GSTS Problem Solving Session

Week 5

1. Use the Cayley graph of the Klein bottle group $K =< x, y \,|\, xy = y^{-1}x >$ to show that $e(K) = 1$. Begin by describing a path between elements $g, h \in \Gamma(K)\backslash C$ for some finite subgraph $C \subseteq \Gamma(K)$ and use that $\Gamma(K)$ contains multiple copies of the Cayley graph of $\mathbb{Z}$.

2. Let $\mathcal{T}$ represent the standard Cayley graph of the free group on two generators and $\partial\mathcal{T}$ denote the ends of $\mathcal{T}$. Show that given an end $e \in \partial\mathcal{T}$, and a vertex $v \in \mathcal{T}$, there is a unique ray r in $\mathcal{T}$ that represents e and is based at v.

3. Prove $\partial\mathcal{T}$ is a totally disconnected, perfect compact metric space. This implies that $\partial\mathcal{T}$ is homeomrophic to the Cantor set.

4. Give a definition for the ends of a general group G.

5. Choose your favorite infinite group G. Discuss the ends of this group and what properties they may have. State and attempt to prove a conjecture about the ends of G.

Some exercises from Office Hours with a Geometric Group Theorist