Exercises

1) Show iii $\Rightarrow$ iv, where

iii) $H^{n+1}(G, -) = 0$

iv) If $0 \to K \to P_{n-1} \to \cdots \to P_0 \to \mathbb{Z}$ is any exact sequence of $\mathbb{Z}G$ modules with each P_i projective, then K is projective

2) Prove that $\operatorname{cd} G = 0 \Rightarrow G$ trivial

3) Let G be of type FP_n, and let M be a G-module which is finitely generated as an abelian group. Show for $i \leq n$ that $H_i(G, M)$ and $H^i(G, M)$ are finitely generated abelian groups

4) [Note: $H^*(G, \mathbb{Z}G)$ admits a right G-action]

If G is a group of type FP_∞, prove that

$$H^*(G, F) \cong H^*(G, \mathbb{Z}G) \otimes_{\mathbb{Z}G} F$$

for any flat $\mathbb{Z}G$-module F

Exercises taken from Brown's "Cohomology of groups"