Problems

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1.
   - Show that BO is a $\mathcal{L}$-space.
   - Show that MO is an E_∞-ring spectrum.
   - Show that if X is a $\mathcal{L}$-space, and $\xi : X \to BO$ is a map of $\mathcal{L}$-spaces, then the Thom spectrum of ξ is an E_∞-ring spectrum.

2. Show that $\mathbb{S}^X = F(X, \mathbb{S})$, for any pointed space X, is an E_∞-ring spectrum.

3. Given an operad pair $(\mathcal{G}, \mathcal{C})$, and a $\mathcal{G}$-space X, show that $\mathbf{C}X$ is a $\mathcal{G}$-space.

4. Let X be E_∞-ring space, construct two different products, $\star$ and $\sharp$, on $H_*(X)$, and show that for $x, y, z \in H_*(X)$ with $\Delta(z) = \sum z' \otimes z''$ we have $(x \star y)\sharp z = \sum(x\sharp z') \star (y\sharp z'')$.

5. Draw a nice picture of the composition rule for the Steiner operad.