GSTS Problem Session

Week 3

1. Recall that $\mathrm{PSL}(2,\mathbb{Z})$ has presentation $\langle a, b \mid a^2, b^3 \rangle$. Show that this presentation is a Dehn presentation.

2. The goal of this problem is to compute the smallest δ for which $\mathbb{H}^2$ is δ-hyperbolic.

   (a) Show that it is enough to consider "ideal" geodesic triangles (geodesic triangles with all three vertices on the boundary).

   (b) Show that it is enough to consider only one ideal triangle. (Hint: how does the isometry group of $\mathbb{H}^2$ act on the boundary?)

   (c) Choose your favorite ideal triangle, and find the smallest δ for which this triangle is δ-thin. The hyperbolic distance formula

   $$\rho(z,w) = 2\tanh^{-1}\left(\frac{|z-w|}{|z-\overline{w}|}\right)$$

   may be helpful.