectra.*

1. *Define a graded object in a symmetric monoidal category $\mathcal{C}$ enriched in spaces (the examples we have in mind are $\mathcal{C}$ = chain complexes or spectra), and the tensor product of graded objects to define a symmetric monoidal category of graded objects $\mathcal{C}^{\mathbb{Z}}$.*

2. *Write down the structure maps and equations between them for an E_∞ / H_∞ object in $\mathcal{C}^{\mathbb{Z}}$. Simplify your answer.*

3. *Let $\mathcal{R} = (R_i)_{i\in\mathbb{Z}}$ be an H^d_∞ object in $\mathcal{C}^{\mathbb{Z}}$. Assume that*

   (a) *There are equivalences $R_i \simeq \sigma^{di} R$ (where $R = R_0$) for each $i \in \mathbb{Z}$.*

   (b) *If $\mu_{i,j} : R_i \otimes R_j \to R_{i+j}$ is the multiplication, then $\Sigma^{i+j}\mu_{0,0} = \mu_{i,j}$ for all i, j.*

   Show that the R is an H^d_∞ object in $\mathcal{C}$.

4. *Show that if we use the commutative operad instead of the E_∞ operad throughout, then these conditions are satisified if and only if R is a commutative monoid and $\mathcal{R} = R[t^\pm]$ is the Laurent polynomial ring over R on a generator in degree d.*

5. *Conclude that if R is a commutative ring, then HR is an H^2_∞ ring spectrum. What facts do we need that we haven't yet shown?*